



La non-stationnarité comme phénomène physique : étude de signaux audios et biomédicaux.

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Outline

1 Présentation de mon parcours

2 Introduction: la non-stationnarité

3 Locally deformed signals: an analysis-based approach

4 Reduction of boundary effects

5 Conclusion

Parcours

Depuis septembre 2021 :
École Normale Supérieure de Lyon
Maître de conférences, au département de physique.

2020–2021 : Duke University
Postdoctoral Associate,
Department of Mathematics.

2016–2019 : Aix-Marseille Université
Doctorant, Institut de Mathématiques de Marseille.

2012–2016 :
École Normale Supérieure de Cachan
Élève au département EEA.

- **2015–2016**
  Master 2, Automatique et Traitement du Signal et des Images
- **2014–2015**
  Master 2, Formations d'enseignants pour le supérieur, spécialité Physique appliquée

1 Présentation de mon parcours

2 Introduction: la non-stationnarité
- Motivations
- Broken stationarity: a class of nonstationarity
- Outline

3 Locally deformed signals: an analysis-based approach

4 Reduction of boundary effects

5 Conclusion

Stationarity: a mathematical concept

Definition (Stationarity)

A random process X is said to be second-order stationary if:

- $\mathbb{E}\{X(t)\} = m, \ \forall t$,
- $\mathbb{E}\{X(t)X(\tau)\} = k(t-\tau), \ \forall (t,\tau)$.

Spectrum:

- Gives the distribution over frequencies of the power of X.
- Many methods to estimate the spectrum from a single realization of the stationary process X (e.g. Welch method).

Nonstationarity

The stationarity assumption is often irrelevant to study real-life signals, such as audio signals, or physiological signals.

$\Rightarrow$ **Questions:**

1. Which classes of nonstationarity should we consider?
2. How should we analyze nonstationarity? In particular, how to extend spectral estimation to nonstationary signals?

Broken stationarity: a class of nonstationarity

Two key ingredients:

1. A zero-mean stationary random process X.
2. A deformation operator that breaks stationarity $\mathcal{T}$.

We observe the "deformed" process Y given by:

$$Y = \mathcal{T}X \ .$$

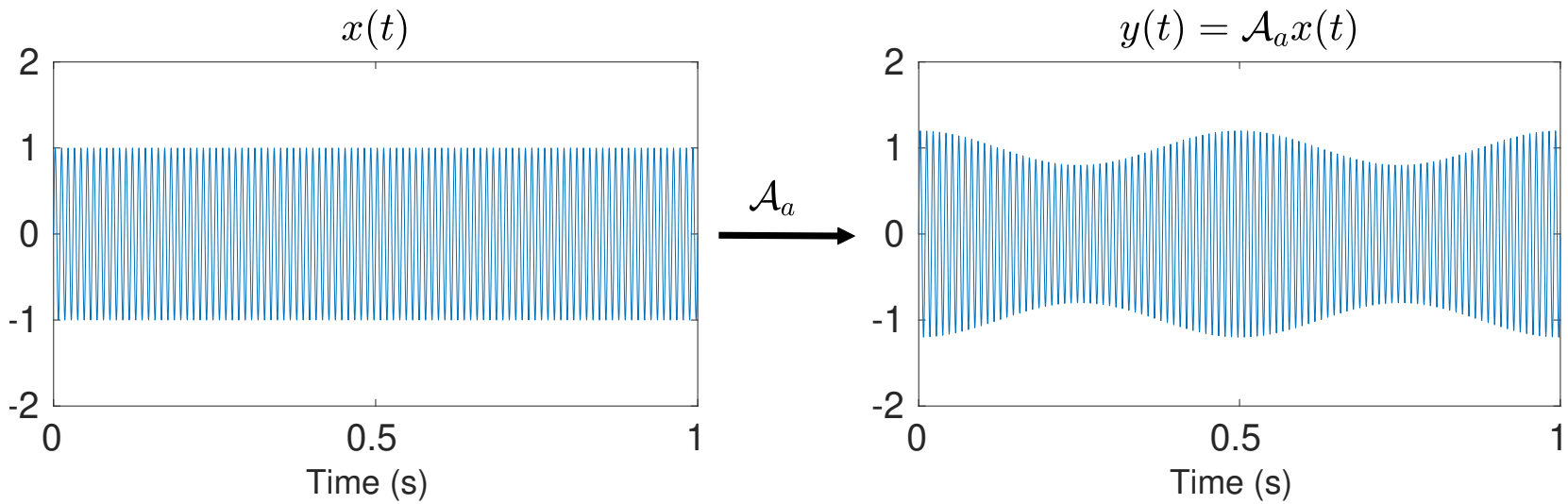
$\Rightarrow$ We limit ourselves to some physically relevant forms of operators.

Deformation operators

- Amplitude modulation

$$\mathcal{A}_\alpha : \qquad \mathcal{A}_\alpha x(t) = \alpha(t)x(t) \ ,$$

with $\alpha \in C^1$ such that $\forall t,\ 0 < \alpha(t) < \infty$.



- Frequency modulation

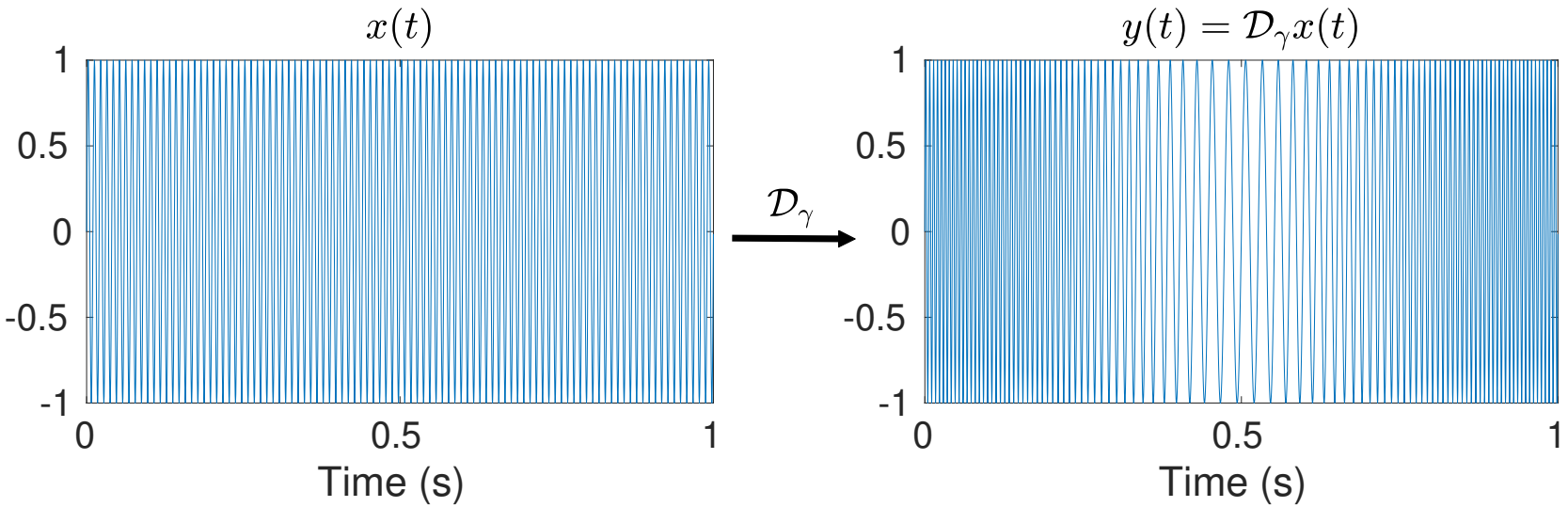
Deformation operators

- Time warping

$$\mathcal{D}_\gamma : \qquad \mathcal{D}_\gamma x(t) = \sqrt{\gamma'(t)}x(\gamma(t)) \ ,$$

where $\gamma \in C^2$ is a strictly increasing function such that

$$0 < \gamma'(t) < \infty, \forall t \ .$$



- Any combination of the above deformations

Goal: spectral estimation of nonstationary signals

From a single realization of the nonstationary random process Y, we aim at estimating simultaneously:

- The spectrum $\mathscr{S}_X$ of the underlying stationary random process X,
- The deformation operator $\mathcal{T}$.

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- Wavelet transform and approximation
- Estimation algorithm: JEFAS
- Applications to audio signals
- Multivariate locally deformed signals

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Model and goal

- **Model**:

$$Y = \mathscr{A}_\alpha \mathscr{D}_\gamma X \ .$$

  where X is a stationary process.

  - Relevant to model physical phenomena, such as Doppler effect, speed variations, or animal vocalizations.

- **Goal**: From a single realization of the process Y, estimate simultaneously:
  - the spectrum $\mathscr{S}_X$ of the underlying stationary process X,
  - the deformation functions α and γ.

Wavelet transform

Definition (Wavelet transform)

$$\mathcal{W}_x(s,\tau) = \langle x, \psi_{s\tau} \rangle \quad \text{avec} \quad \psi_{s\tau}(t) = 2^{-s/2}\psi\left(2^{-s}(t-\tau)\right) \ ,$$

where ψ is the analysis wavelet.

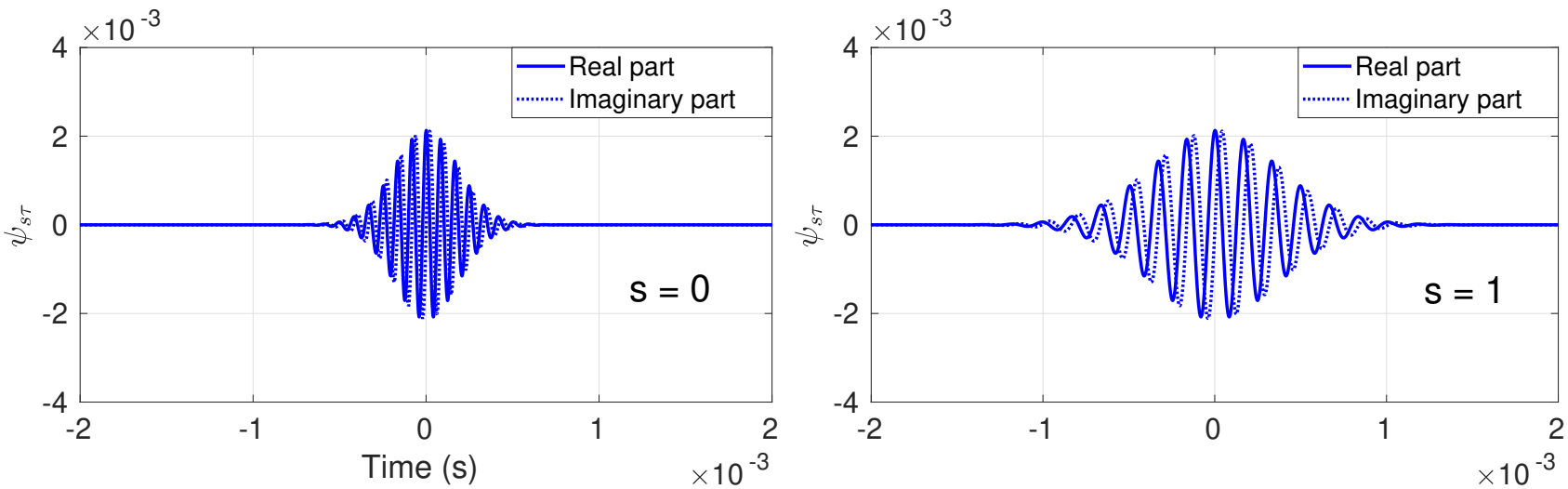
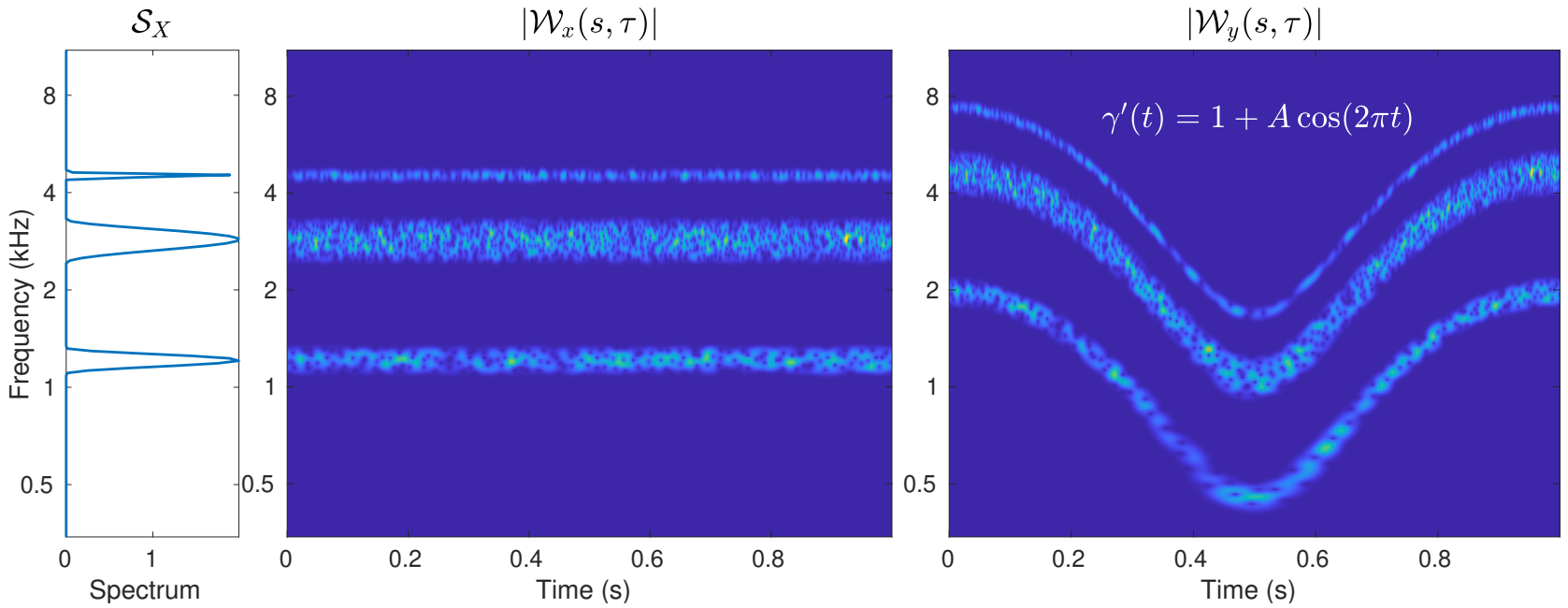


Figure: "Sharp wavelet" for two different values of s.

Approximated behavior



Approximation theorem

The wavelet transforms of X and Y are approximately related by:

$$\mathcal{W}_Y(s,\tau) \approx \alpha(\tau)\mathcal{W}_X\left(s + \log_2(\gamma'(\tau)), \gamma(\tau)\right) \ .$$

The error term $\epsilon = \mathcal{W}_Y - \widetilde{\mathcal{W}}_Y$ is a zero-mean random process, whose variance $\mathbb{E}\left\{|\epsilon(s,\tau)|^2\right\}$ depends on the regularity of α and γ', and the speed of decay of ψ.

Estimation algorithm: JEFAS

The JEFAS (*Joint Estimation of Frequency, Amplitude and Spectrum*) algorithm consists in an alternating estimation.

Initializations:

- Initialize the power spectrum estimate.
- Initialize the amplitude modulation by a constant.
- $k \leftarrow 1$

while stopping criterion $=$ FALSE **do**

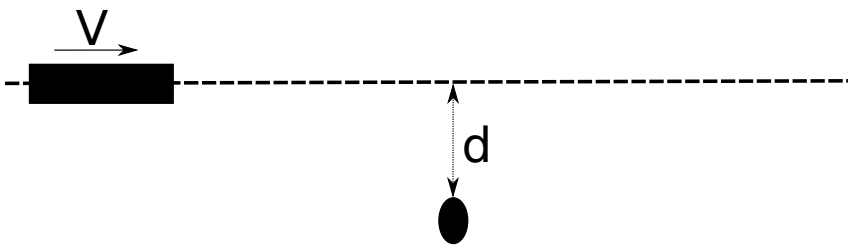
- Time warping: Estimate $\tilde{\alpha}^{(k)}$ by ML, $\forall \tau$.
- Amplitude modulation: Estimate $\tilde{\gamma}^{(k)}$ by ML, $\forall \tau$.
- Spectrum: Estimate $\tilde{\mathscr{S}}_X^{(k)}$ from the "rectified" wavelet transform.
- $k \leftarrow k+1$

end while

Doppler effect

Assumptions:

- A source emits a stationary sound.
- The source follows a uniform linear motion, at speed V.
- From a fixed station, we record the sound emitted by the source.

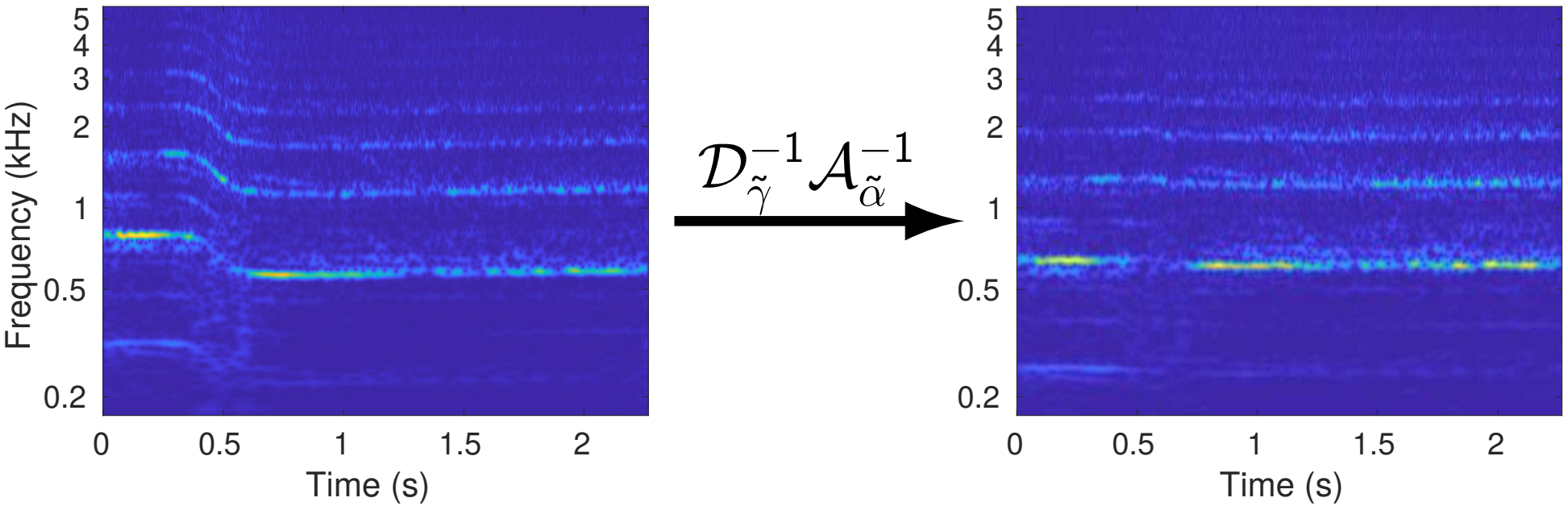


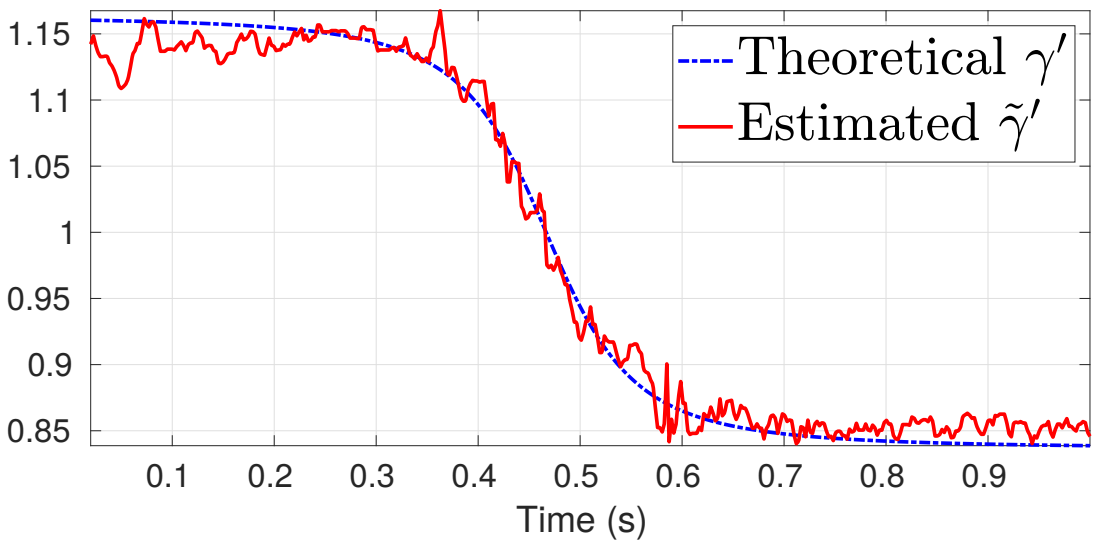
$\Rightarrow$ Due to the Doppler effect, the sound we receive is time-warped, with:

$$\gamma'(t) = \frac{c^2}{c^2 - V^2}\left(1 - \frac{V^2 t}{\sqrt{d^2(c^2 - V^2) + (cVt)^2}}\right).$$

An example:

Doppler effect





Comparison to the theoretical function with: $d = 5$ m and $V = 54$ m/s.

Spectral analysis of a broadband wind sound

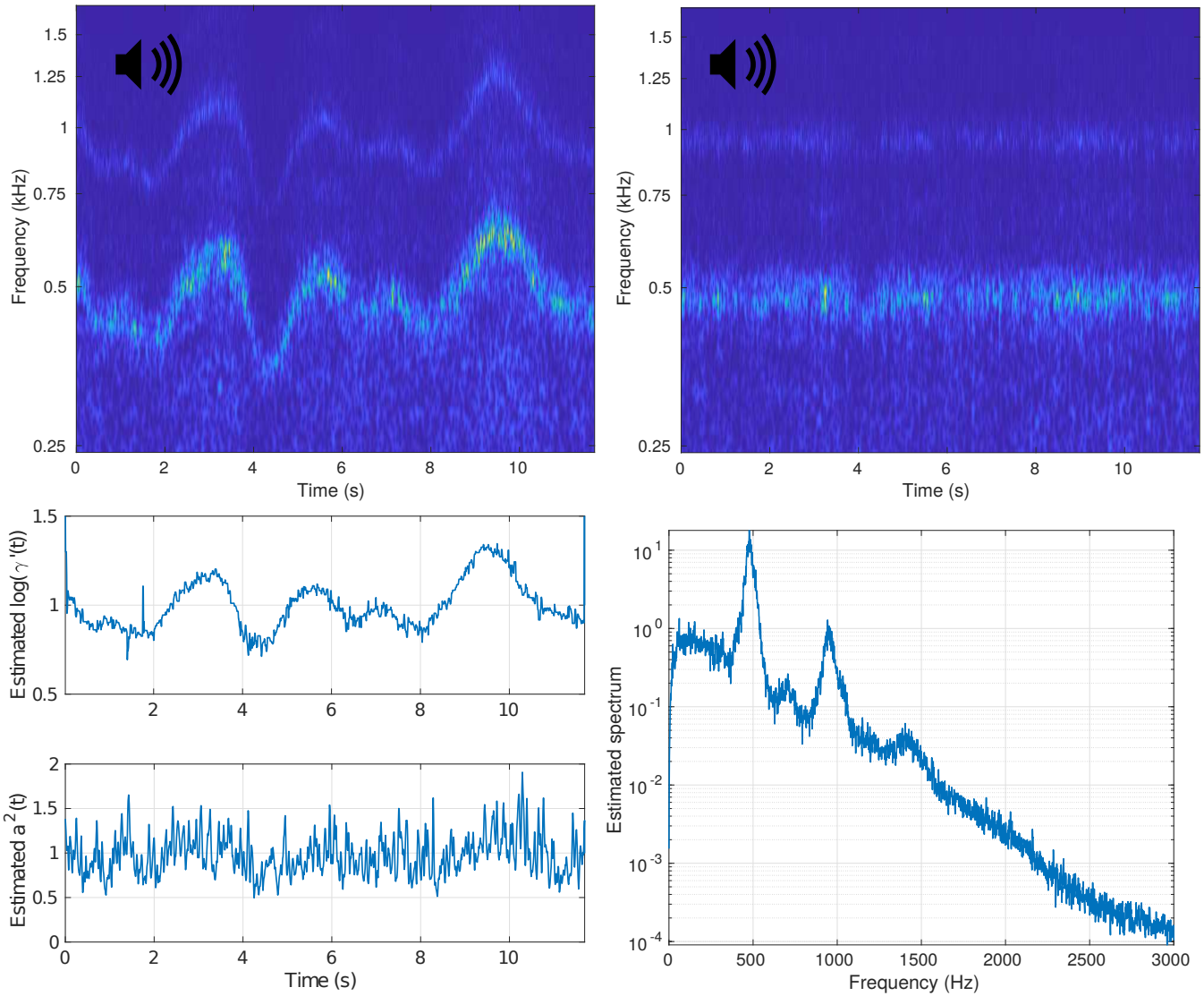


Figure: Top: Scalograms of the original signal (left) and the estimated stationary signal (right). Bottom left: estimated time warping and amplitude modulation. Bottom right: estimated spectrum.

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  - Model
  - Estimation algorithm: JEFAS-BSS
  - Illustrations on synthetic mixtures

4 Reduction of boundary effects

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Model and goal (determined case)

- **Model**: Multivariate extension of the previous model, i.e.:

$$\begin{pmatrix} Z_1 \\ \vdots \\ Z_N \end{pmatrix} = \begin{pmatrix} \mathcal{A}_{\alpha_{11}} & \cdots & \mathcal{A}_{\alpha_{1N}} \\ \vdots & \ddots & \vdots \\ \mathcal{A}_{\alpha_{N1}} & \cdots & \mathcal{A}_{\alpha_{NN}} \end{pmatrix} \begin{pmatrix} \mathcal{D}_{\gamma_1} & & \\ & \ddots & \\ & & \mathcal{D}_{\gamma_N} \end{pmatrix} \begin{pmatrix} X_1 \\ \vdots \\ X_N \end{pmatrix}$$

  where X_j are uncorrelated stationary processes.

  - Relevant to model linear mixtures of different nonstationary sources.

- **Goal**: From a single realization of the multivariate process $(Z_1, \ldots, Z_N)$, estimate simultaneously:
  - the spectra $\mathscr{S}_{X_j}$ of the underlying stationary processes X_j,
  - the deformation functions α_{ij} and γ_j.

Reformulation of the problem

Spectral estimation problem

$\Leftrightarrow$

Doubly nonstationary Blind Source Separation (BSS)

Mixture model

At time t, we observe the multivariate signal $\mathbf{z}(t) \in \mathbb{R}^N$

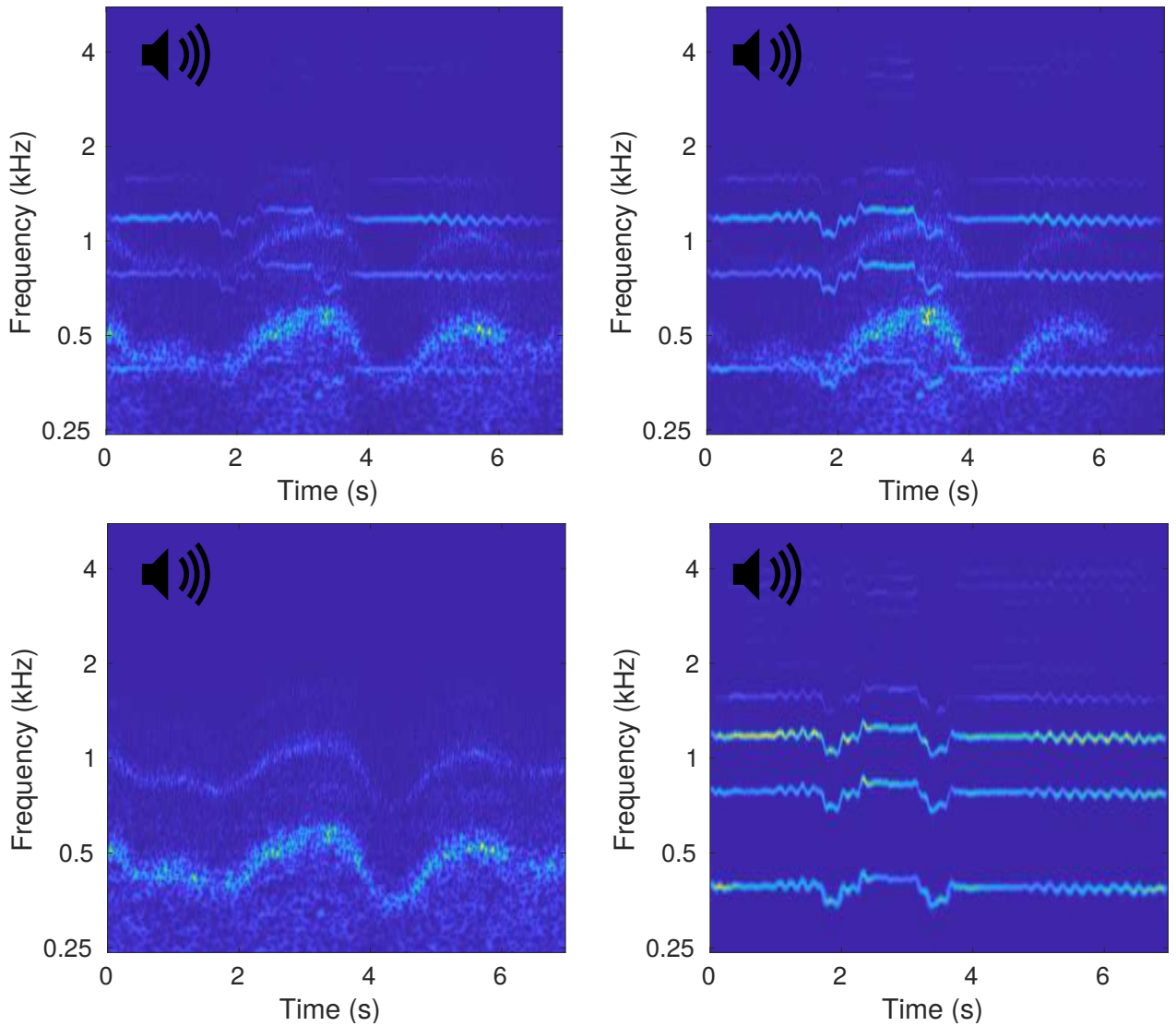
$$\mathbf{z}(t) = \mathbf{A}(t)\mathbf{y}(t) \ ,$$

where:

- $\mathbf{A}(t) \in \mathbb{R}^{N \times N}$ is the mixing matrix,
- $\mathbf{y}(t) \in \mathbb{R}^N$ is such that

$$y_i(t) = \mathcal{D}_{\gamma_i} x_i(t) \ .$$

Illustration on a synthetic mixture of audio signals



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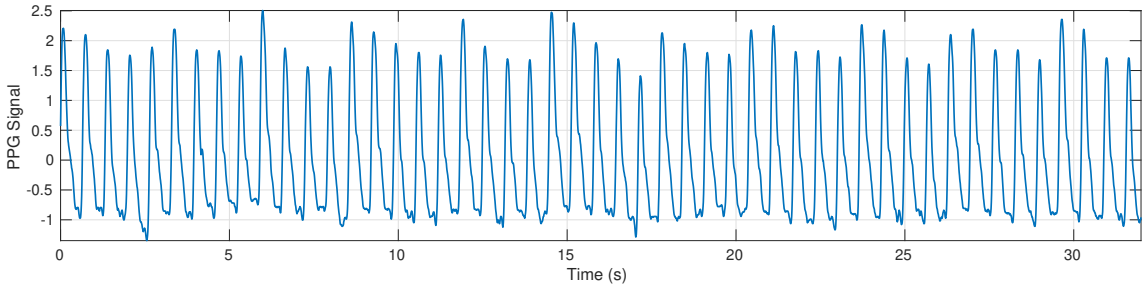
4 Reduction of boundary effects
- Motivations
- Boundary effects
- A boundary-effect reduction algorithm
- Numerical results

5 Conclusion

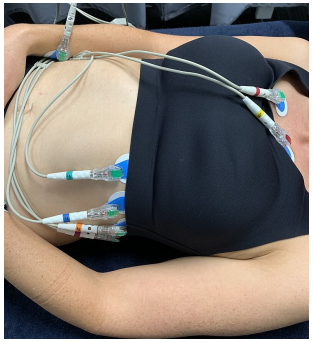
Biomedical signals: Cardiac signals

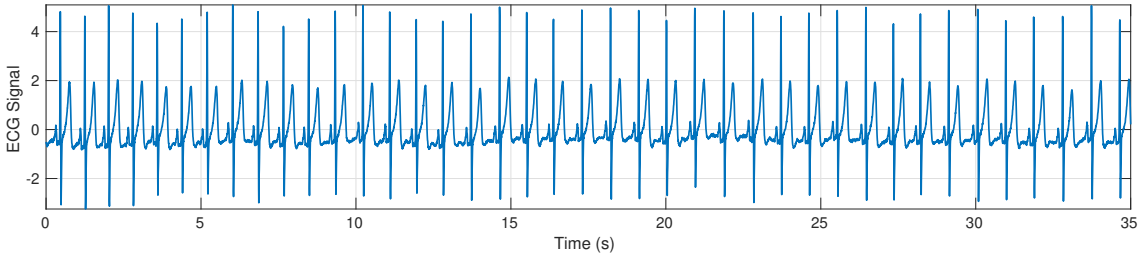
- Blood volume changes $\leftrightarrow$ Photoplethysmogram (PPG)



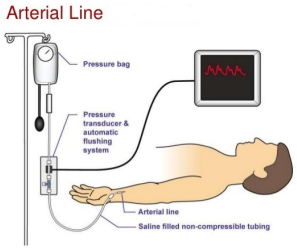


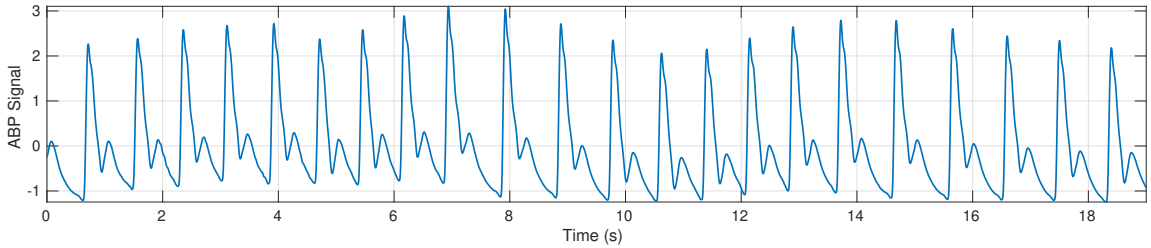
- Electrical activity of the heart $\leftrightarrow$ Electrocardiogram (ECG)





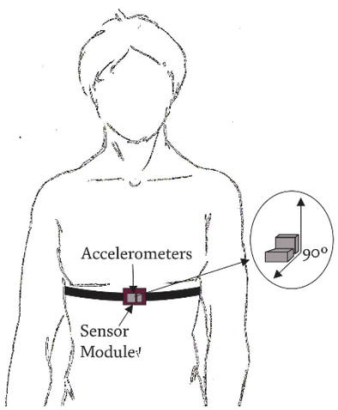
- Arterial blood pressure (ABP)

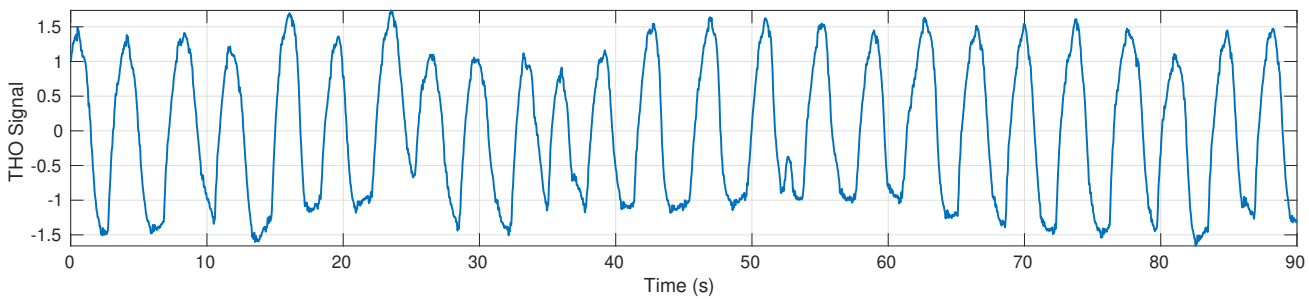




Biomedical signals: Respiratory signals

- Thoracic movement recorded by a piezoelectric sensor (THO)



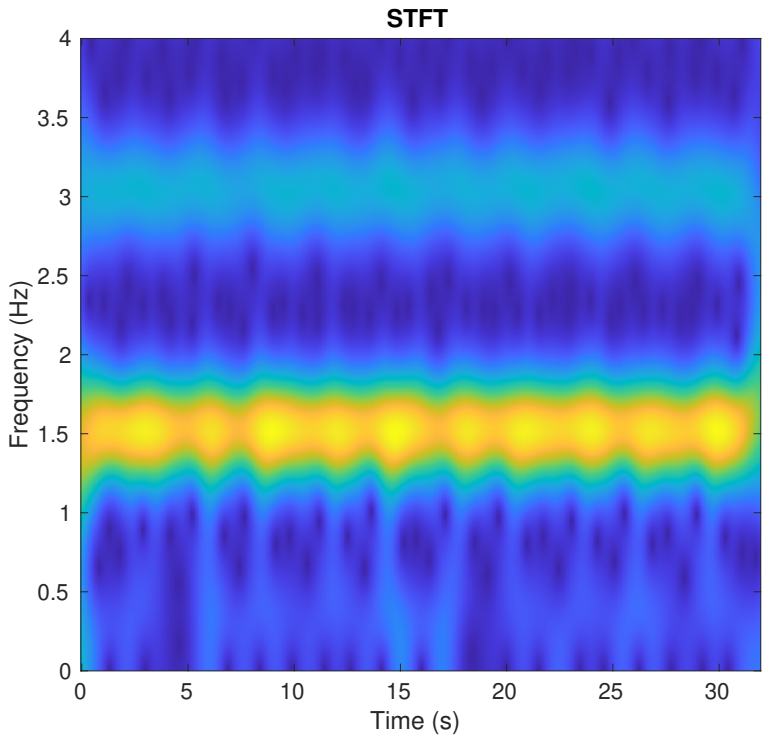


Time-frequency analysis

Goal: Visualize the evolution of the instantaneous frequencies forming the signal.

⇒ Time-frequency representations.

- Short-Time Fourier Transform (STFT);
- Synchrosqueezing Transform (SST);
- Reassignment (RS);
- ...

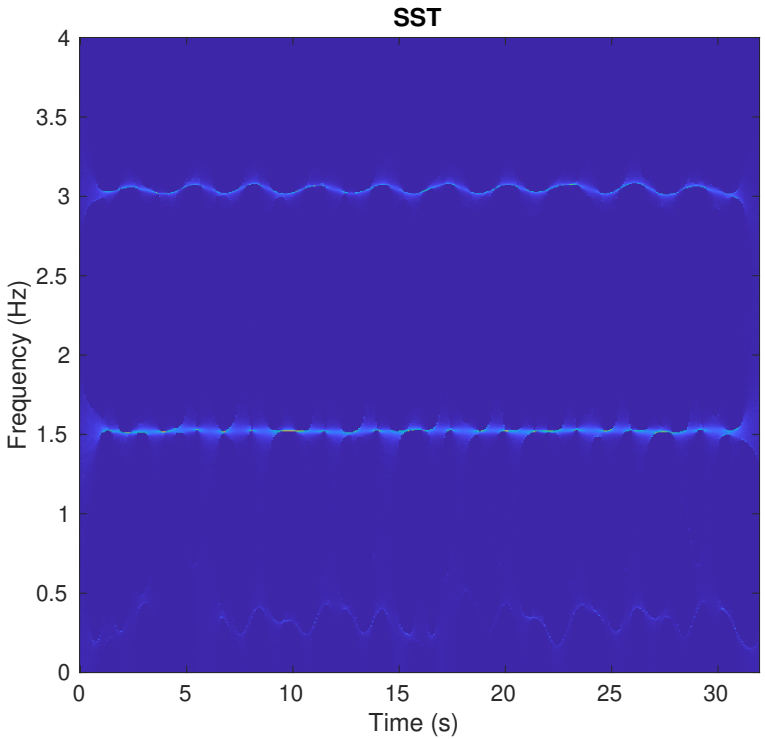


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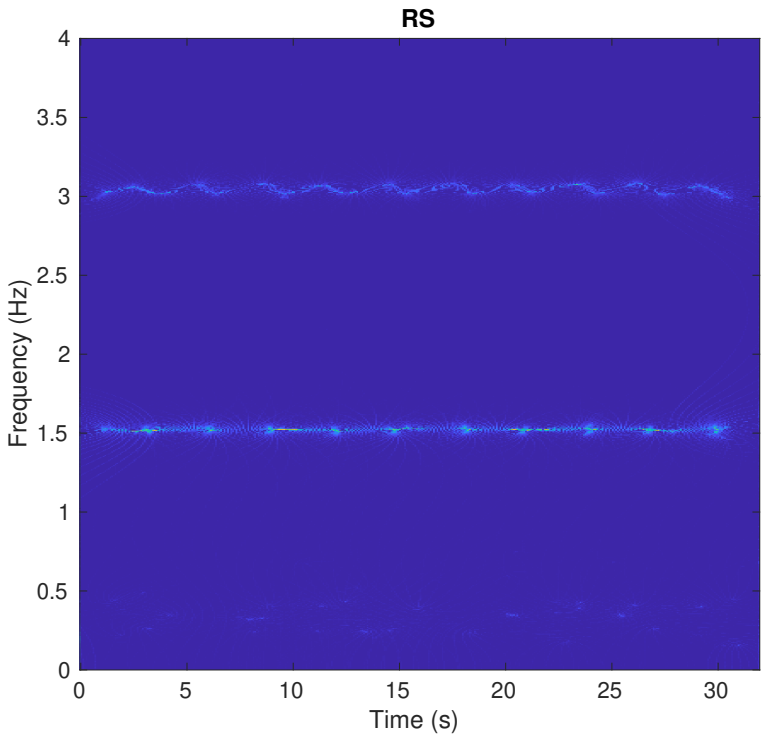


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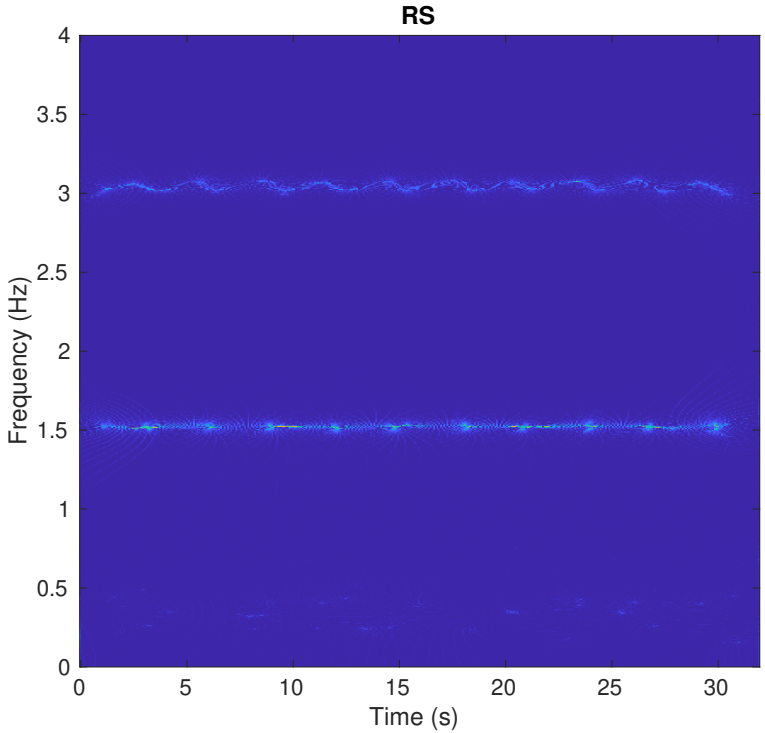


Time-frequency analysis

Goal: Visualize the evolution of the instantaneous frequencies forming the signal.

⇒ Time-frequency representations.

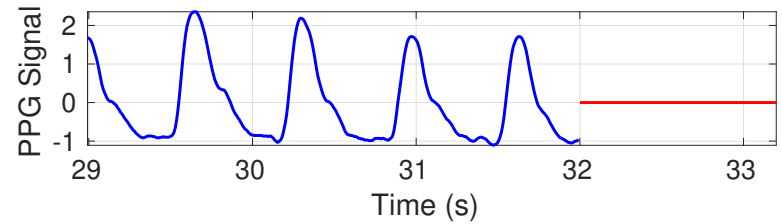
- Short-Time Fourier Transform (STFT);
- Synchrosqueezing Transform (SST);
- Reassignment (RS);
- ...

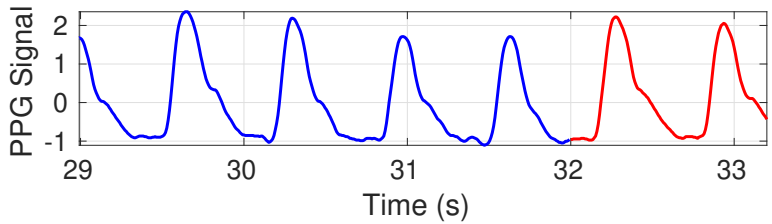


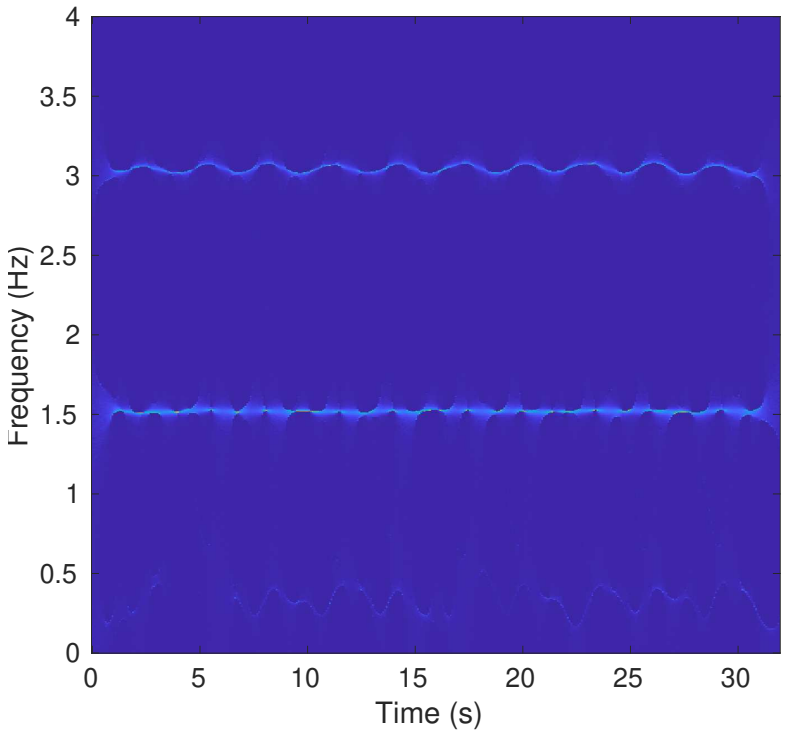
Boundary effects

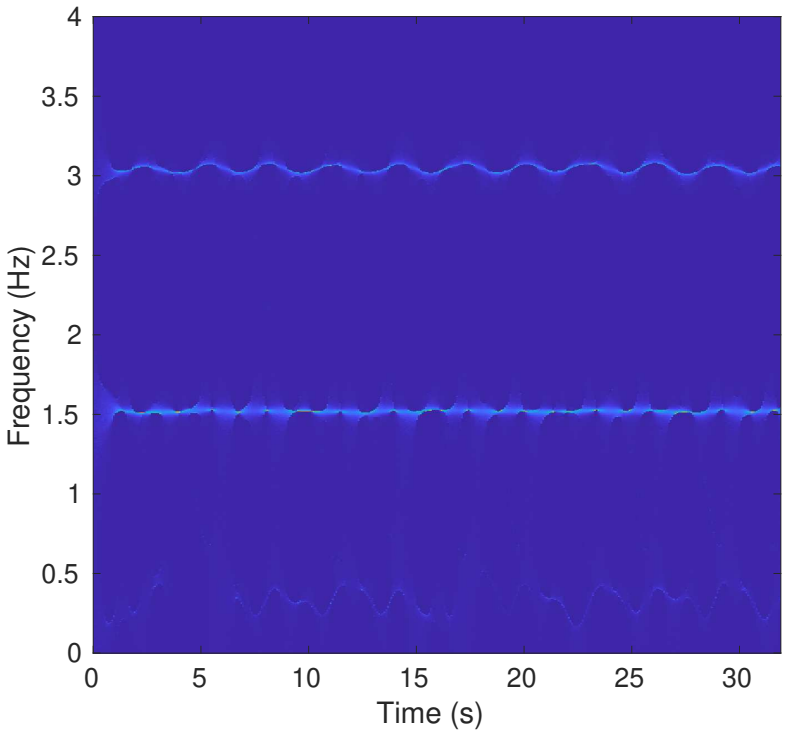
- Time-frequency analysis is based on windowing techniques.
- Time-frequency representations are thus sensitive to boundaries.
- The accuracy of these representation is deteriorated near boundaries.

$\Longrightarrow$ **Boundary effects**

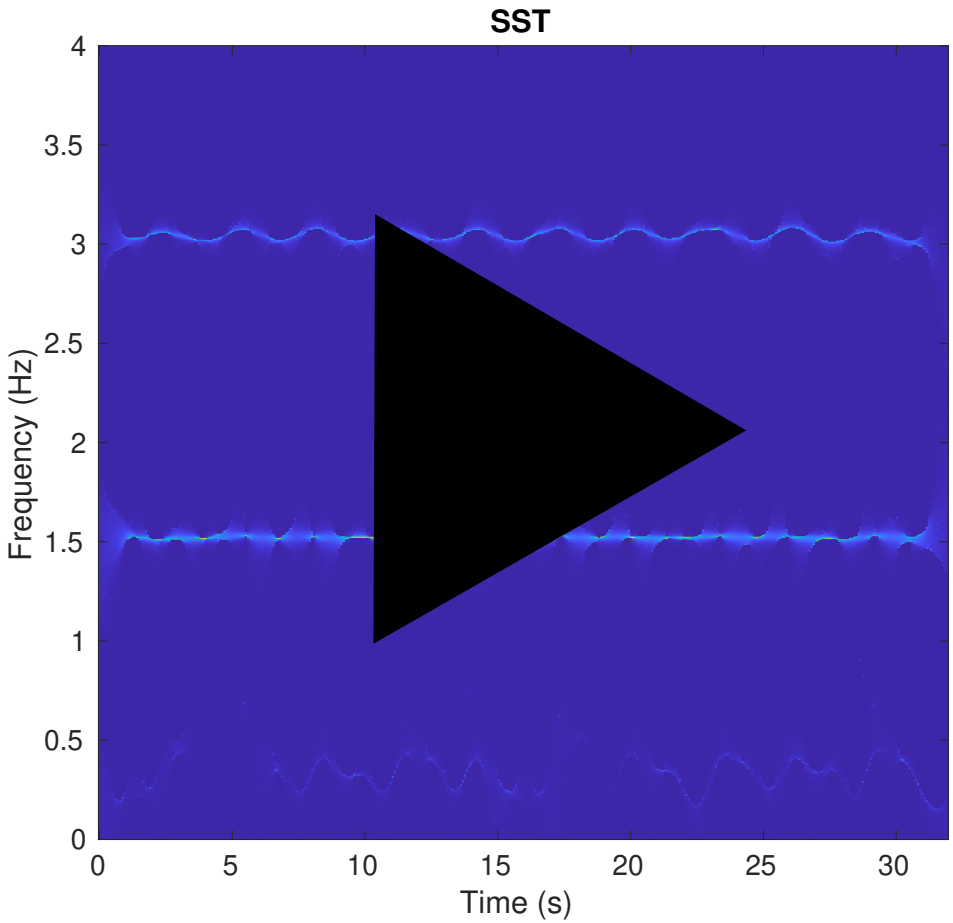








Real-time time-frequency analysis



Goal: Being able to display real-time boundary-free time-frequency representations.

Strategy

Two-step strategy:

1. Forecast a sufficiently long extension of the signal to push signal's boundaries;
2. Perform a time-frequency analysis on the extended signal, until the current time.

$\Rightarrow$ How can we forecast the biomedical signals that interest us?

Extension step

Idea: Take advantage of the shape of the signals we consider to establish a **fast** and **accurate** forecasting method.

Adaptive Harmonic Model

- $\mathbf{x} \in \mathbb{R}^N$ is the observed signal;
- $\mathbf{w} \in \mathbb{R}^N$ is a Gaussian white noise;
- $\mathbf{z} \in \mathbb{R}^N$ is an oscillatory deterministic signal such that

$$\mathbf{z}[n] = \sum_{j=1}^{J} a_j[n] \cos\left(2\pi\phi_j[n]\right) ,$$

where a_j and ϕ'_j describe how large and fast the signal oscillates.

Then, we assume that

$$\mathbf{x} = \mathbf{z} + \sigma\mathbf{w} .$$

Extension step

To determine a prediction of the signal of length L, we estimate the dynamical behavior of the oscillating signal from its previous values.

1. We extract a set $\{\mathbf{x}_1, \ldots, \mathbf{x}_K\}$ of K subsignals of length $M > L$ from $\mathbf{x}$.
2. Establishing a dynamical model consists in determining the relationship that binds $\mathbf{x}_{k+1}$ to $\mathbf{x}_k$, that is finding a function f so that

$$\mathbf{x}_{k+1} = f(\mathbf{x}_k), \quad \forall k \in \{1, \ldots, K-1\} .$$

Extension step

Here, we consider here a naive dynamical model, assuming that we have

$$\mathbf{x}_{k+1} = \mathbf{A}\mathbf{x}_k \,, \quad \forall k \in \{1, \ldots, K-1\} \,,$$

where $\mathbf{A} \in \mathbb{R}^{M \times M}$.

- Classical strategy in the study of dynamical systems: *linearization of a nonlinear system*.
- Fast and simple estimation of the parameters characterizing the dynamical model. Indeed:

$$\tilde{\mathbf{A}} = \mathbf{Y}\mathbf{X}^T(\mathbf{X}\mathbf{X}^T)^{-1} \,.$$

Least square estimator

where $\mathbf{X} = \begin{pmatrix} \mathbf{x}_1 & \cdots & \mathbf{x}_{K-1} \end{pmatrix}$ and $\mathbf{Y} = \begin{pmatrix} \mathbf{x}_2 & \cdots & \mathbf{x}_K \end{pmatrix}$.

- The forecasting of $\mathbf{x}_{k+\ell}$ is obtained by recursively applying the linear relation:

$$\tilde{\mathbf{x}}_{K+\ell} = \underbrace{\tilde{\mathbf{A}}\tilde{\mathbf{A}} \cdots \tilde{\mathbf{A}}}_{\ell \text{ times}} \mathbf{x}_K = \tilde{\mathbf{A}}^\ell \mathbf{x}_K \,.$$

$\Longrightarrow$ Fast forecasting strategy. Is it accurate?

Theoretical forecasting error

Fast forecasting strategy. Is it accurate?

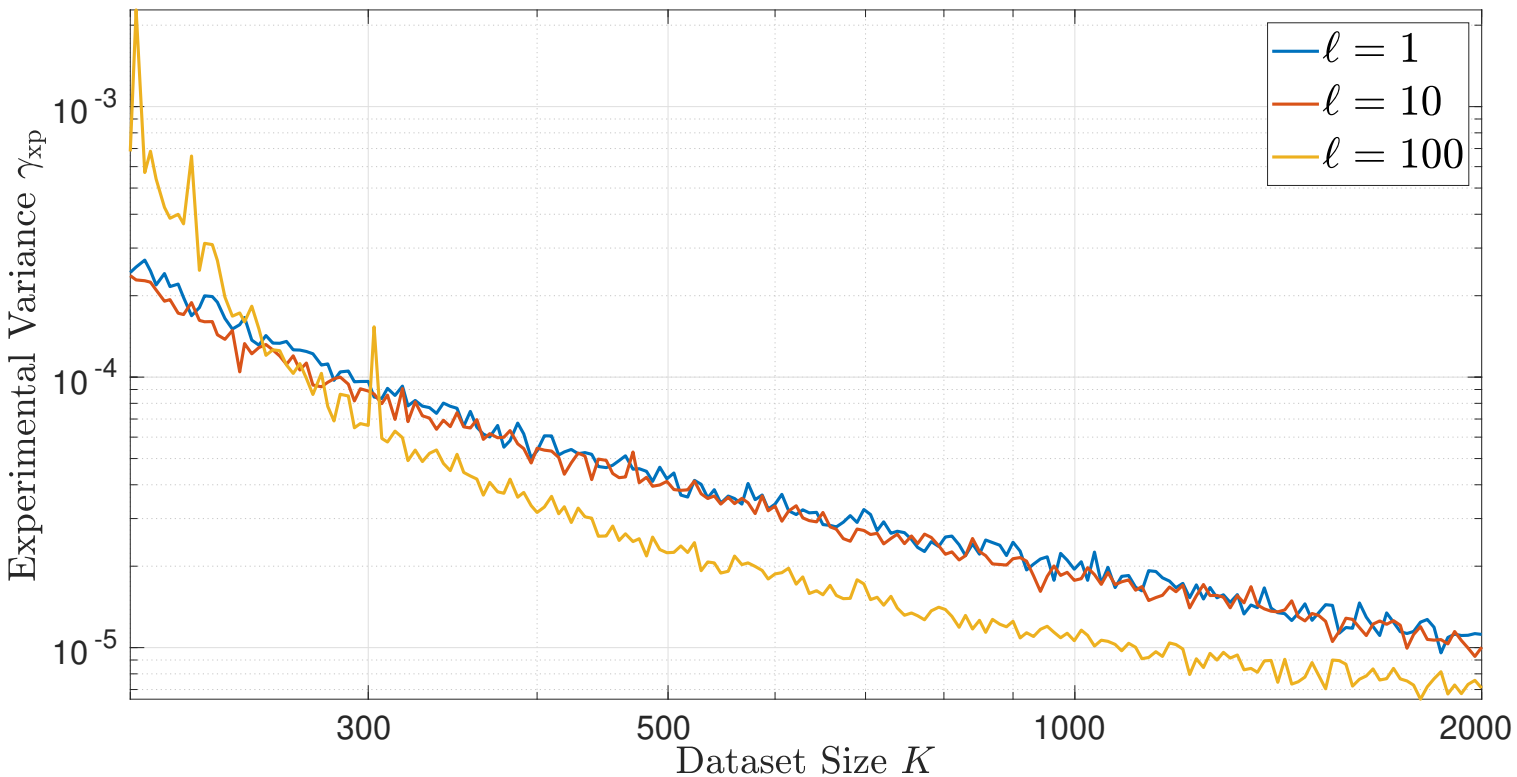
$\Longrightarrow$ **YES!**

We theoretically proved that the bias and the variance of the estimation error remain controlled as long as:

- the noise variance is small enough;
- $K \to \infty$ i.e. the dataset is large.

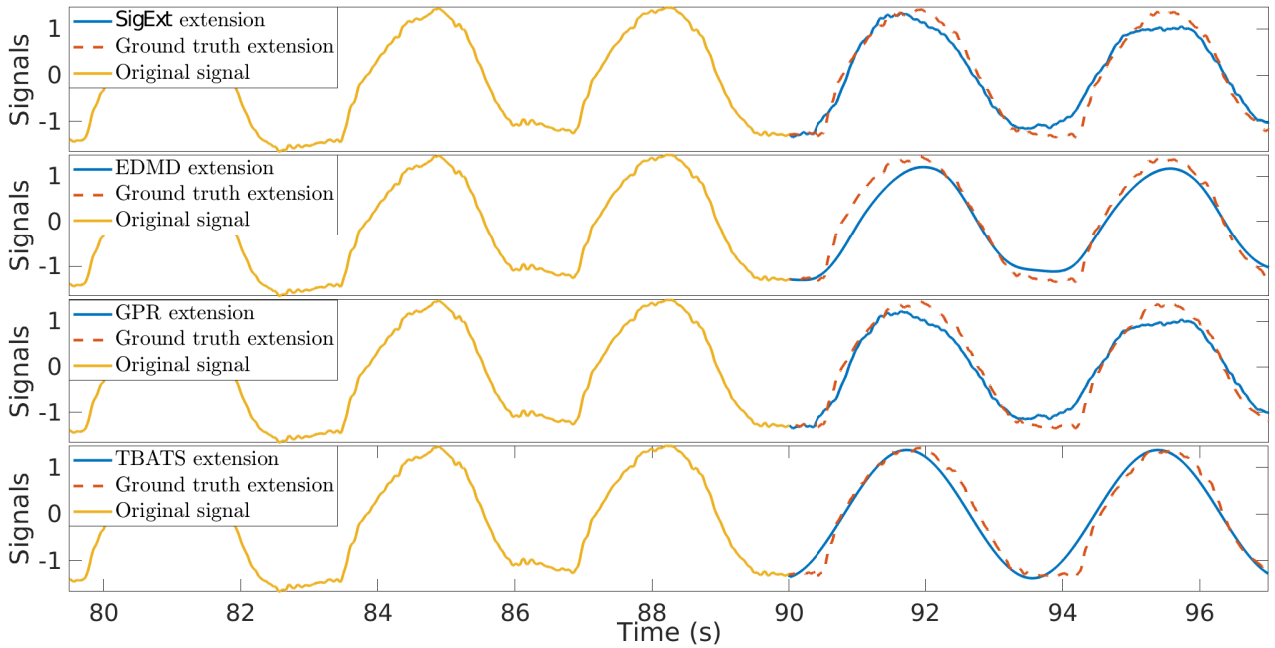
Illustration of the theoretical results on a simulated signal

- Evolution of the experimental forecasting variance in function of the dataset size for three different values the forecasting sample index ℓ.



Applications to Real Physiological Signals

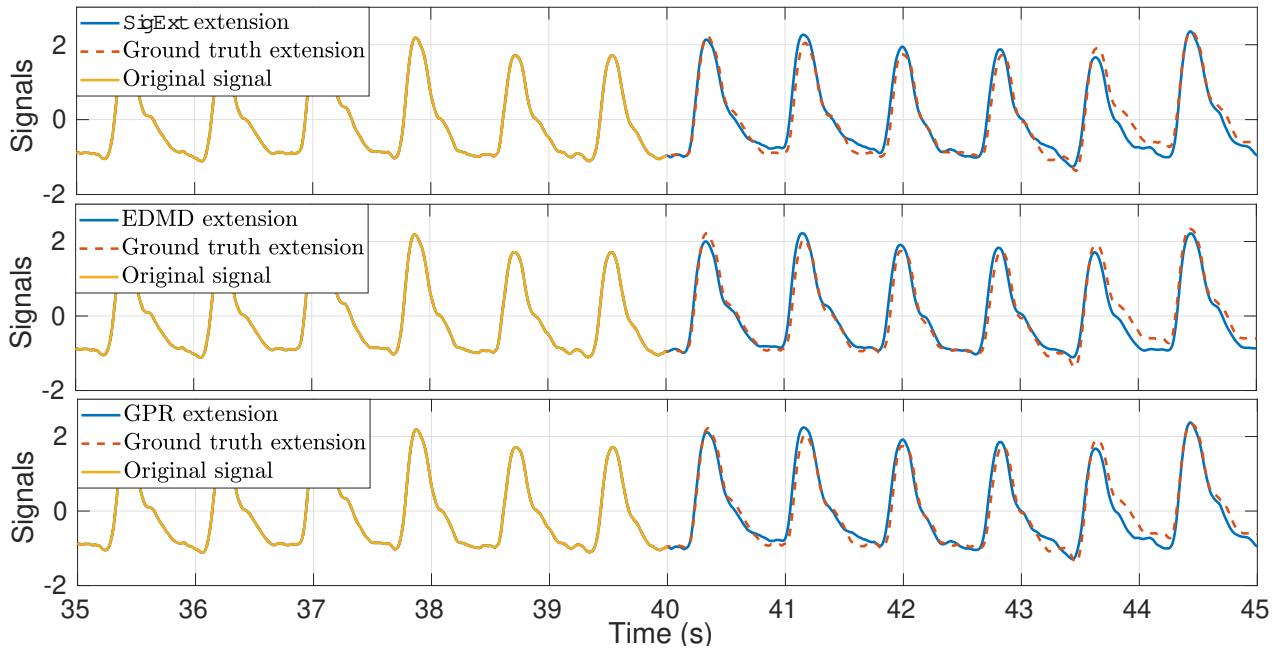
Respiratory signal



Extension method	MSE	Performance index D (mean $\pm$ SD)			
		STFT	SST	RS	ConceFT
SigExt	0.292 $\pm$4.438	0.370 $\pm$0.623	0.408 $\pm$0.436	0.866 $\pm$0.879	0.423 $\pm$0.344
Symmetric	0.044 $\pm$0.111	1.162 $\pm$0.893	1.173 $\pm$0.886	1.022 $\pm$0.281	1.144 $\pm$0.579
EDMD	0.026 $\pm$0.112	0.359 $\pm$0.266	0.422 $\pm$0.282	0.828 $\pm$0.248	0.449 $\pm$0.296
GPR	0.331 $\pm$4.858	0.391 $\pm$0.853	0.411 $\pm$0.406	0.897 $\pm$1.140	0.430 $\pm$0.364

Applications to Real Physiological Signals

PPG signal



Extension method	MSE	Performance index D (mean $\pm$ SD)			
		STFT	SST	ConceFT	RS
SigExt	0.018 $\pm$0.014	0.280 $\pm$0.107	0.309 $\pm$0.112	0.367 $\pm$0.183	0.534 $\pm$0.160
Symmetric	0.037 $\pm$0.007	1.168 $\pm$0.390	1.209 $\pm$0.340	1.310 $\pm$0.140	0.983 $\pm$0.304
EDMD	0.012 $\pm$0.005	0.289 $\pm$0.126	0.319 $\pm$0.134	0.375 $\pm$0.163	0.503 $\pm$0.163
GPR	0.018 $\pm$0.013	0.276 $\pm$0.106	0.303 $\pm$0.110	0.361 $\pm$0.165	0.544 $\pm$0.157

Real-Time Implementation

- f_s: sampling frequency
- t_{forecast}: time to compute the forecasting of L oncoming samples
- t_{SST}: time to compute one column of the synchrosqueezing transform
- H: hop size (number of samples between each successive column of the synchrosqueezing transform)

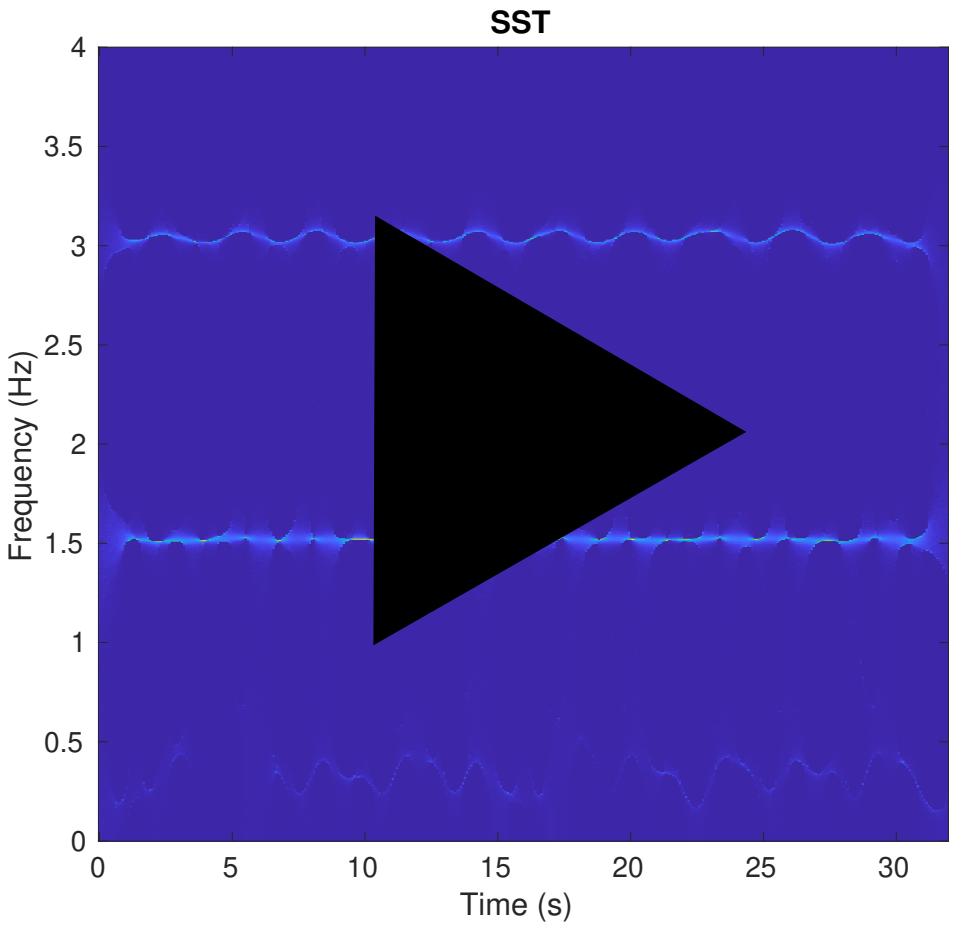
A general rule to determine the acceptable values of H for real-time implementation of update the boundary-free TF representation is

$$t_{\text{forecast}} + \left\lceil \frac{L}{H} \right\rceil t_{\text{SST}} < \frac{H}{f_s} \, .$$

Applications to Real Physiological Signals

PPG signal : real-time implementation

In this example, taking $H \geq 8$ samples is sufficient to ensure the feasibility of real-time implementation. It thus allows a maximum overlap of 98.4% of the window length.



1 Présentation de mon parcours

2 Introduction: la non-stationnarité

3 Locally deformed signals: an analysis-based approach

4 Reduction of boundary effects

5 Conclusion

Conclusion

Summary:

- Broken stationarities:

$$Y = \mathcal{T}X$$

  - Locally deformed signals
  - Multivariate locally deformed signals
  - Locally harmonic signals
- Spectral estimation: Simultaneous estimation of the spectrum $\mathscr{S}_X$ and the deformation operator $\mathcal{T}$.
- Reduction of boundary effects: Forecasting of the variation of the deformation operator $\mathcal{T}$.

Questions?

Cross synthesis

$y_1(t) = \alpha_1(t)x_1(\gamma_1(t)) \Longrightarrow$ Spectrum estimation of $\mathscr{S}_{x_1}$

Cross synthesis:
$y_{1\to 2}(t) = \tilde{\alpha}_2(t)\tilde{x}_1(\tilde{\gamma}_2(t))$

$y_2(t) = \alpha_2(t)x_2(\gamma_2(t)) \Longrightarrow$ Deformations estimations of α_2 and γ_2

		Deformation		
		Song	Wind	Formula 1
Spectrum	Song			
	Wind			
	Formula 1			

D'autres thématiques de recherche

2 Traitement du signal appliqué aux sciences sociales

- **Évaluation de la synchronie sociale** (*projet en cours*)

