

Two Applications of Signal Processing to Physics Problems

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Outline

- 1 My background
- 2 Nonstationarity
- 3 Correction of Doppler effect
- 4 Estimation of impact times

Background

Since September 2021 :

École Normale Supérieure de Lyon

Maître de conférences,

LP and Department of Physics.



2020–2021 : Duke University

Postdoctoral Associate,

Department of Mathematics.



2016–2019 : Aix-Marseille Université

PhD student,

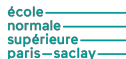
Institut de Mathématiques de Marseille.



2012–2016 :

École Normale Supérieure de Cachan

Student, Electrical Engineering Department.



- **2015–2016**

Master, Image and Signal Processing

- **2014–2015**

Master, Applied Physics (teaching)

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Stationarity: a mathematical concept

Definition (Stationarity)

A random process X is said to be second-order stationary if:

- $\mathbb{E}\{X(t)\} = m, \forall t,$
- $\mathbb{E}\{X(t)X(\tau)\} = k(t - \tau), \forall(t, \tau).$

Spectrum:

- Gives the distribution over frequencies of the power of X .
- Many methods to estimate the spectrum from a single realization of the stationary process X (e.g. Welch method).

Nonstationarity

The stationarity assumption is often irrelevant to study real-life signals, such as audio signals, or physiological signals.

⇒ **Questions:**

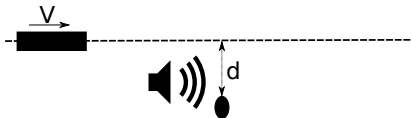
- 1 Which classes of nonstationarity should we consider?
- 2 How should we analyze nonstationarity?

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Doppler effect

Assumptions:

- A source emits a stationary sound $x(t)$.
- The source follows a uniform linear motion, at speed V .
- From a fixed station, we record the sound $y(t)$ emitted by the source.



⇒ Due to the Doppler effect, the sound we receive is time-warped:

$$y(t) = x(\gamma(t)) ,$$

with

$$\gamma'(t) = \frac{c^2}{c^2 - V^2} \left(1 - \frac{V^2 t}{\sqrt{d^2(c^2 - V^2) + (cVt)^2}} \right) .$$

Wavelet transform

Wavelet transform

$$\mathcal{W}_x(s, \tau) = \int_{\mathbb{R}} x(t) \psi_{s\tau}^*(t) dt \quad \text{with} \quad \psi_{s\tau}(t) = \frac{1}{\sqrt{s}} \psi\left(\frac{t - \tau}{s}\right),$$

where ψ is the analysis wavelet.

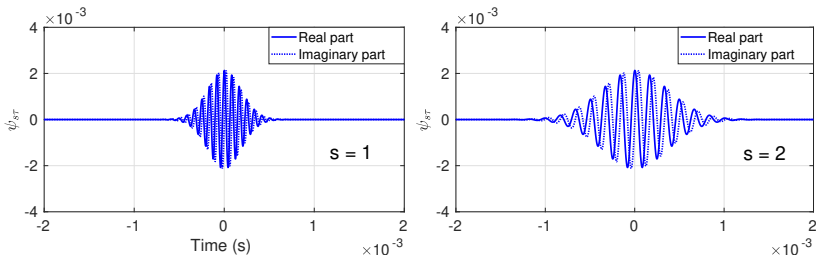
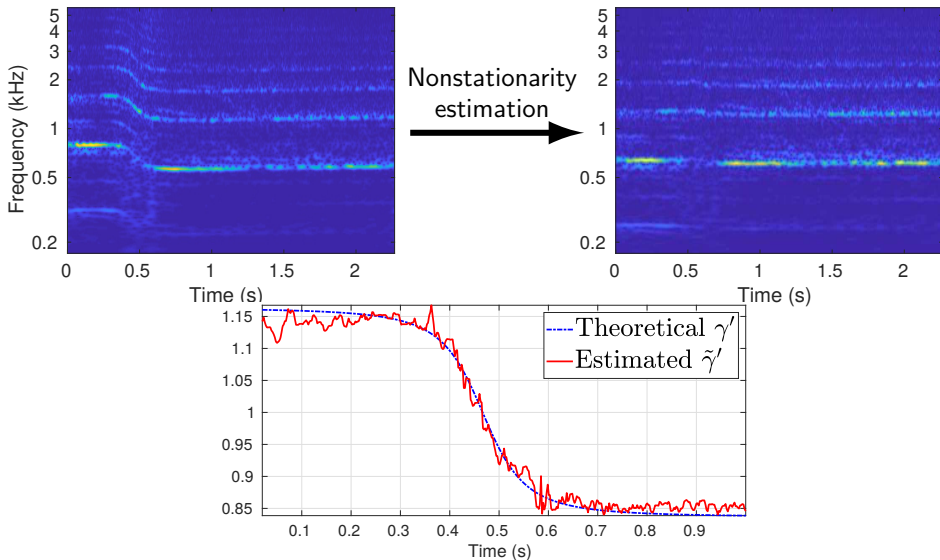


Figure: "Sharp wavelet" for two different values of s .

Doppler effect

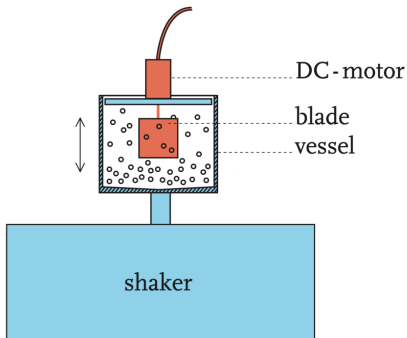
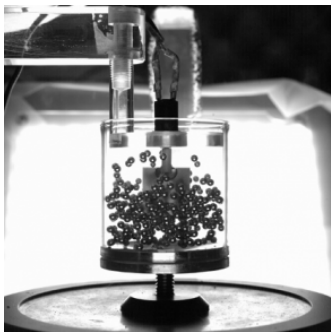


Comparison to the theoretical function with: $d = 5$ m and $V = 54$ m/s.

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 - Problem

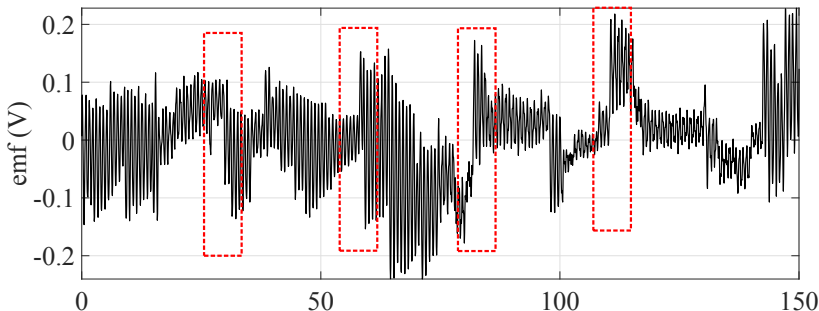
Experimental setting

M. Lagoin, C. Crauste-Thibierge, and A. Naert, Human-scale Brownian Ratchet: A Historical Thought Experiment, Phys. Rev. Lett. **129**, 120606 (2022).



Goal: Determine the instants when the beads hit the blade.

EMF Signal



Model

$$e(t) = \sum_p a_p s(t - t_p),$$

- a_p is the magnitude of the p th impact;
- t_p is the time of the p th impact;
- s is the impulse response of the system.

Time-frequency

Short time Fourier transform (STFT)

$$\mathcal{G}_x(\nu, \tau) = \int_{\mathbb{R}} x(t) g_{\nu\tau}^*(t) dt \quad \text{with} \quad g_{\nu\tau}(t) = g(t - \tau) e^{i2\pi\nu t},$$

where g is the analysis window.

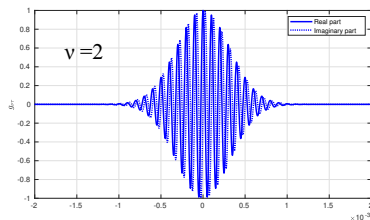
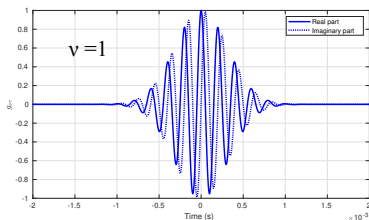
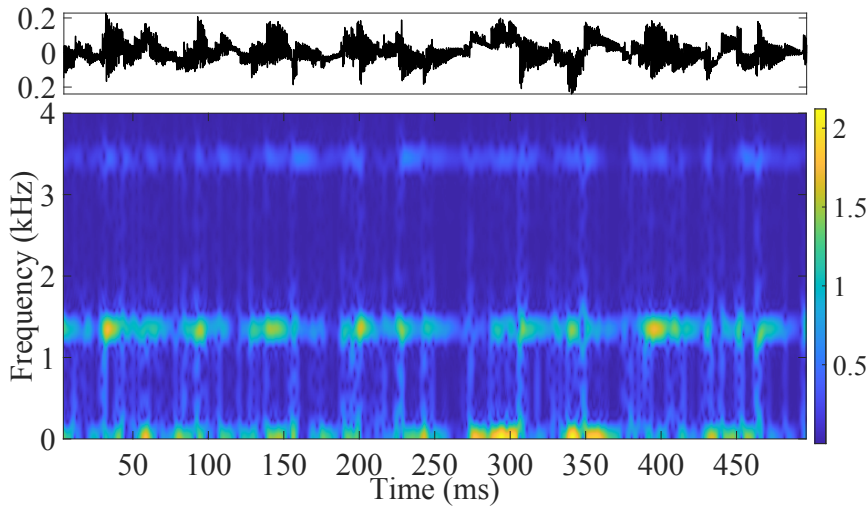


Figure: Modulated window for two different values of ν .

Time-frequency

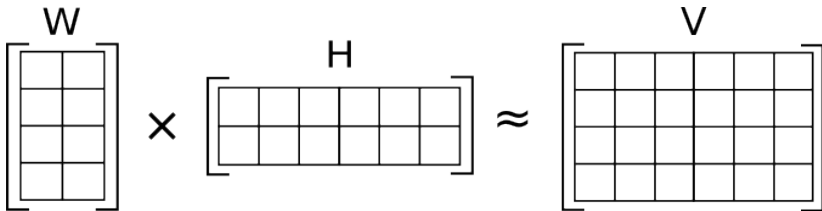


Nonnegative Matrix Factorization (NMF)

Idea: Low rank decomposition of the spectrogram $\mathbf{S}$ (= squared modulus of the STFT)

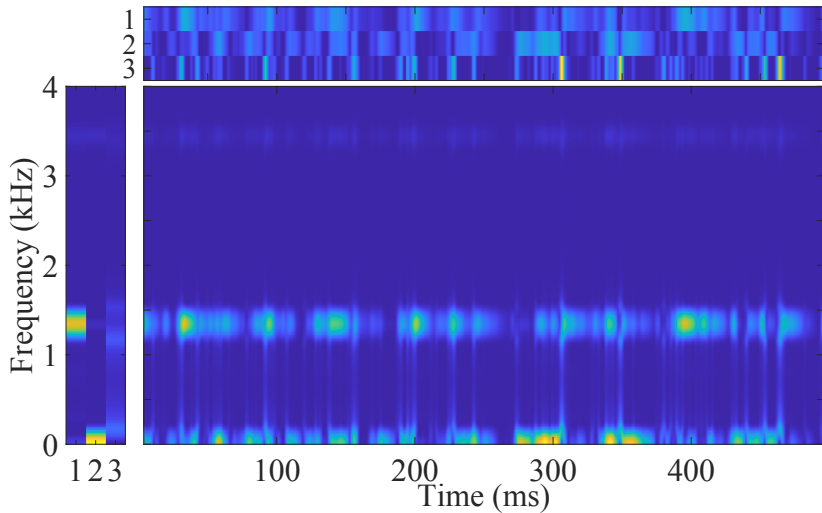
Method: Find $\mathbf{W}$ and $\mathbf{H}$ two matrices of nonnegative elements such that

$$\mathbf{S} \approx \mathbf{W}\mathbf{H}.$$

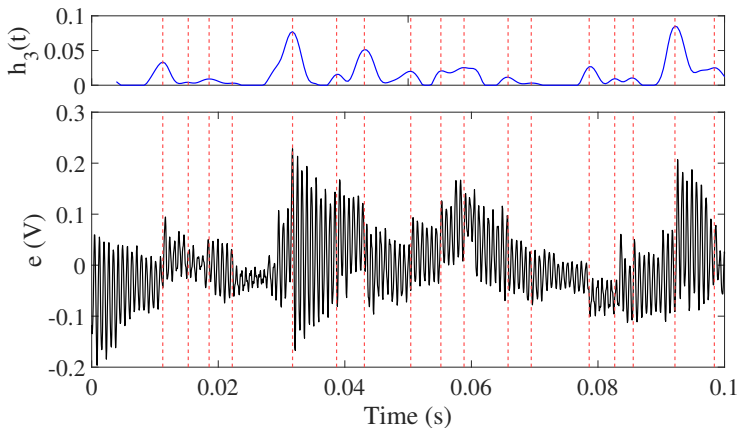


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NMF of the spectrogram



Third component of the NNMF



Questions?