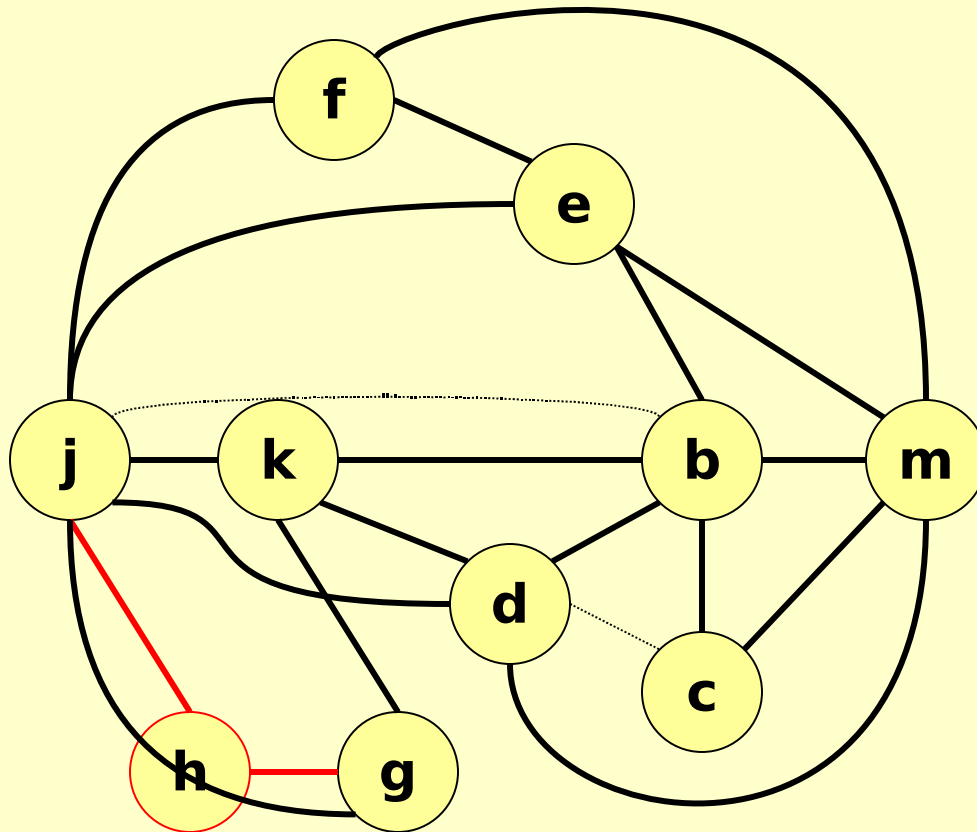




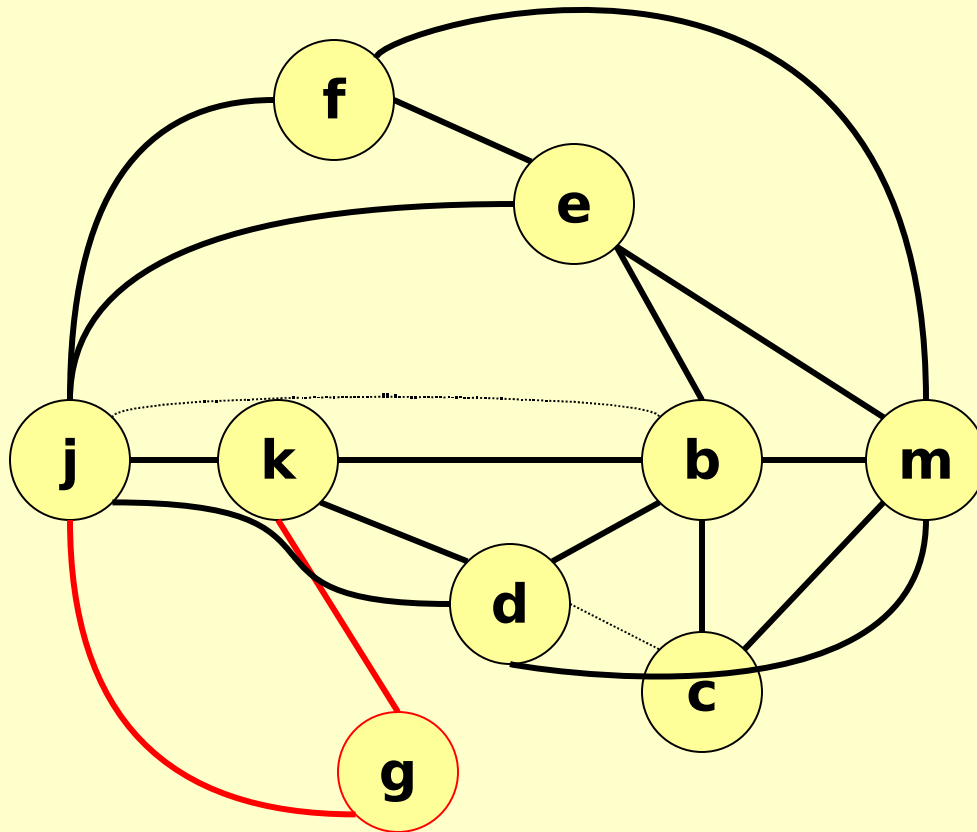
# Example: Simplify (K=4)



**stack**  
**(h,no-spill)**



# Example: Simplify (K=4)

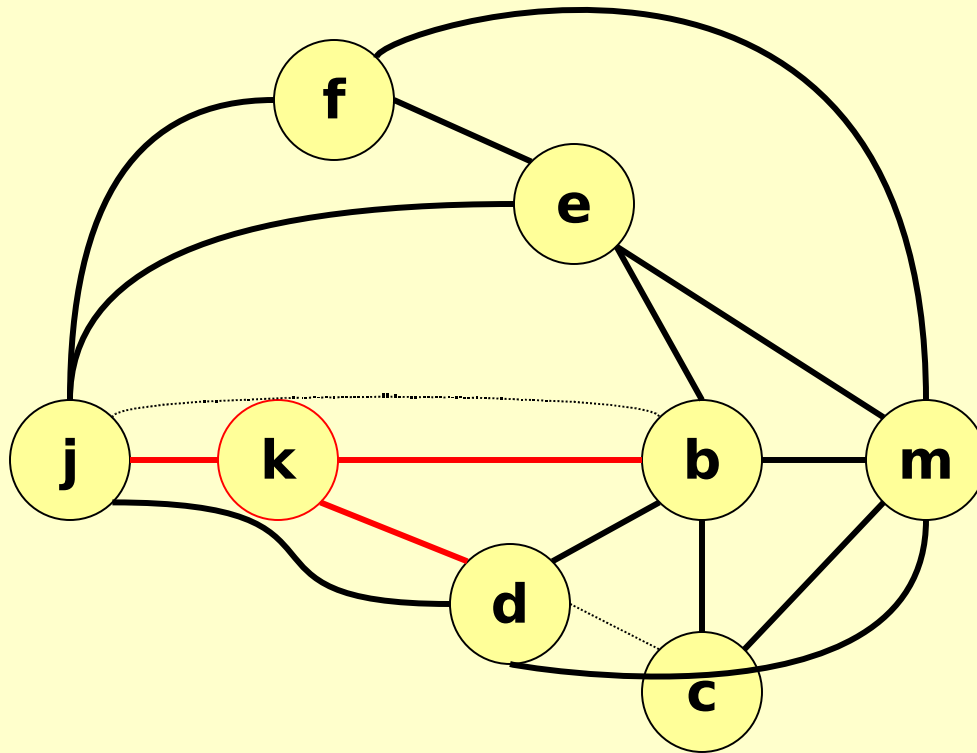


**stack**

(g, no-spill)  
(h, no-spill)



# Example: Simplify (K=4)

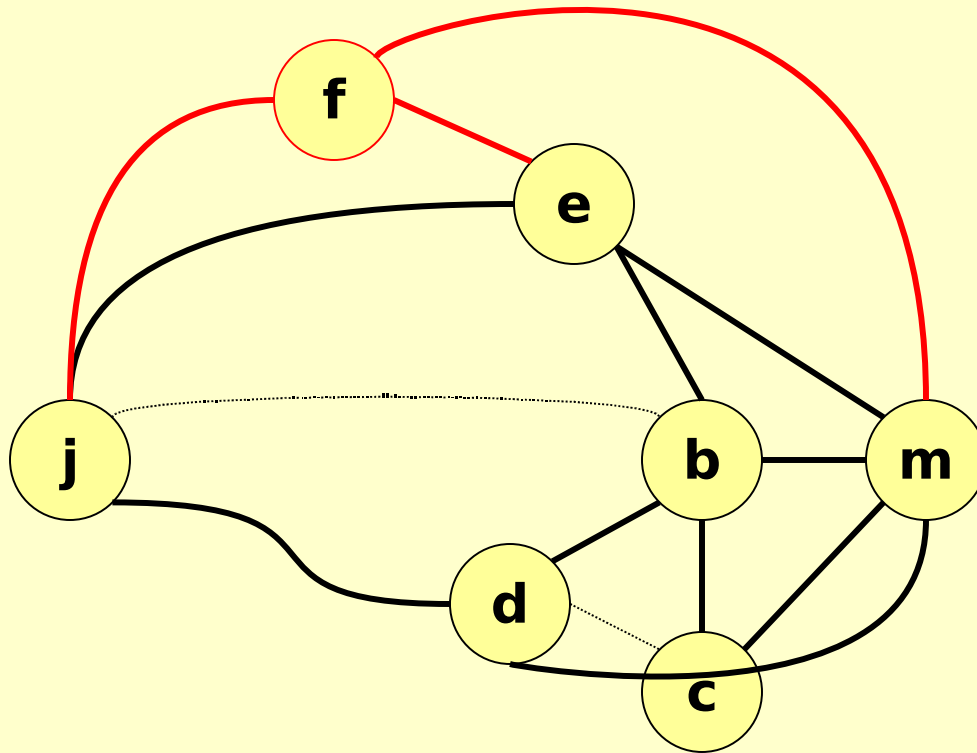


**stack**

(k, no-spill)  
(g, no-spill)  
(h, no-spill)



# Example: Simplify (K=4)

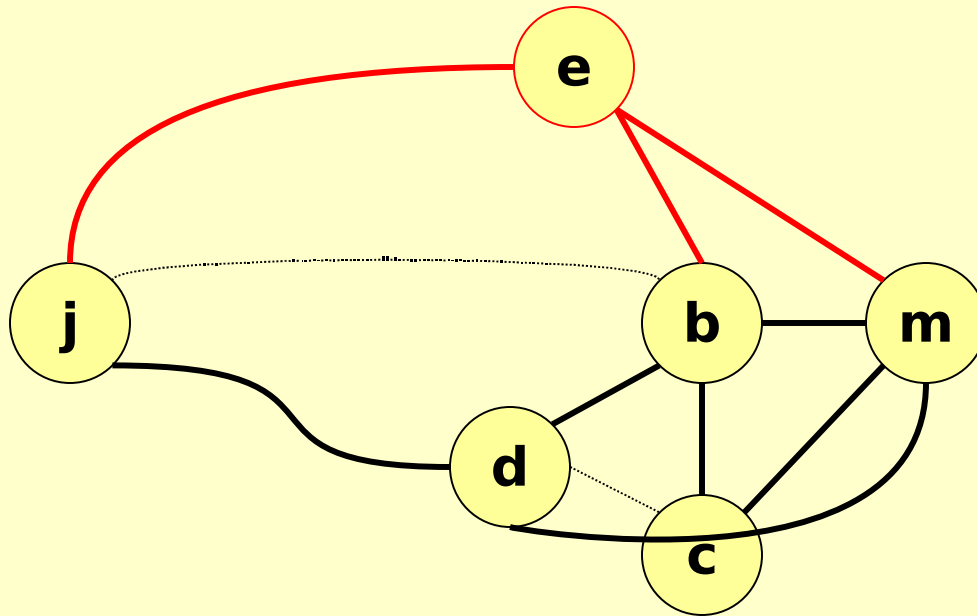


**stack**

(f, no-spill)  
(k, no-spill)  
(g, no-spill)  
(h, no-spill)



# Example: Simplify (K=4)



**stack**

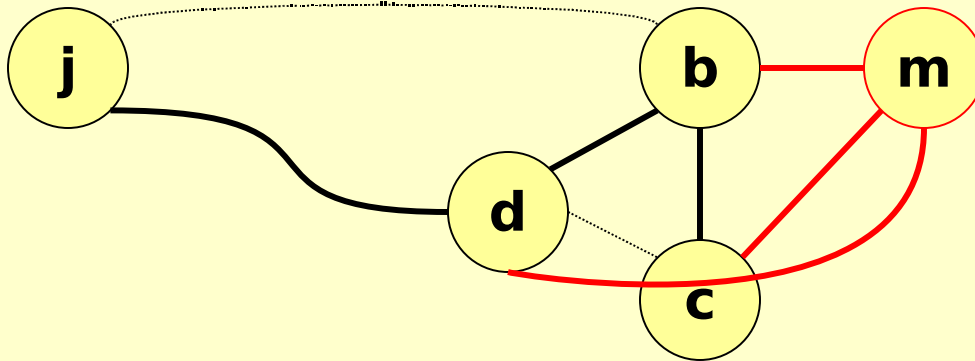
**(e, no-spill)**  
**(f, no-spill)**  
**(k, no-spill)**  
**(g, no-spill)**  
**(h, no-spill)**



# Example: Simplify (K=4)

**stack**

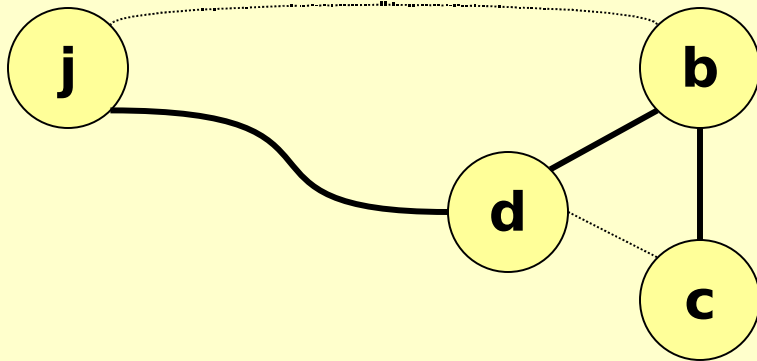
**(m, no-spill)**  
**(e, no-spill)**  
**(f, no-spill)**  
**(k, no-spill)**  
**(g, no-spill)**  
**(h, no-spill)**



# Example: Coalesce (K=4)

stack

(m, no-spill)  
(e, no-spill)  
(f, no-spill)  
(k, no-spill)  
(g, no-spill)  
(h, no-spill)



Why can't we simplify?

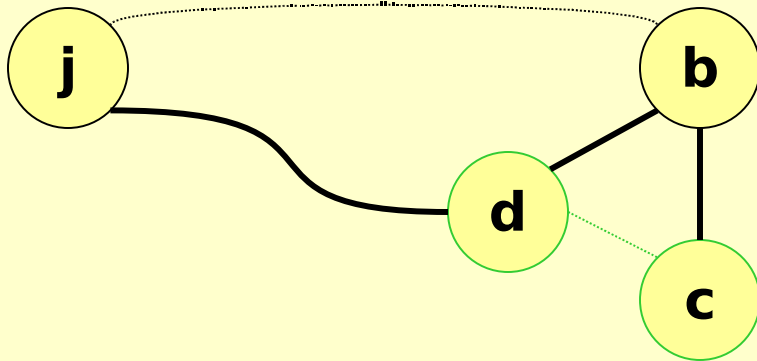
Cannot simplify move-related nodes.



# Example: Coalesce (K=4)

**stack**

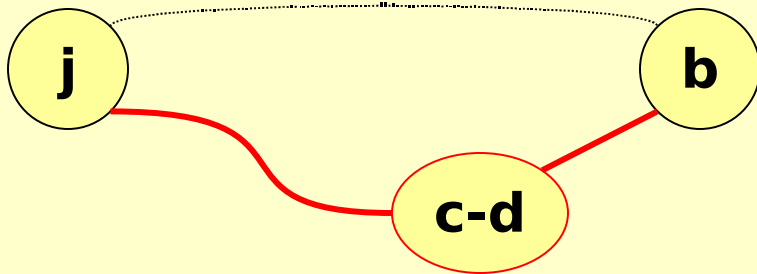
**(m, no-spill)**  
**(e, no-spill)**  
**(f, no-spill)**  
**(k, no-spill)**  
**(g, no-spill)**  
**(h, no-spill)**



# Example: Simplify (K=4)

**stack**

**(c-d, no-spill)**  
**(m, no-spill)**  
**(e, no-spill)**  
**(f, no-spill)**  
**(k, no-spill)**  
**(g, no-spill)**  
**(h, no-spill)**



# Example: Coalesce (K=4)

**stack**

**(c-d, no-spill)**  
**(m, no-spill)**  
**(e, no-spill)**  
**(f, no-spill)**  
**(k, no-spill)**  
**(g, no-spill)**  
**(h, no-spill)**



# Example:

## Simplify (K=4) greedy-4-colorable

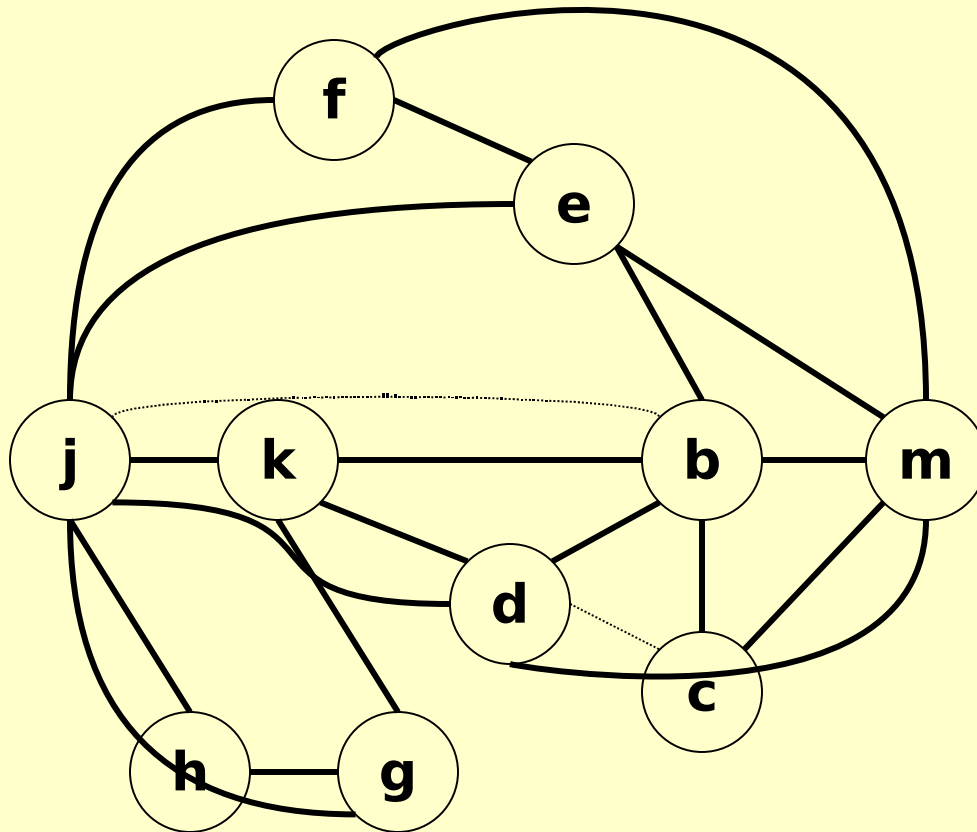
**stack**

**(b-j, no-spill)**  
**(c-d, no-spill)**  
**(m, no-spill)**  
**(e, no-spill)**  
**(f, no-spill)**  
**(k, no-spill)**  
**(g, no-spill)**  
**(h, no-spill)**

**b-j**



# Example: Select (K=4)



stack

**(b-j, no-spill)**  
**(c-d, no-spill)**  
**(m, no-spill)**  
**(e, no-spill)**  
**(f, no-spill)**  
**(k, no-spill)**  
**(g, no-spill)**  
**(h, no-spill)**

R1

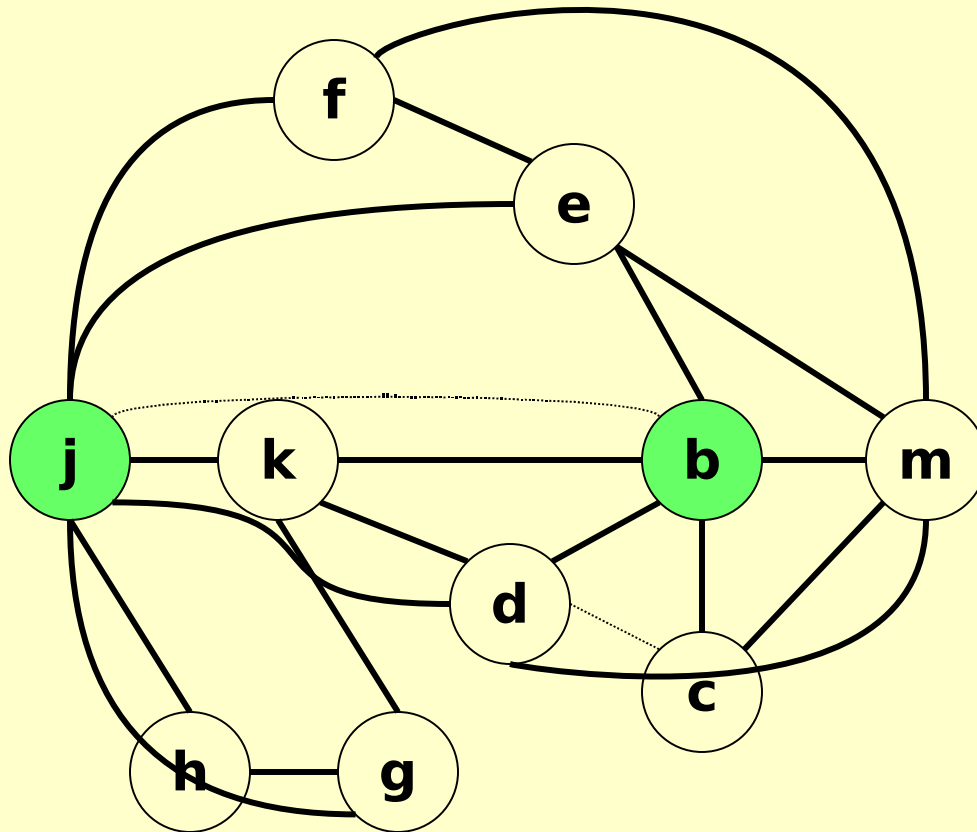
R2

R3

R4



# Example: Select (K=4)



stack

**(b-j, no-spill)**  
**(c-d, no-spill)**  
**(m, no-spill)**  
**(e, no-spill)**  
**(f, no-spill)**  
**(k, no-spill)**  
**(g, no-spill)**  
**(h, no-spill)**

**R1**

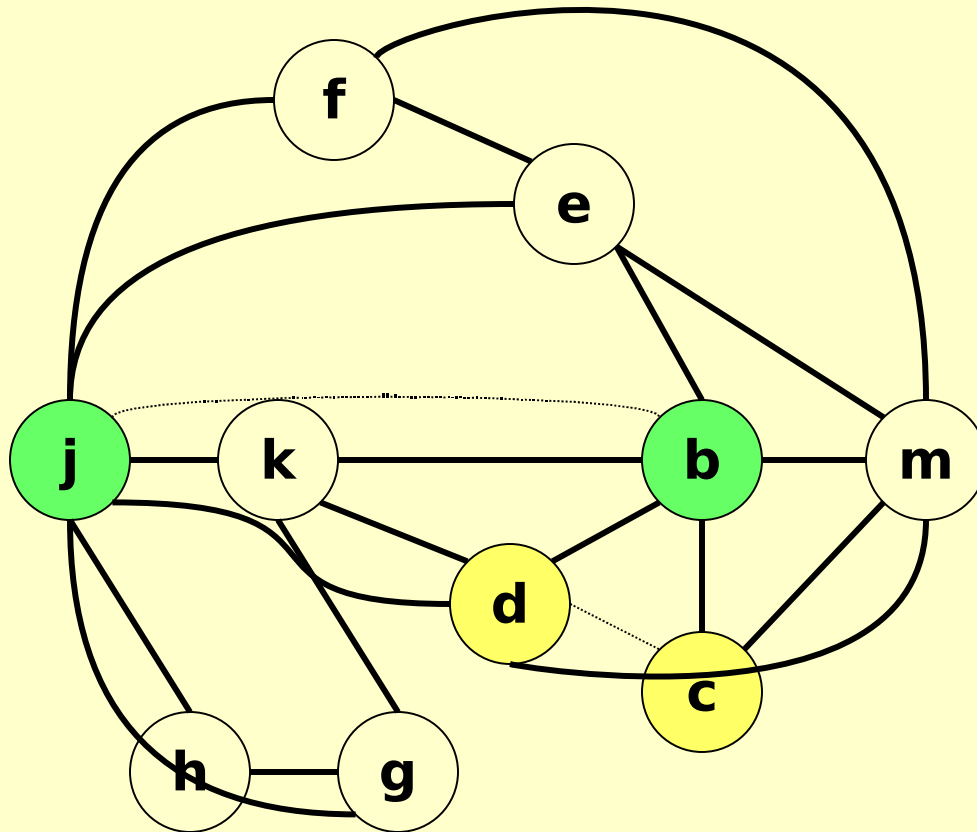
**R2**

**R3**

**R4**



# Example: Select (K=4)



stack

(b-j, no-spill)  
**(c-d, no-spill)**  
(m, no-spill)  
(e, no-spill)  
(f, no-spill)  
(k, no-spill)  
(g, no-spill)  
(h, no-spill)

R1

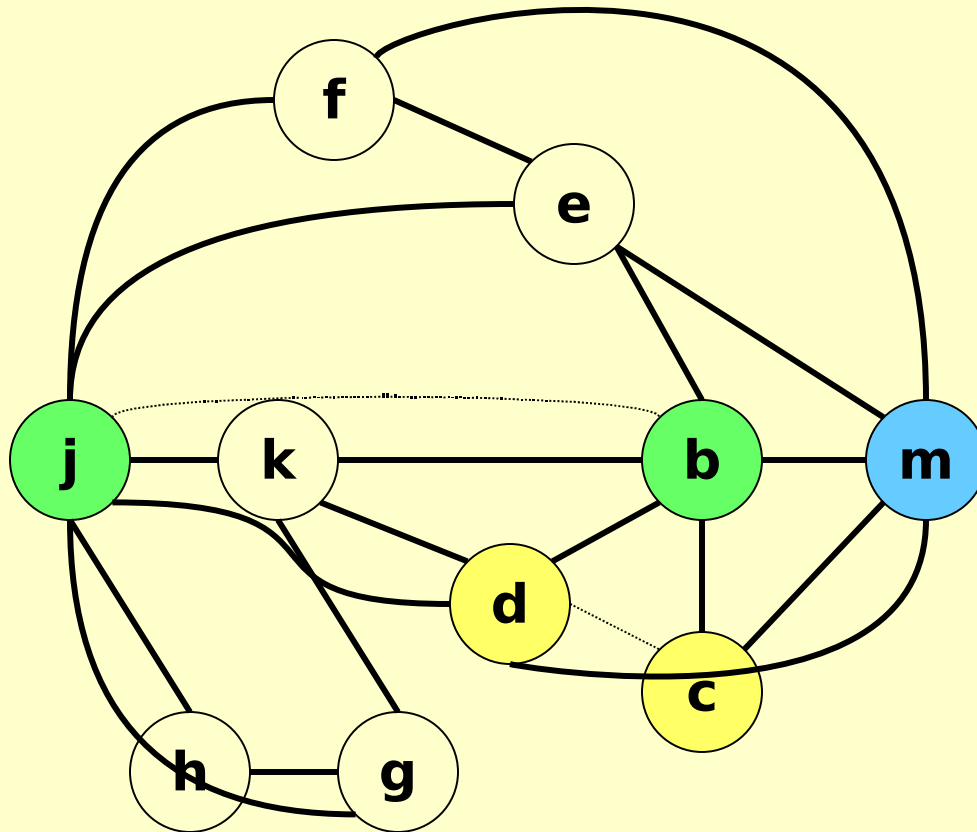
R2

R3

R4



# Example: Select (K=4)



stack

(b-j, no-spill)  
(c-d, no-spill)  
**(m, no-spill)**  
(e, no-spill)  
(f, no-spill)  
(k, no-spill)  
(g, no-spill)  
(h, no-spill)

R1

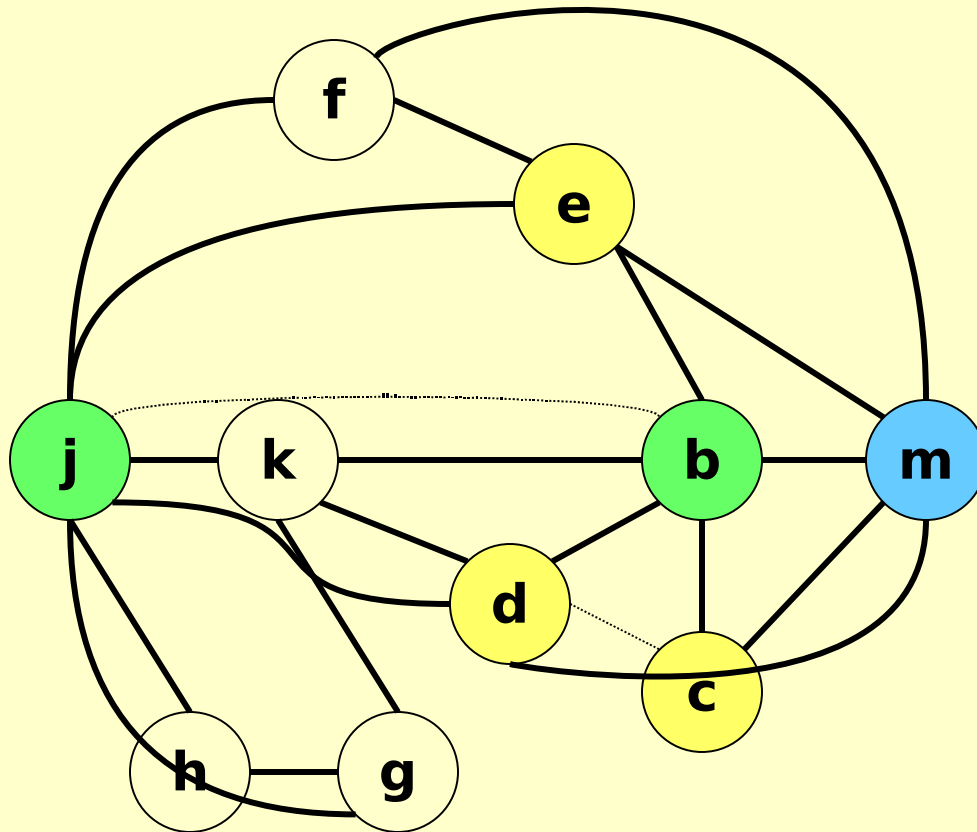
R2

R3

R4



# Example: Select (K=4)



stack

(b-j, no-spill)  
(c-d, no-spill)  
(m, no-spill)  
**(e, no-spill)**  
(f, no-spill)  
(k, no-spill)  
(g, no-spill)  
(h, no-spill)

R1

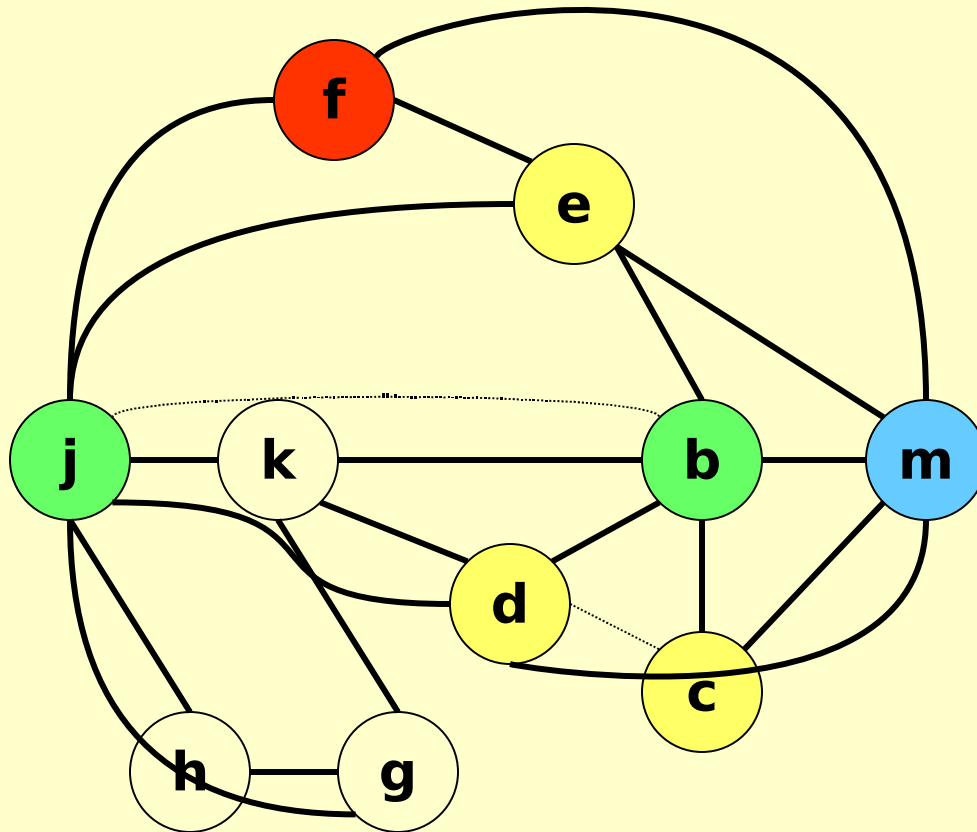
R2

R3

R4



# Example: Select (K=4)



stack

(b-j, no-spill)  
(c-d, no-spill)  
(m, no-spill)  
(e, no-spill)  
**(f, no-spill)**  
(k, no-spill)  
(g, no-spill)  
(h, no-spill)

R1

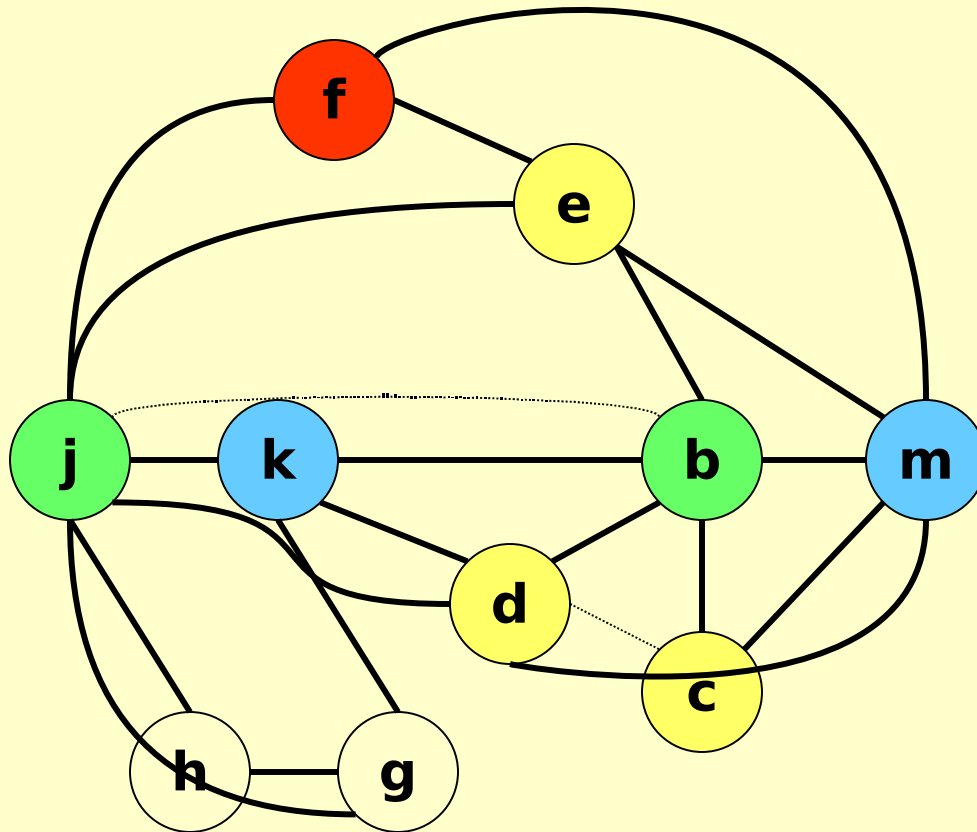
R2

R3

R4



# Example: Select (K=4)



stack

(b-j, no-spill)  
(c-d, no-spill)  
(m, no-spill)  
(e, no-spill)  
(f, no-spill)  
**(k, no-spill)**  
(g, no-spill)  
(h, no-spill)

R1

R2

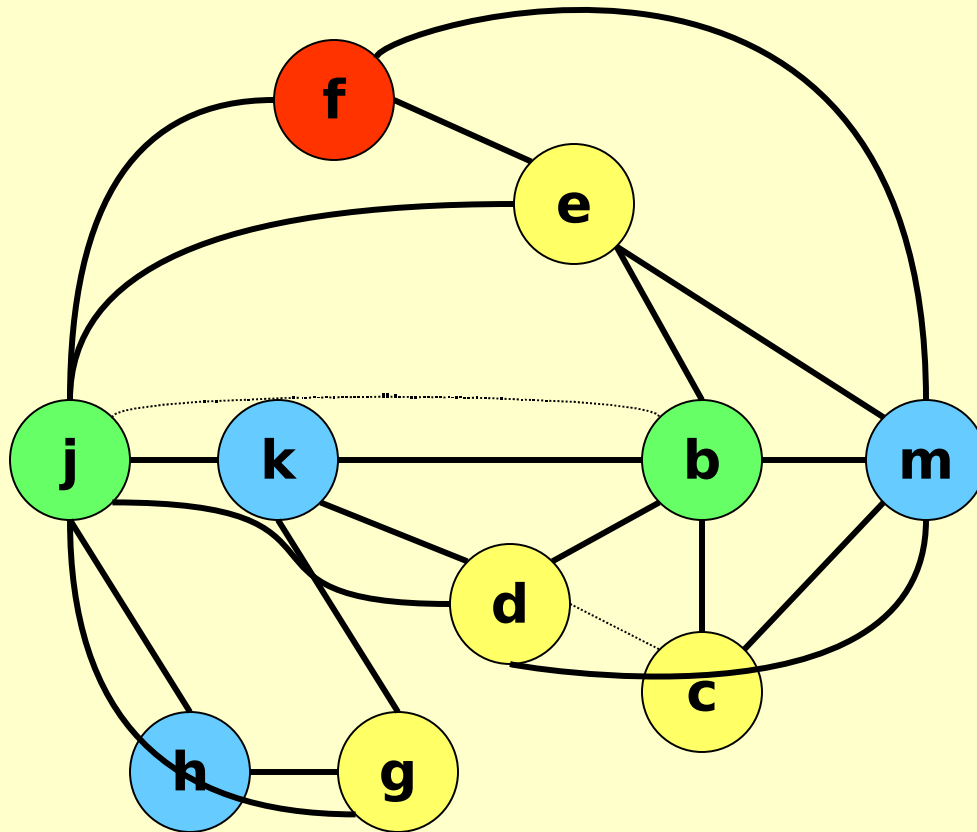
R3

R4





# Example: Select (K=4)



stack

(b-j, no-spill)  
(c-d, no-spill)  
(m, no-spill)  
(e, no-spill)  
(f, no-spill)  
(k, no-spill)  
(g, no-spill)  
**(h, no-spill)**

R1

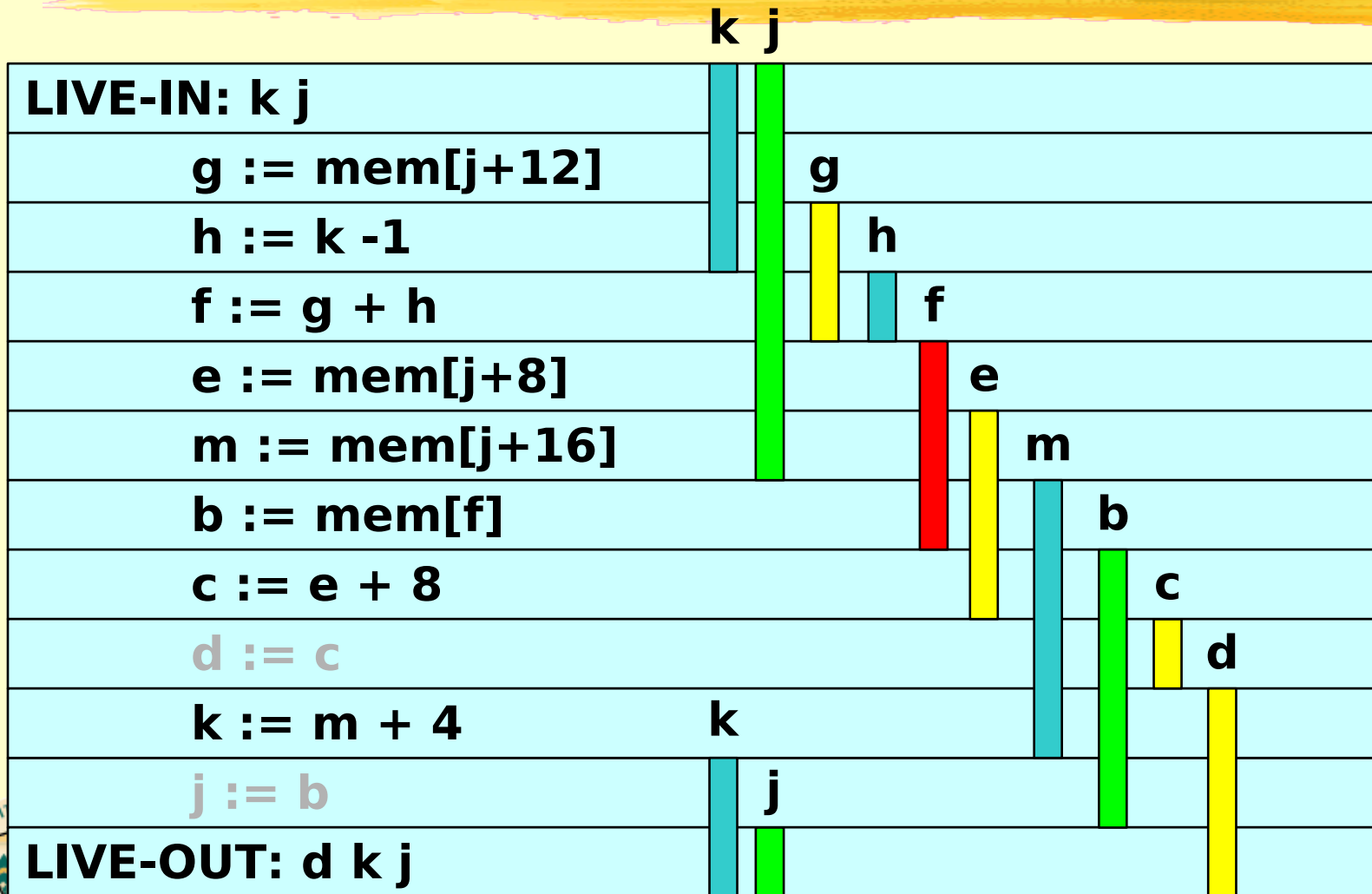
R2

R3

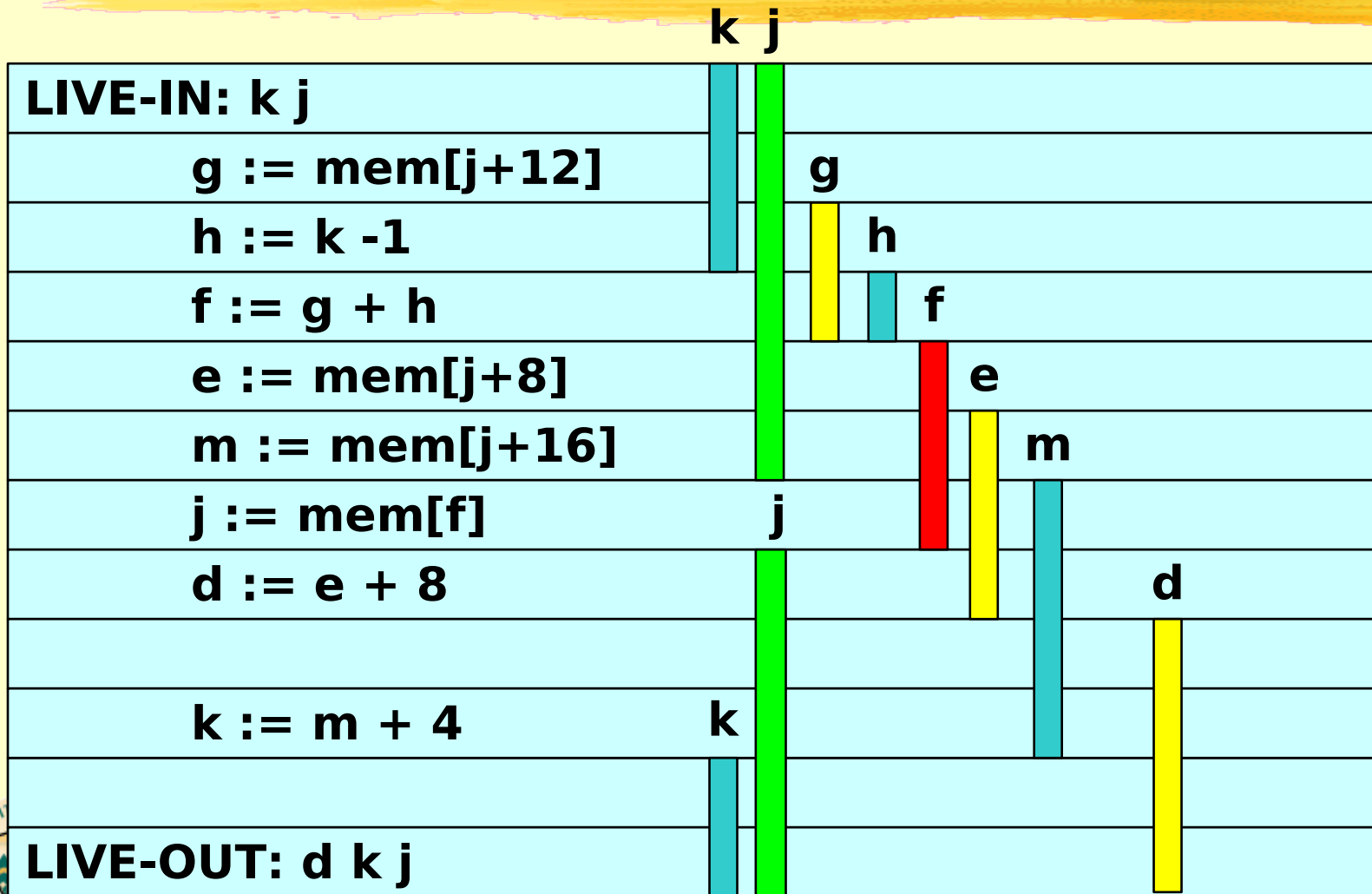
R4




# Example: Allocation with 4 registers



# Example: Allocation with 4 registers

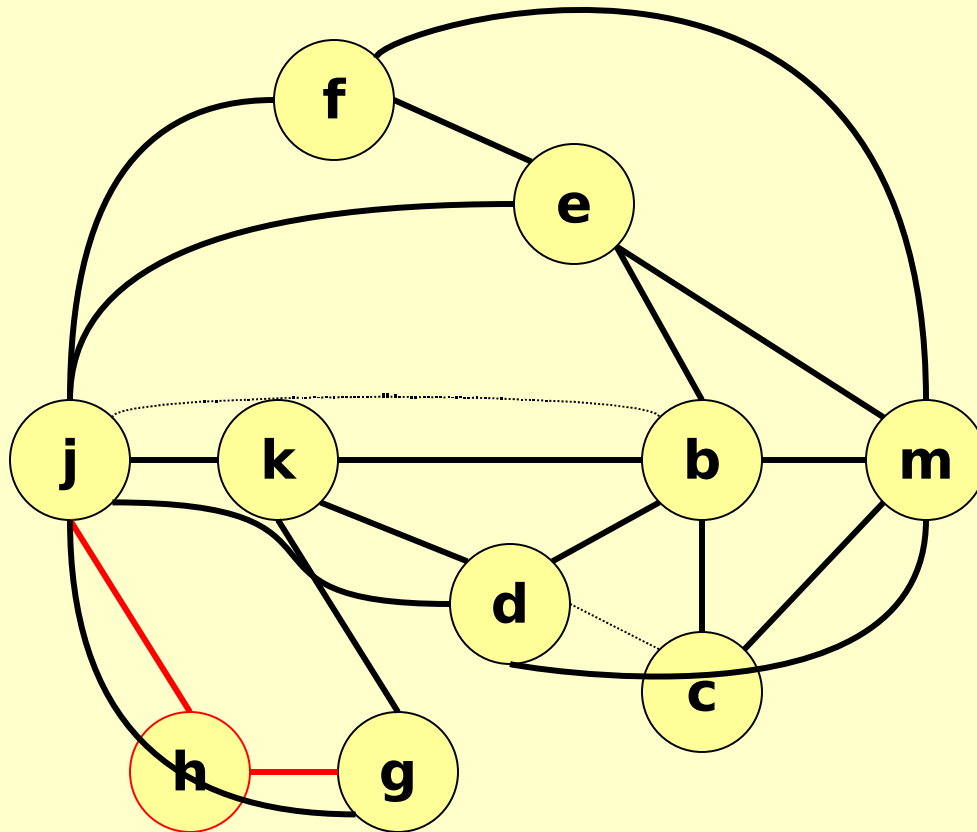




Could we do the allocation in  
the previous example with 3  
registers?



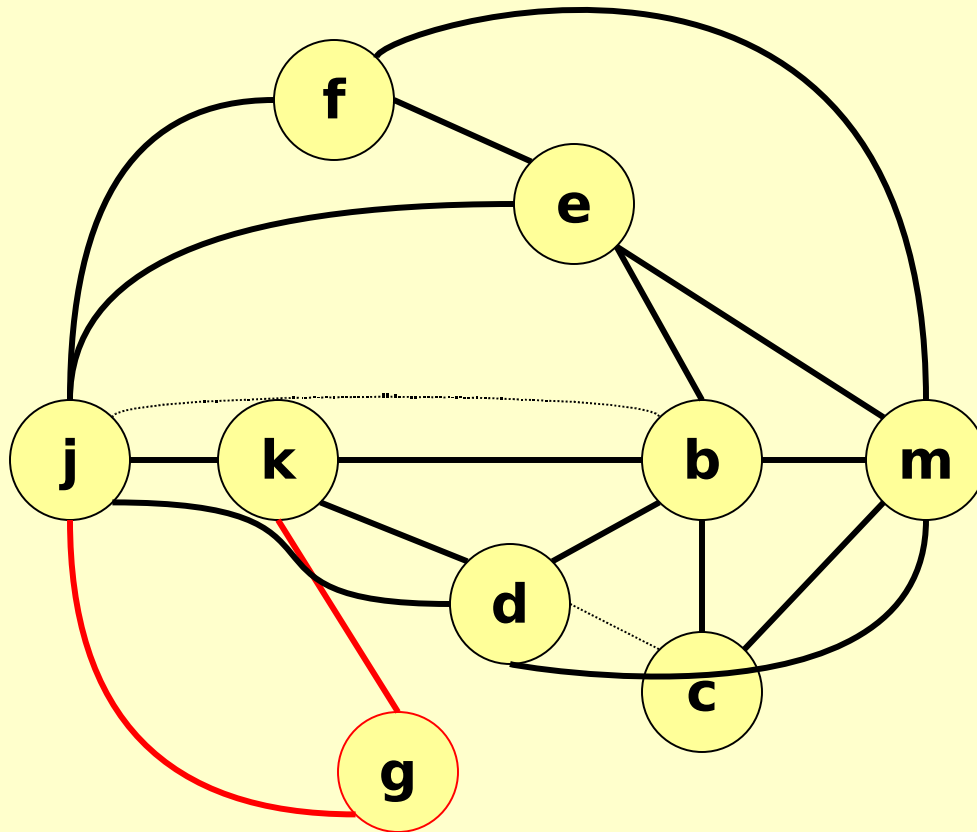
# Example: Simplify (K=3)



**stack**  
**(h,no-spill)**



# Example: Simplify (K=3)

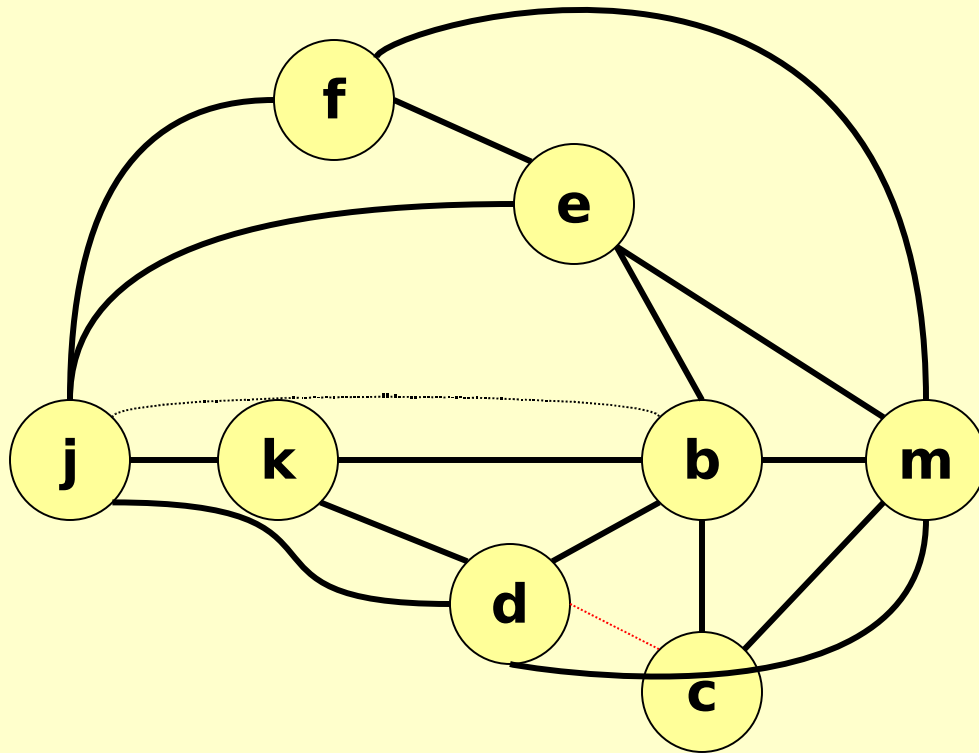


**stack**

**(g, no-spill)**  
**(h, no-spill)**



# Example: Freeze (K=3)



**stack**

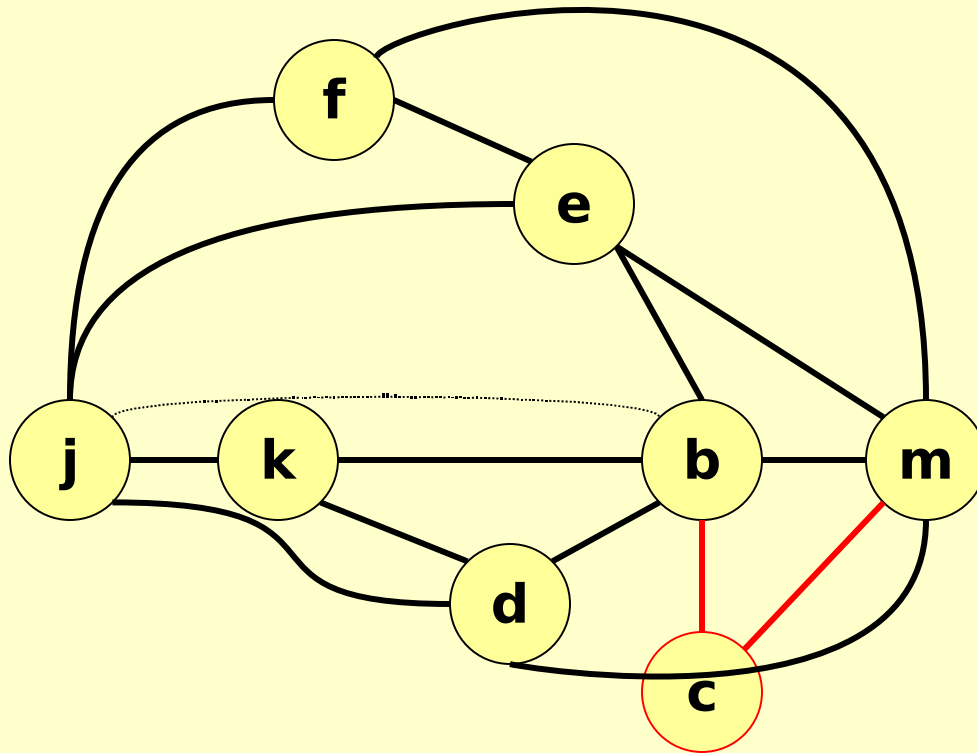
(g, no-spill)  
(h, no-spill)

Coalescing may make things worse (not always).

George's rule would coalesce the move d-c, Briggs' rule would freeze.



# Example: Simplify (K=3)

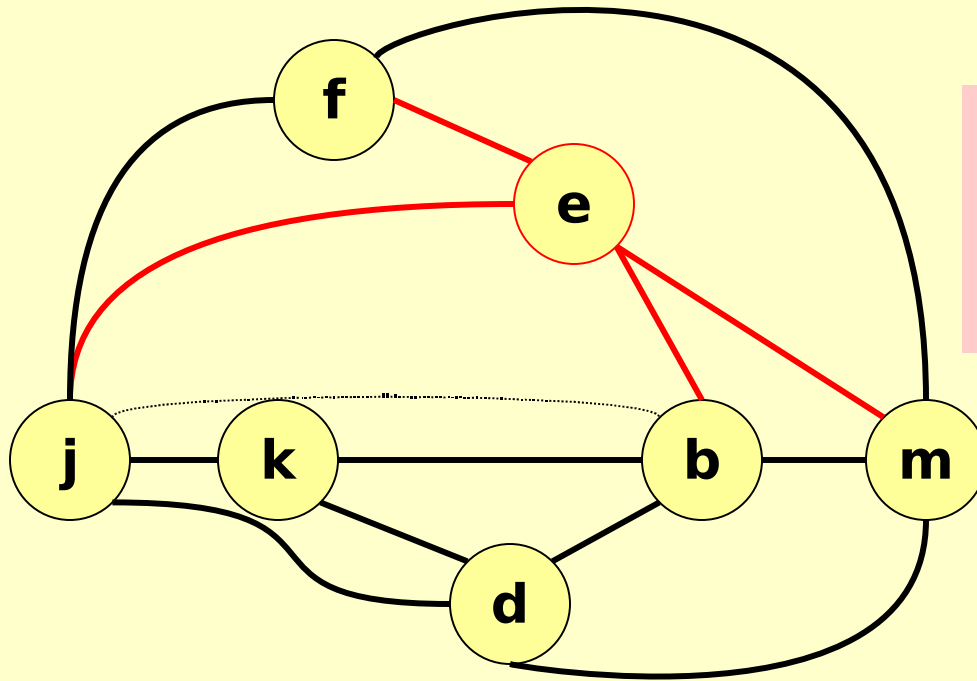


**stack**

**(c, no-spill)**  
**(g, no-spill)**  
**(h, no-spill)**



# Example: Potential Spill (K=3)



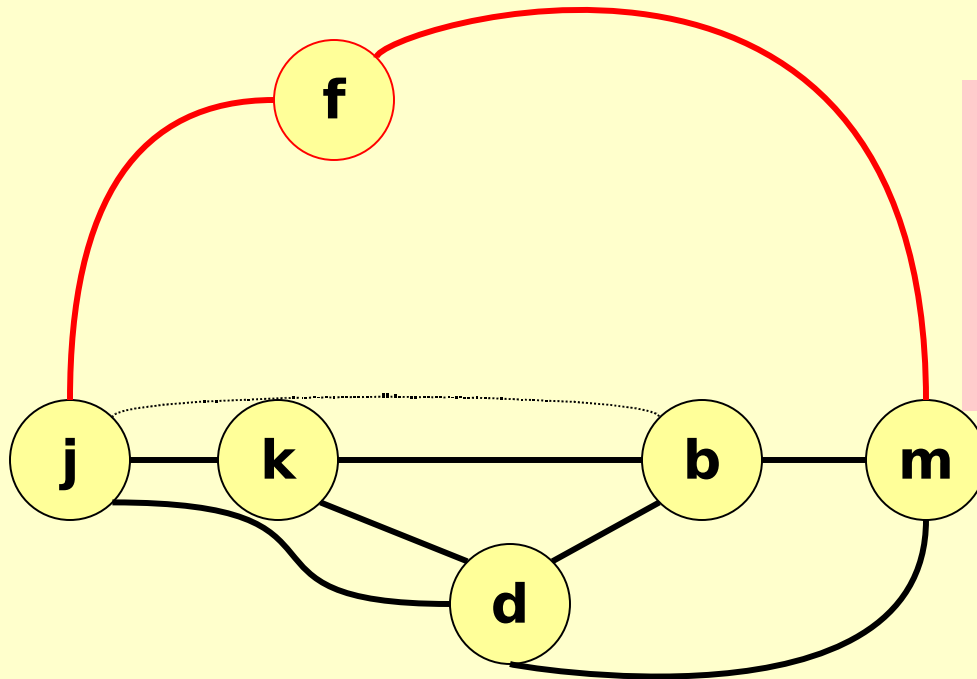
**stack**

(e, may-spill)  
(c, no-spill)  
(g, no-spill)  
(h, no-spill)

Neither coalescing nor freezing help us. At this point we should use some profitability analysis to choose a node as *may-spill*.



# Example: Simplify (K=3)

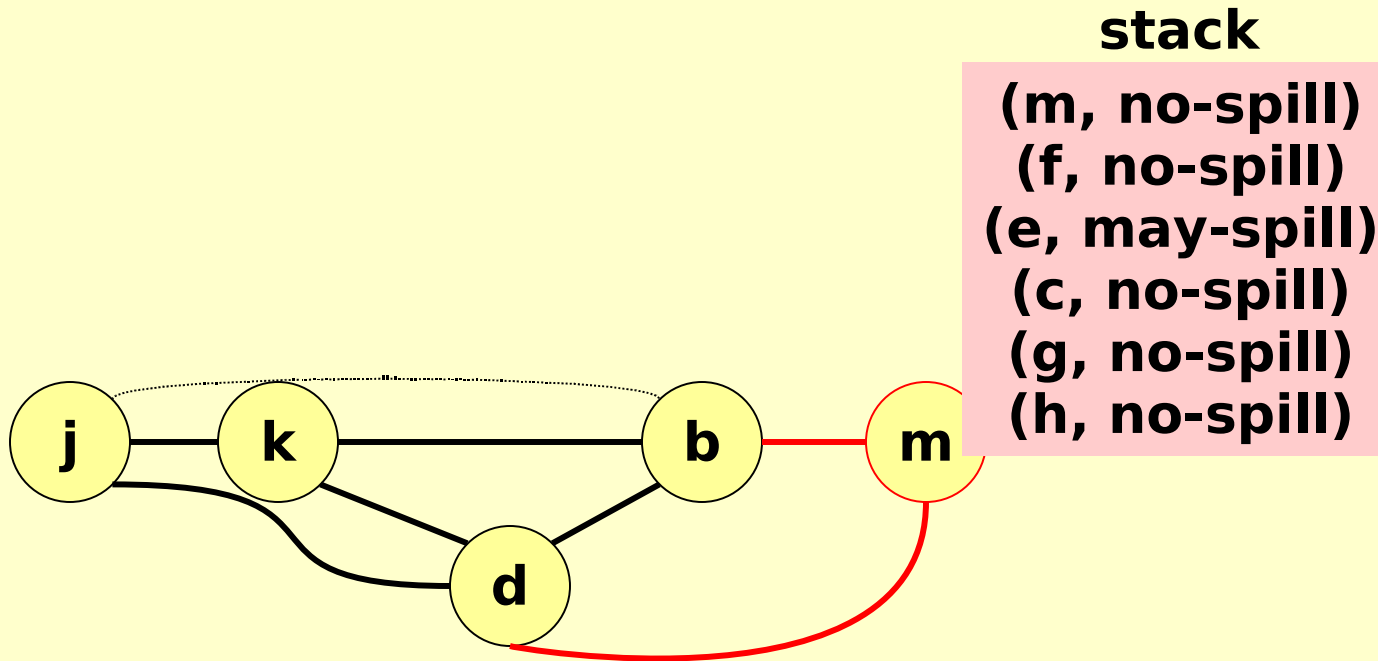


**stack**

**(f, no-spill)**  
**(e, may-spill)**  
**(c, no-spill)**  
**(g, no-spill)**  
**(h, no-spill)**



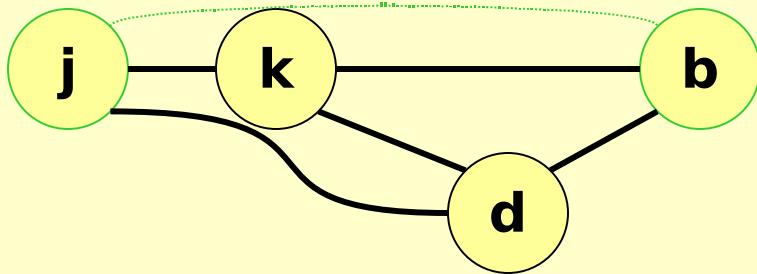
# Example: Simplify (K=3)



# Example: Coalesce (K=3)

**stack**

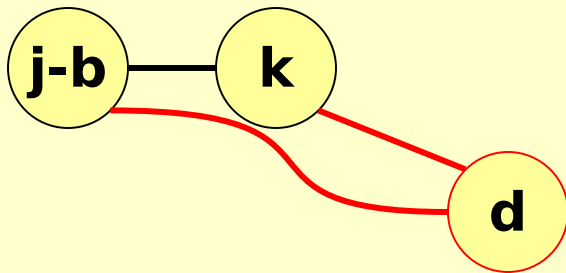
(m, no-spill)  
(f, no-spill)  
(e, may-spill)  
(c, no-spill)  
(g, no-spill)  
(h, no-spill)



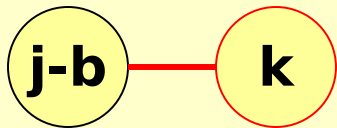
# Example: Coalesce (K=3)

**stack**

**(d, no-spill)**  
**(m, no-spill)**  
**(f, no-spill)**  
**(e, may-spill)**  
**(c, no-spill)**  
**(g, no-spill)**  
**(h, no-spill)**



# Example: Coalesce (K=3)



**stack**

**(k, no-spill)**  
**(d, no-spill)**  
**(m, no-spill)**  
**(f, no-spill)**  
**(e, may-spill)**  
**(c, no-spill)**  
**(g, no-spill)**  
**(h, no-spill)**



# Example: Coalesce (K=3)

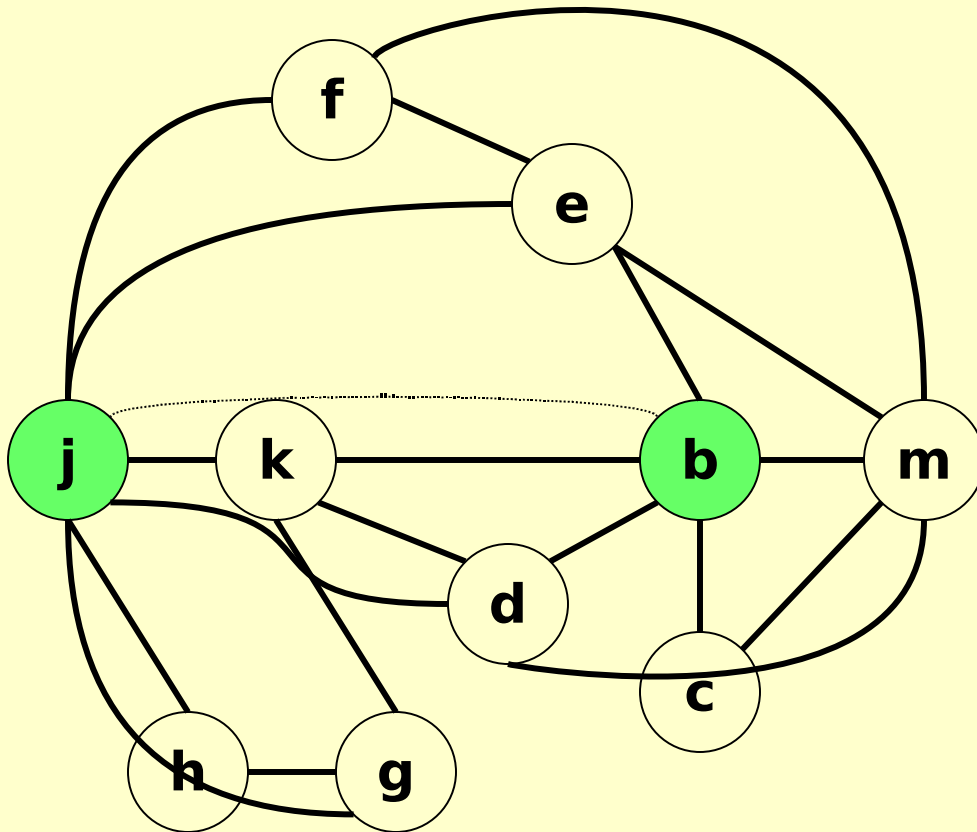
**stack**

**(j-b, no-spill)**  
**(k, no-spill)**  
**(d, no-spill)**  
**(m, no-spill)**  
**(f, no-spill)**  
**(e, may-spill)**  
**(c, no-spill)**  
**(g, no-spill)**  
**(h, no-spill)**

**j-b**



# Example: Select (K=3)



stack

**(j-b, no-spill)**

**(k, no-spill)**

**(d, no-spill)**

**(m, no-spill)**

**(f, no-spill)**

**(e, may-spill)**

**(c, no-spill)**

**(g, no-spill)**

**(h, no-spill)**

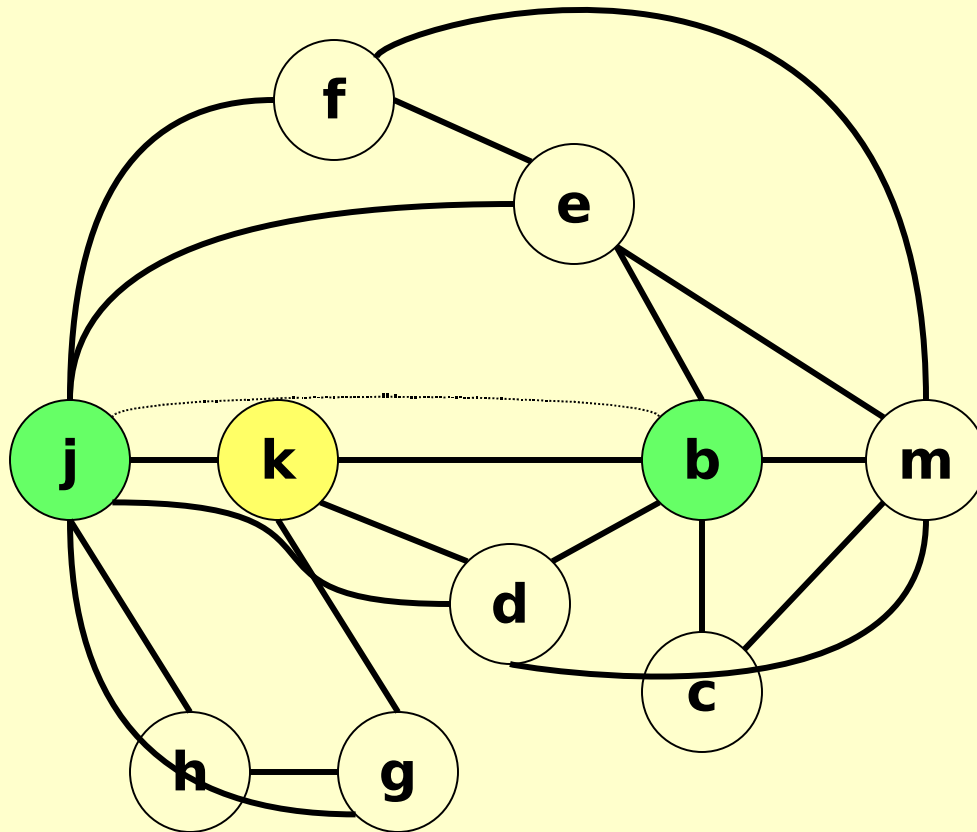
**R1**

**R2**

**R3**



# Example: Select (K=3)



stack

(j-b, no-spill)  
**(k, no-spill)**  
(d, no-spill)  
(m, no-spill)  
(f, no-spill)  
(e, may-spill)  
(c, no-spill)  
(g, no-spill)  
(h, no-spill)

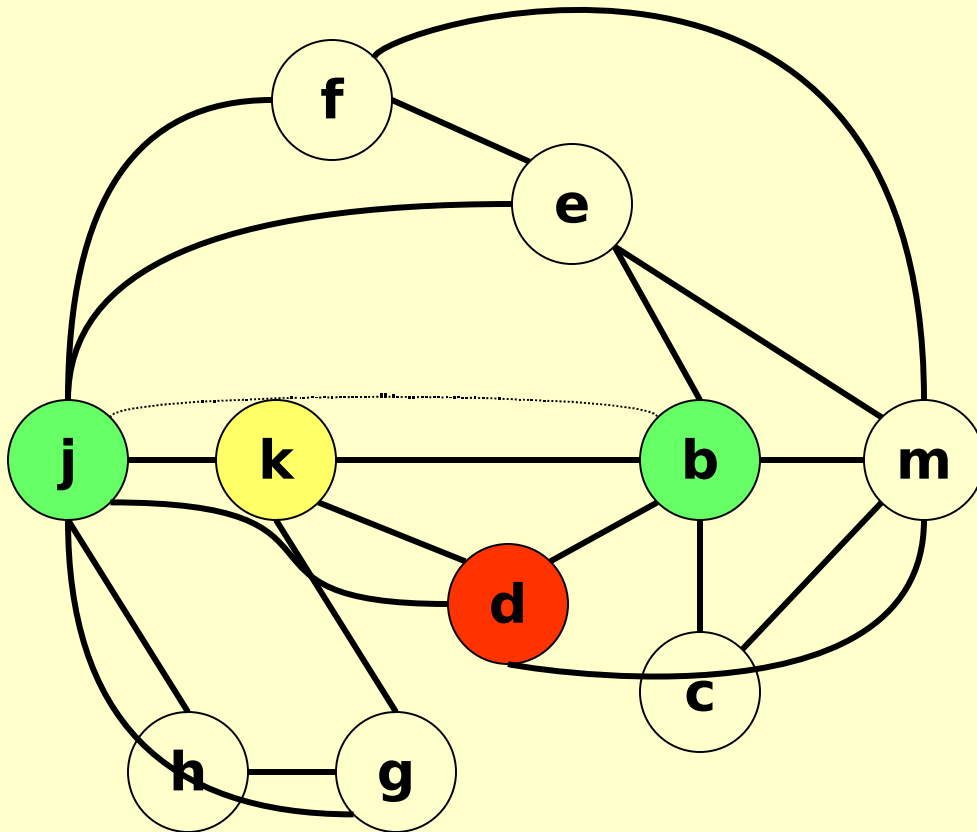
R1

R2

R3



# Example: Select (K=3)



stack

(j-b, no-spill)  
(k, no-spill)  
**(d, no-spill)**  
(m, no-spill)  
(f, no-spill)  
(e, may-spill)  
(c, no-spill)  
(g, no-spill)  
(h, no-spill)

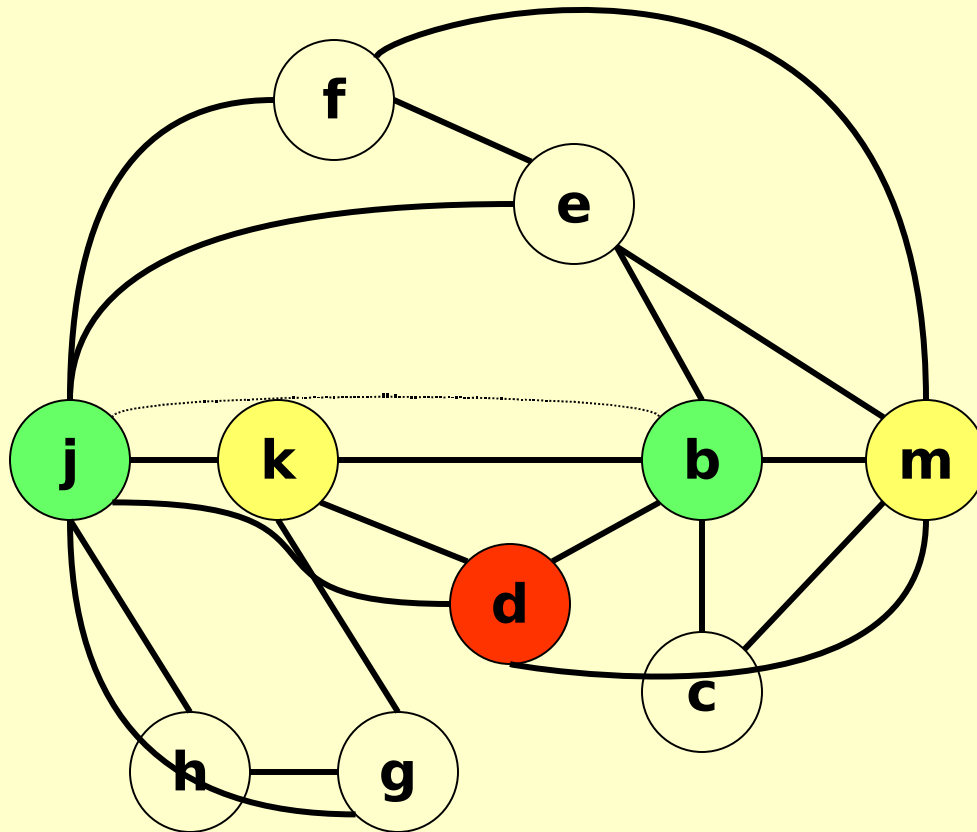
R1

R2

R3



# Example: Select (K=3)



stack

(j-b, no-spill)  
(k, no-spill)  
(d, no-spill)  
**(m, no-spill)**  
(f, no-spill)  
(e, may-spill)  
(c, no-spill)  
(g, no-spill)  
(h, no-spill)

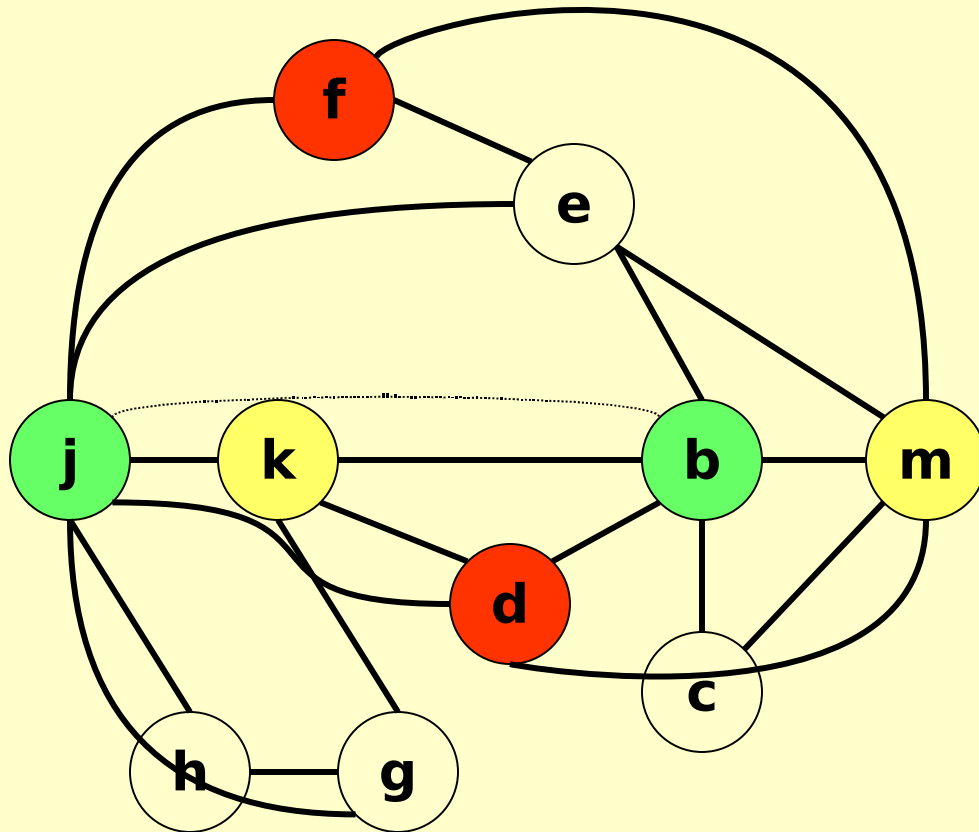
R1

R2

R3



# Example: Select (K=3)



stack

(j-b, no-spill)  
(k, no-spill)  
(d, no-spill)  
(m, no-spill)  
**(f, no-spill)**  
(e, may-spill)  
(c, no-spill)  
(g, no-spill)  
(h, no-spill)

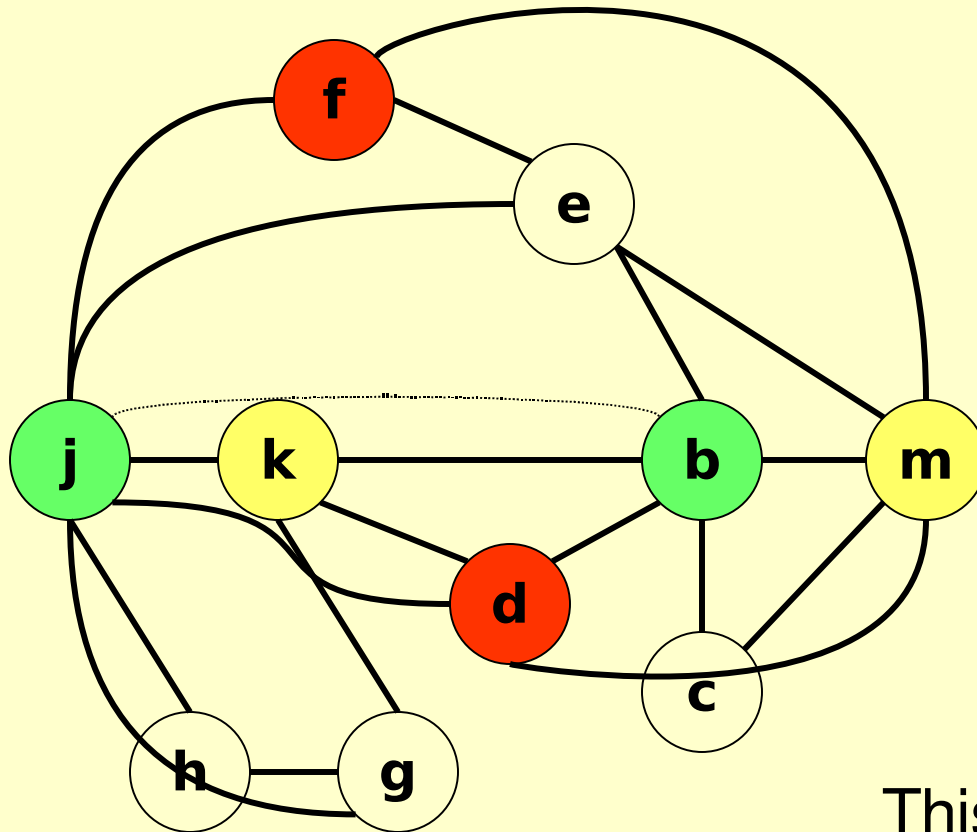
R1

R2

R3



# Example: Select (K=3)



stack

(j-b, no-spill)  
(k, no-spill)  
(d, no-spill)  
(m, no-spill)  
(f, no-spill)  
**(e, may-spill)**  
(c, no-spill)  
(g, no-spill)  
(h, no-spill)

R1

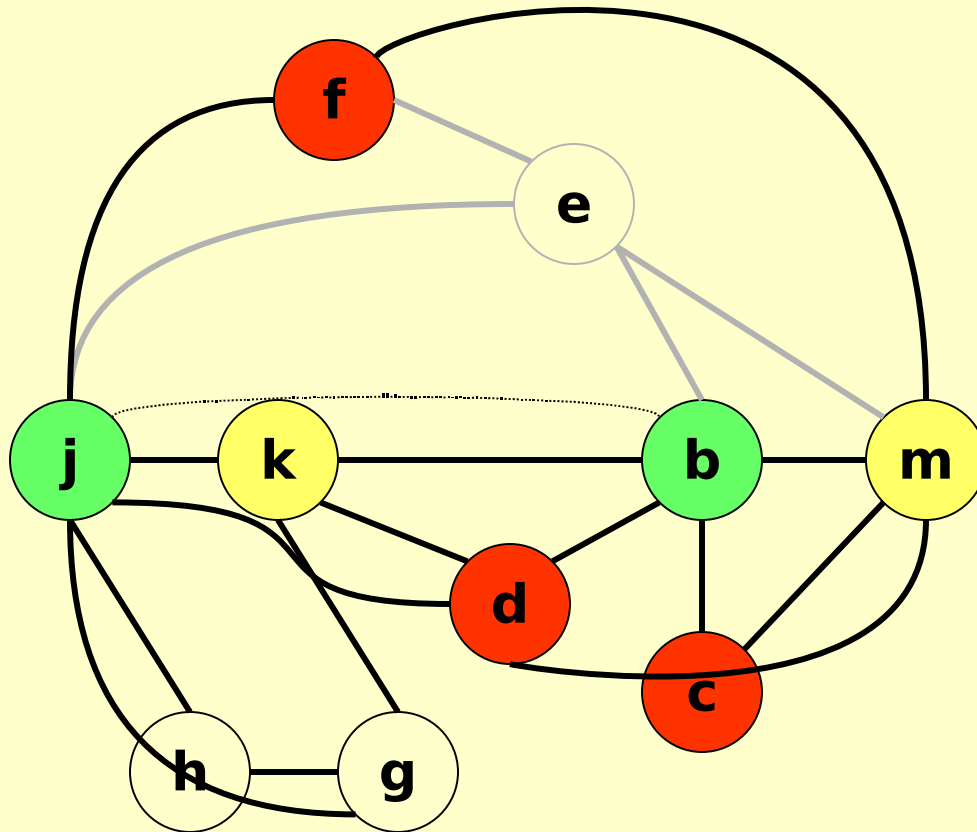
R2

R3

This is when our optimism could  
have paid off.



# Example: Select (K=3)



stack

(j-b, no-spill)  
(k, no-spill)  
(d, no-spill)  
(m, no-spill)  
(f, no-spill)  
(e, may-spill)  
**(c, no-spill)**  
(g, no-spill)  
(h, no-spill)

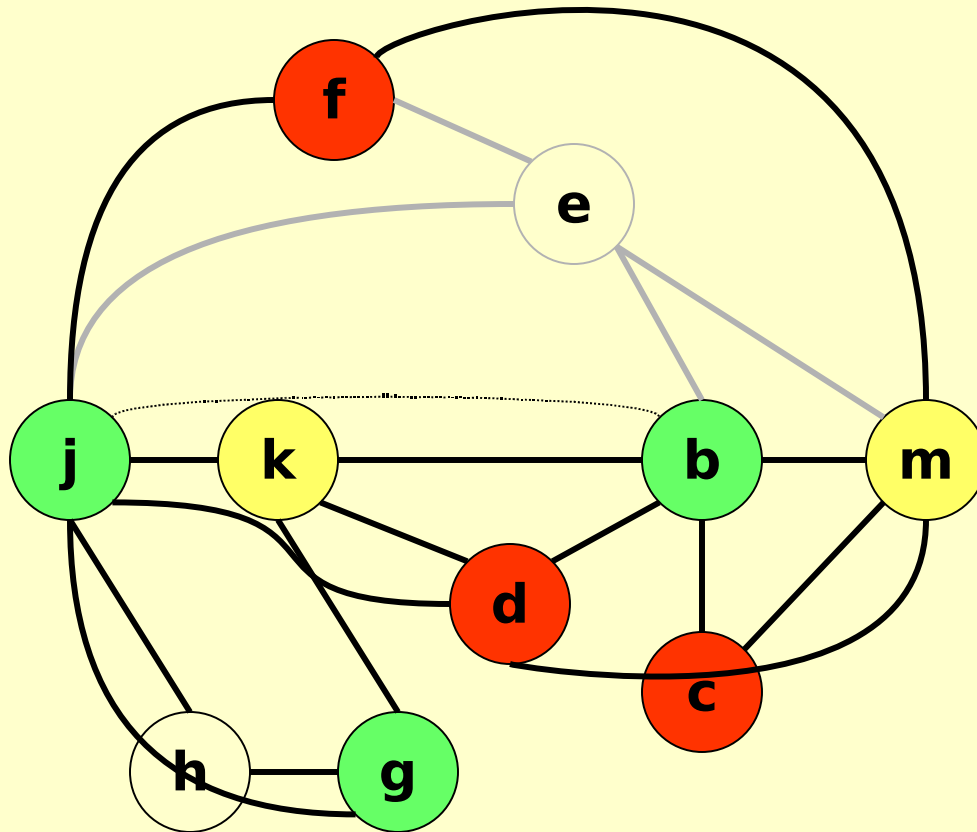
R1

R2

R3



# Example: Select (K=3)



stack

(j-b, no-spill)  
(k, no-spill)  
(d, no-spill)  
(m, no-spill)  
(f, no-spill)  
(e, may-spill)  
(c, no-spill)  
**(g, no-spill)**  
(h, no-spill)

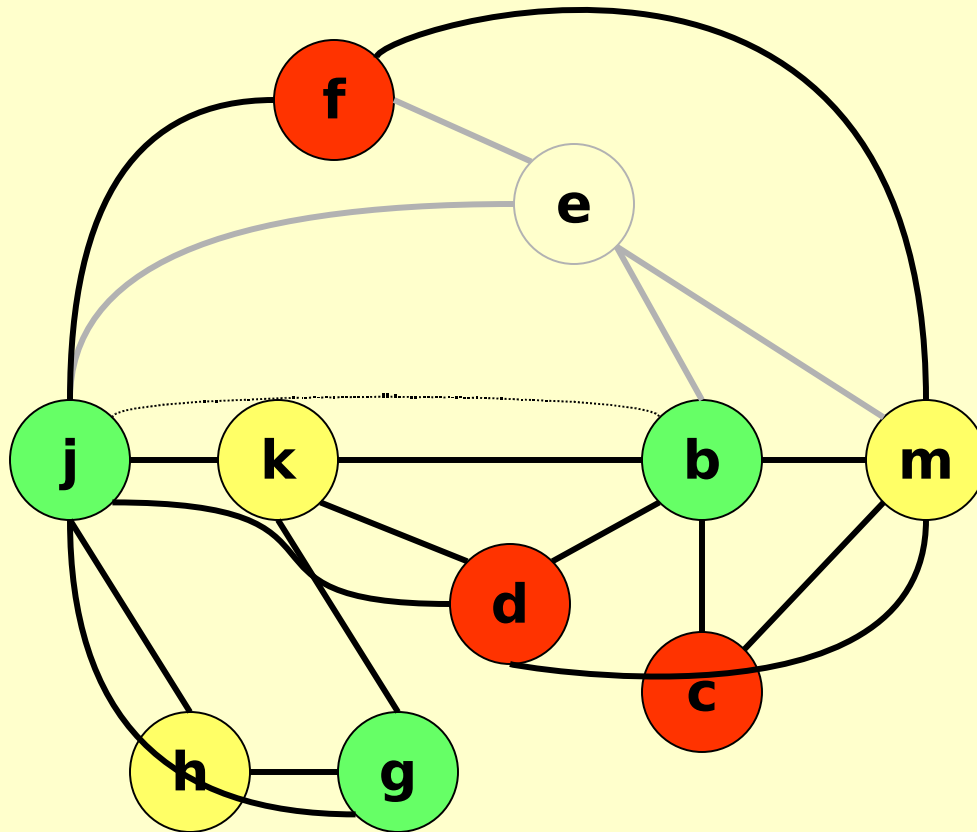
R1

R2

R3



# Example: Select (K=3)



stack

(j-b, no-spill)  
(k, no-spill)  
(d, no-spill)  
(m, no-spill)  
(f, no-spill)  
(e, may-spill)  
(c, no-spill)  
(g, no-spill)  
(h, no-spill)

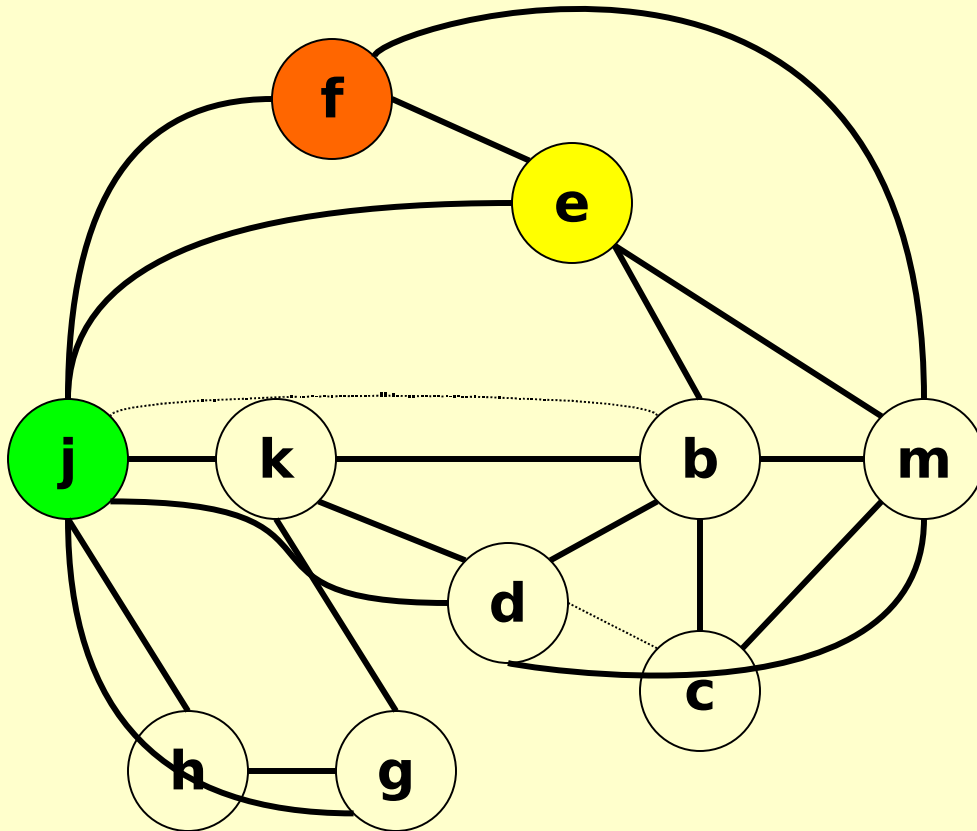
R1

R2

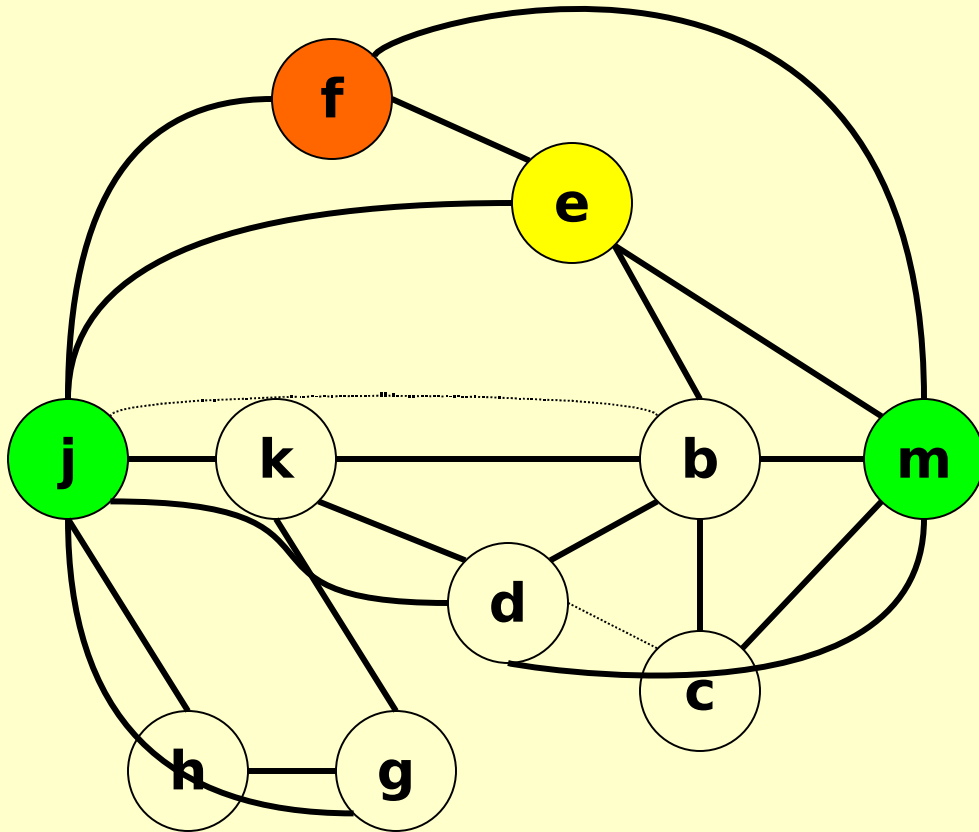
R3



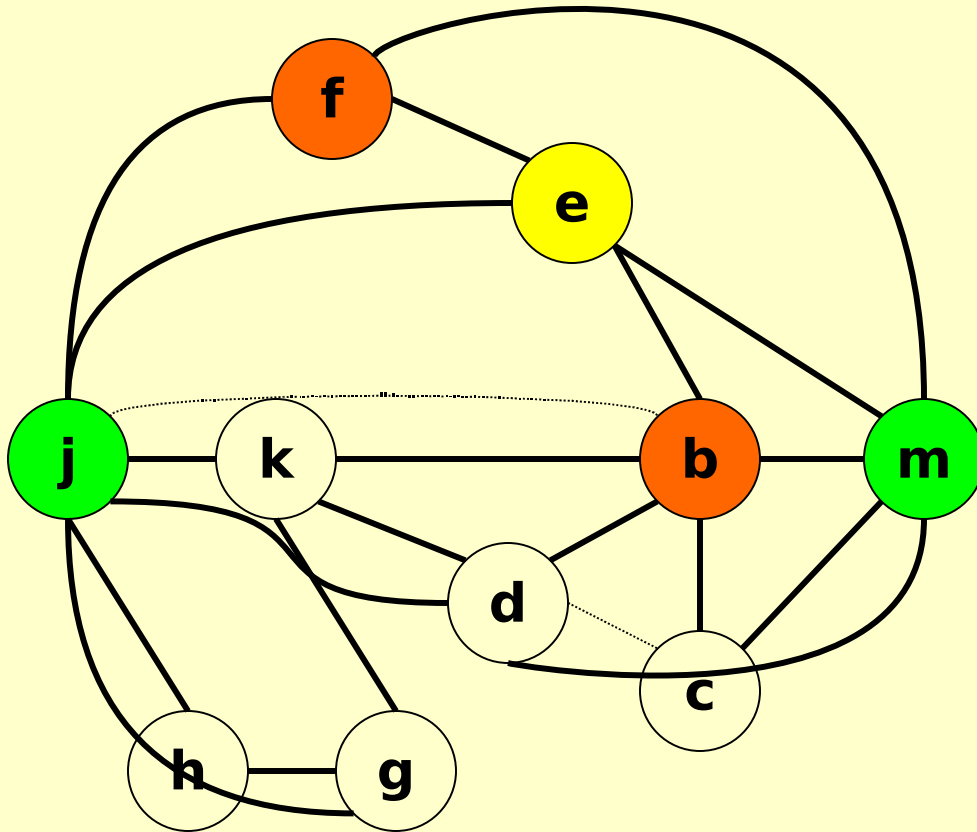
# So, is it possible for $K=3$ ?



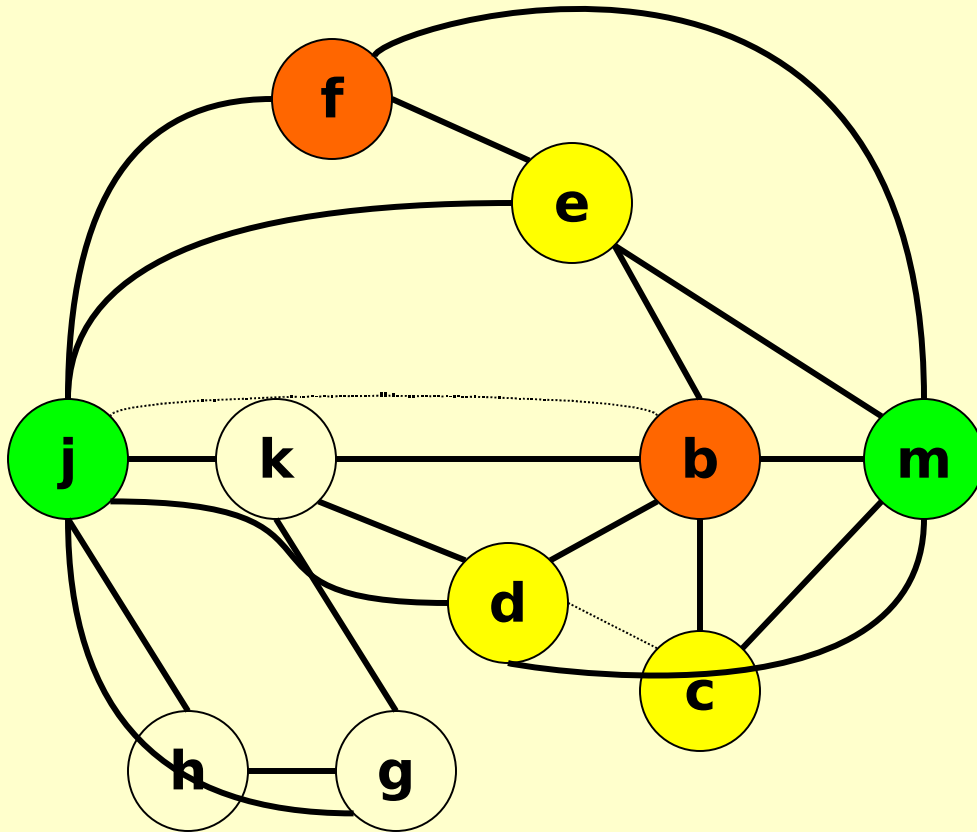
# Example: Simplify (K=3)



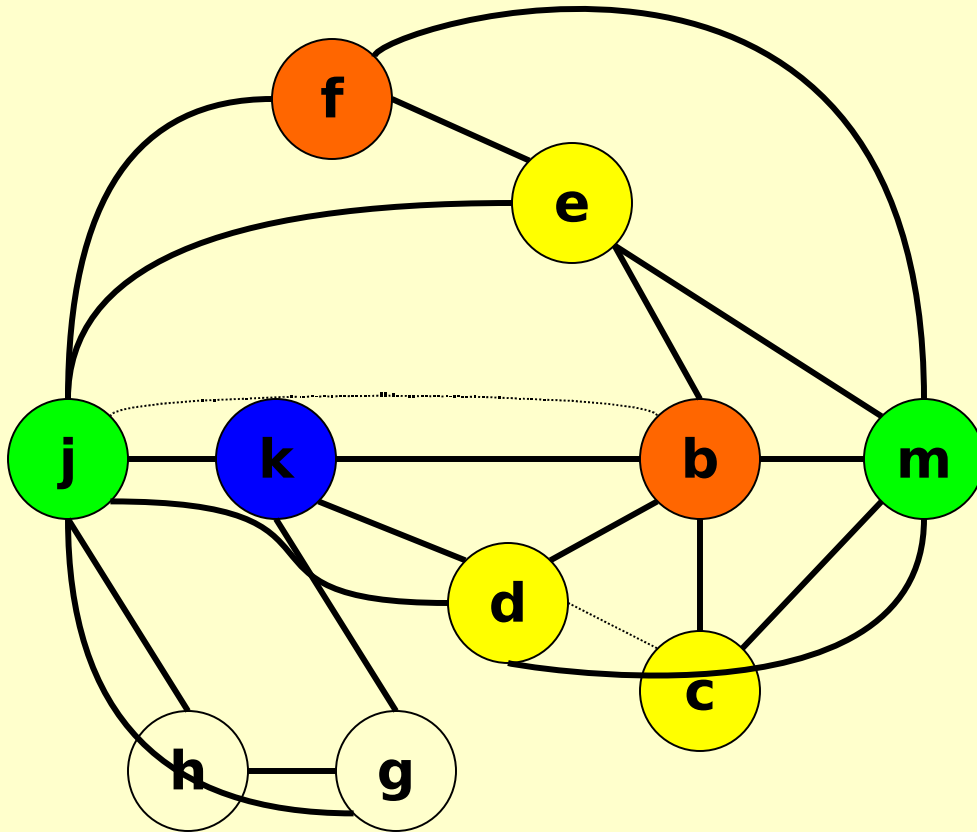
# Example: Simplify (K=3)



# Example: Simplify (K=3)



# Example: Simplify (K=3)

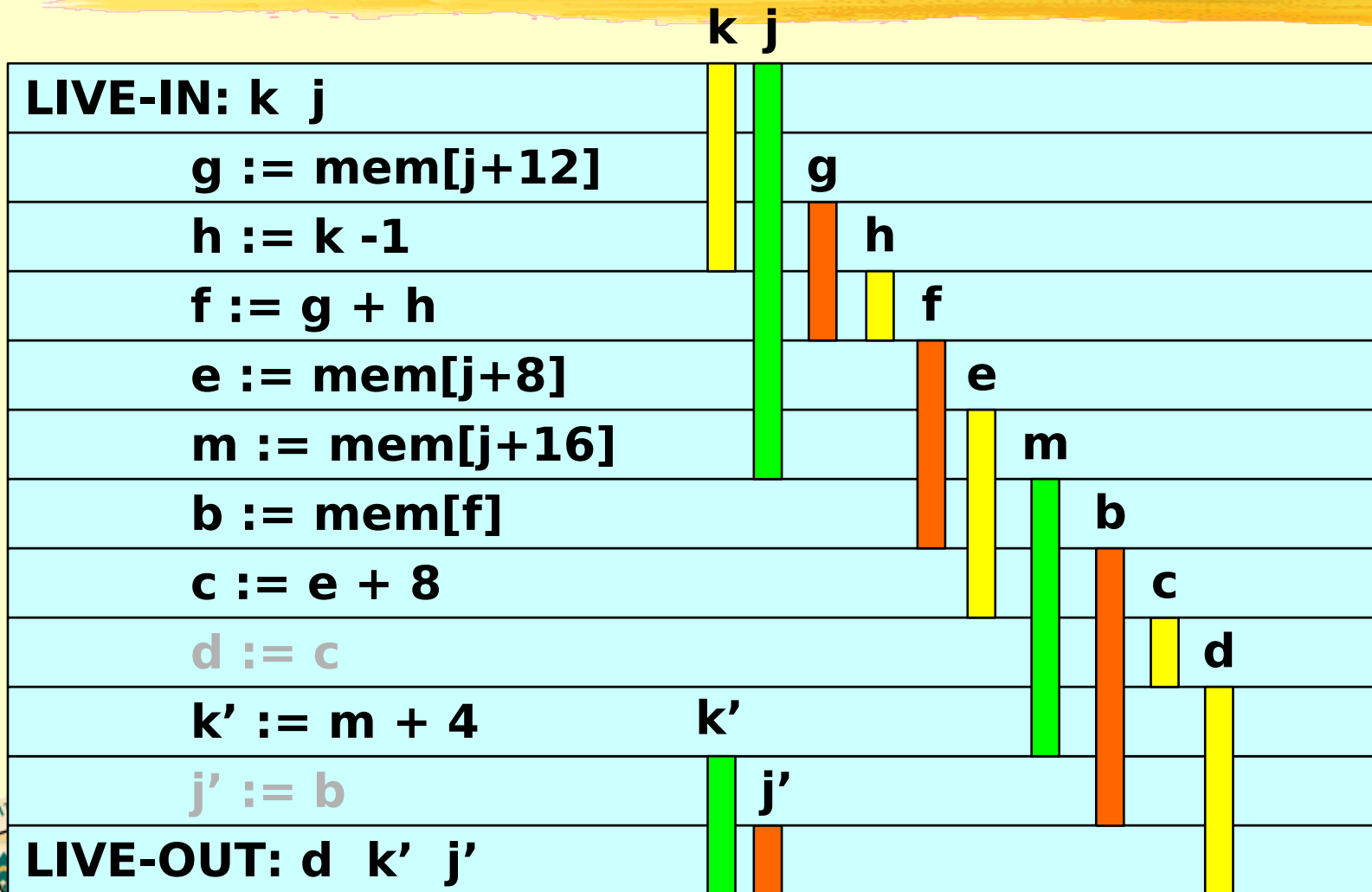


**Impossible!**

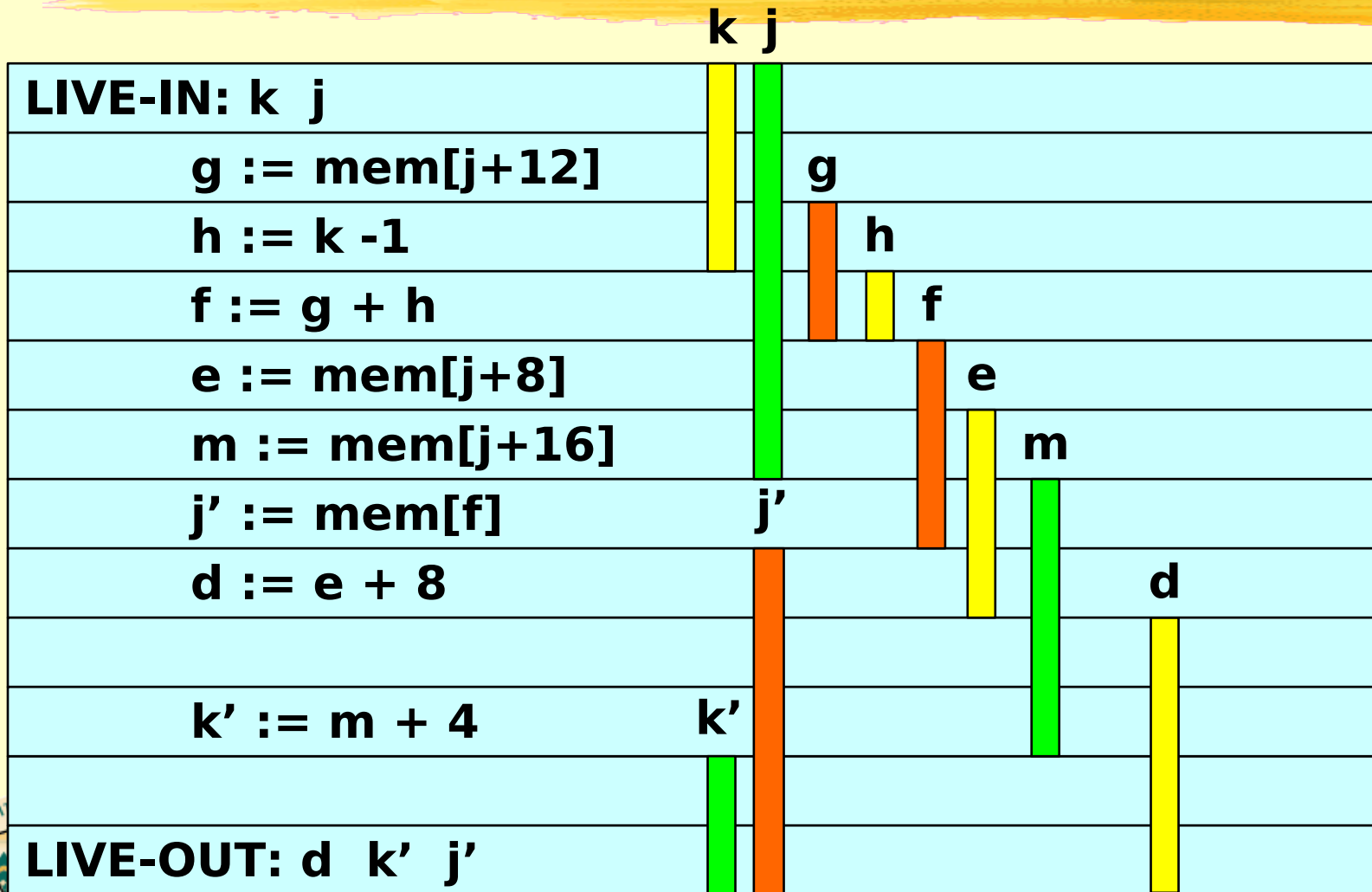
But only 3 variables  
are live at any time...  
there may be a way?



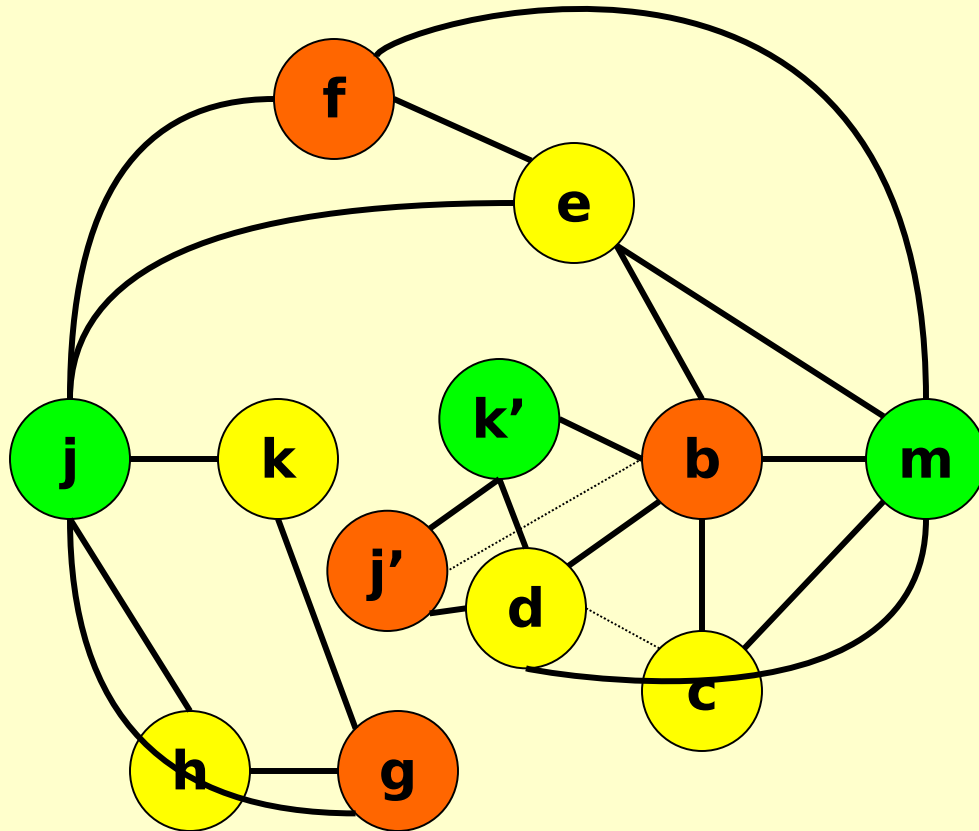
# Example as basic block: 3 Registers by renaming k & j



# Example as basic block: 3 Registers by renaming k & j



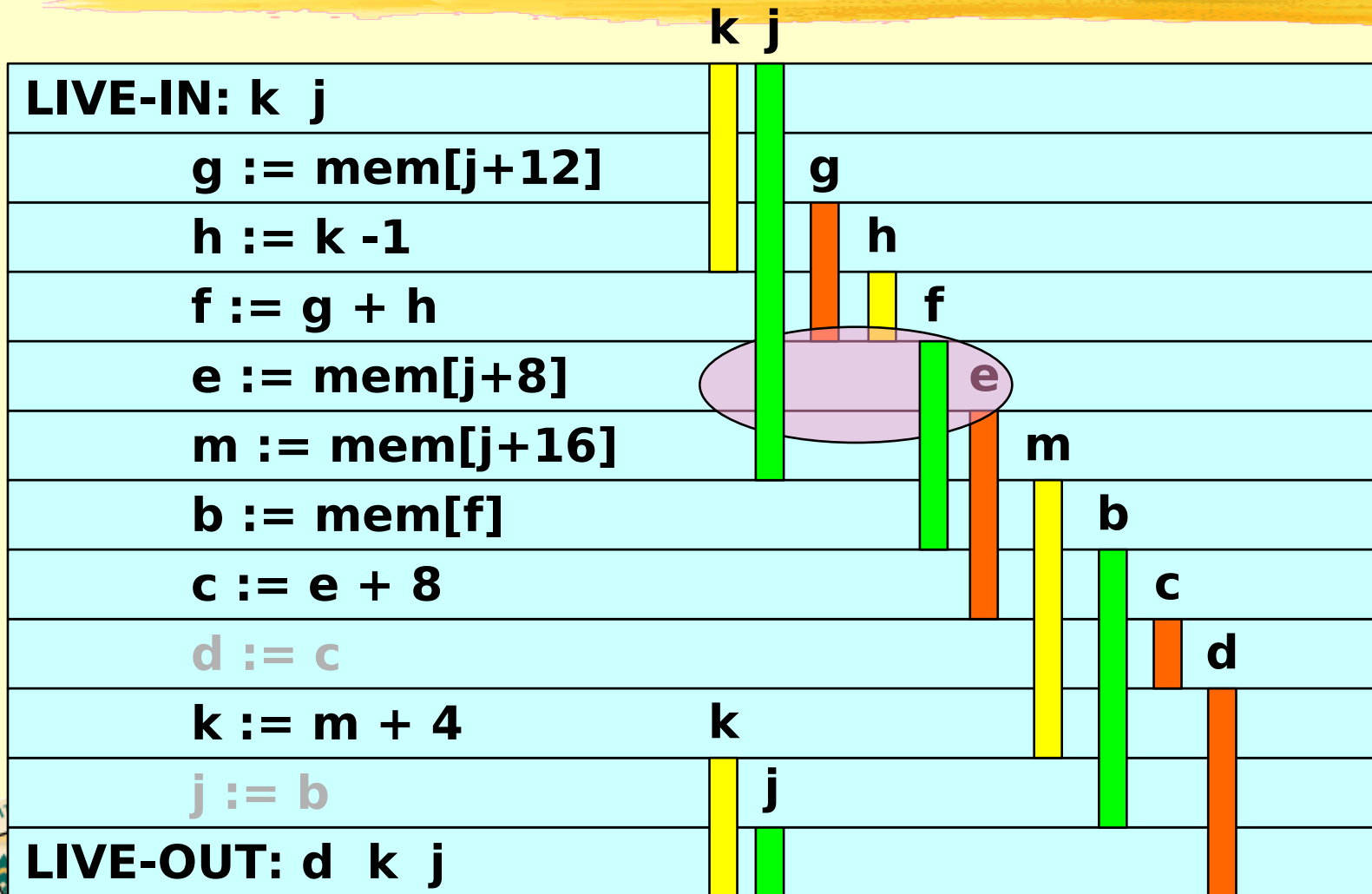
# Example as basic block: A 3-coloring of the graph



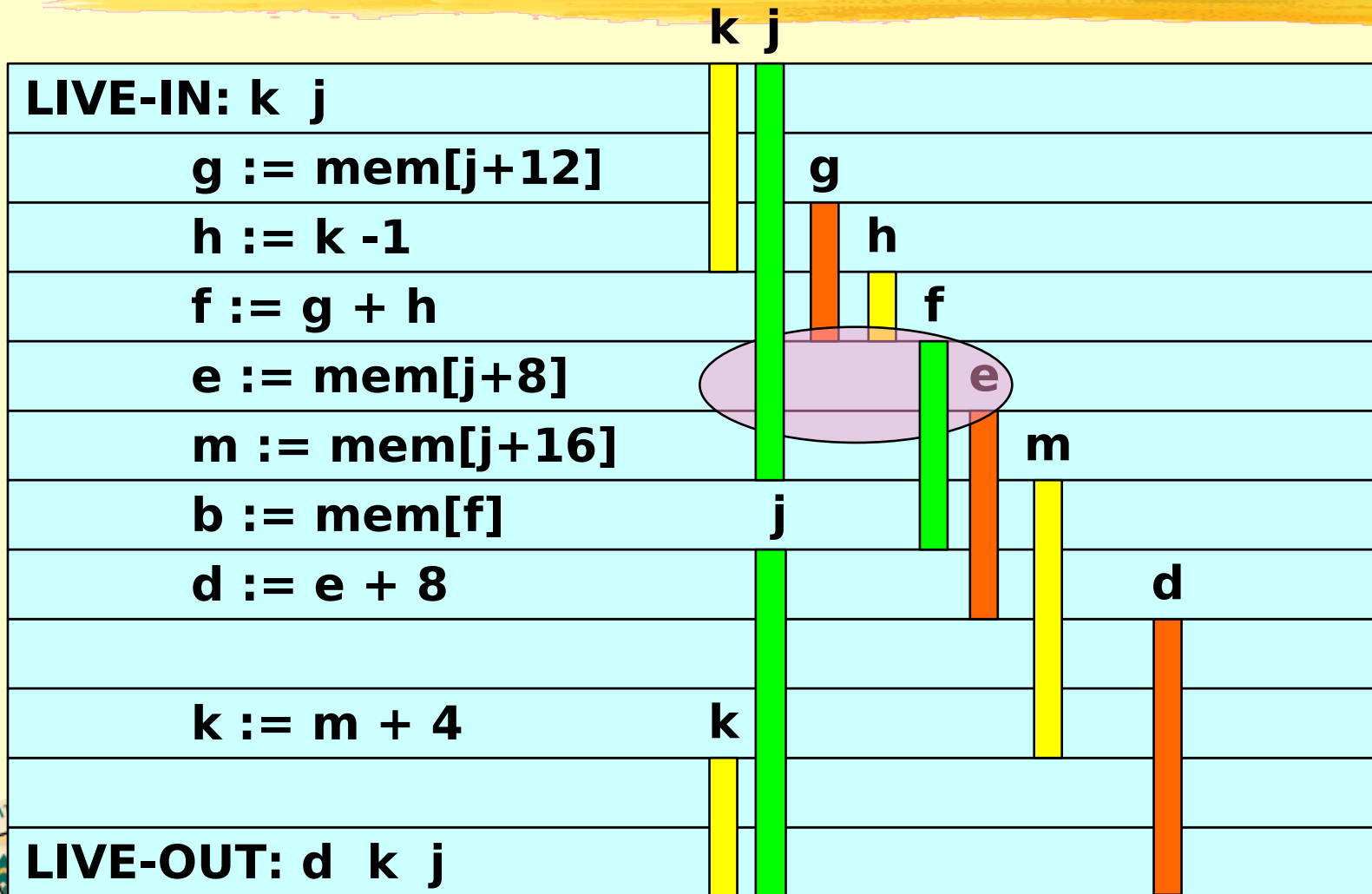
The two assignments of k (resp. j) can be placed in two different registers.



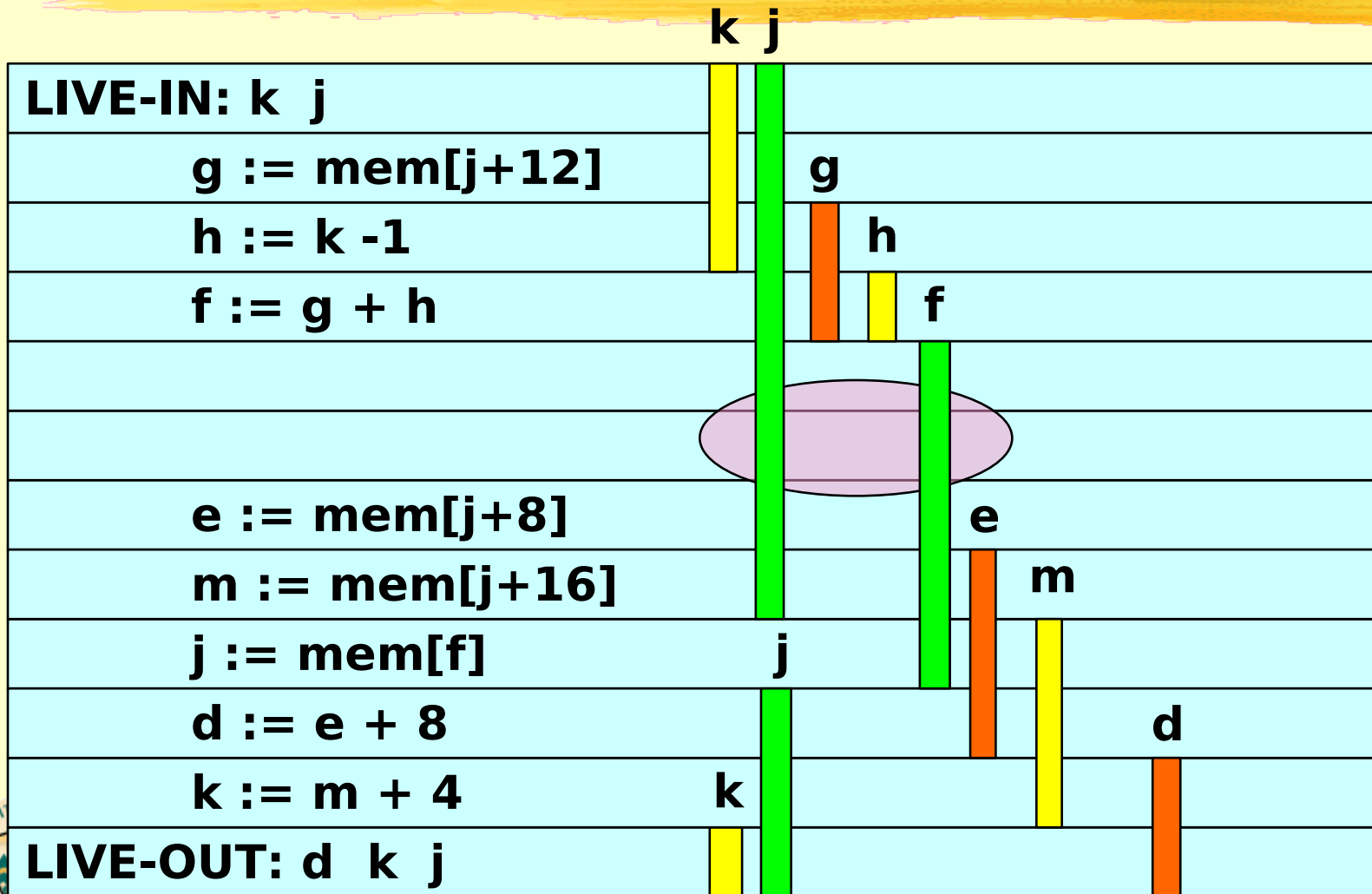
# Example as a loop: 3 Registers are enough!



# Example as a loop: 3 Registers are enough!



# Example as a loop: 3 Registers are enough!



# Example as a loop: 3 Registers are enough!

