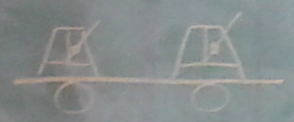
MP 36 : Couplage des oscillateurs

I Introduction



⇒ Synchronisation

Suite = Approximation LINEAIRE

$\rightsquigarrow \quad a\frac{d^2x_1}{dt^2} + b\frac{dx_1}{dt} + cx_1 = f(x_2)$

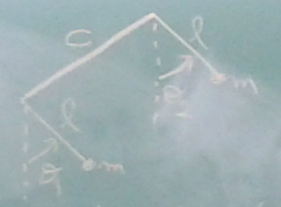
Dépendance de f	Mécanique	Electricité
$\ddot{x}_2$	Inertiel	Inductif
$\dot{x}_2$	Dissipatif	Résistif
x_2	Elastique	Capacitif

Action sur : - Amplitude
- Phase

N oscillateurs → N modes

I Pendules couplés

$$\begin{cases} I\ddot{\theta}_1 = -mgl\,\theta_1 - C(\theta_2 - \theta_1) \\ I\ddot{\theta}_2 = -mgl\,\theta_2 - C(\theta_1 - \theta_2) \end{cases}$$



$m = (1,000 \pm 0,002)$ kg (0,2%) $\quad g = 9,810 \pm 0,005$ N kg^{-1} (0,5%) $\quad l = 40 \pm 0,5$ cm (1,5%)

$\Rightarrow mgl = 3,9 \pm 0,1$ J (2,5%)

1) Pendule libre $\quad f_0 = (\quad \pm \quad)$ mHz (1/4)

$\omega_0 = \sqrt{\frac{mgl}{I}} \quad I = (\quad \pm \quad) 10^{-1}$ kg.m^2 (1/4)

$\frac{\Delta I}{I} = \frac{\Delta(mgl)}{mgl} + 2\frac{\Delta\omega_0}{\omega_0}$

2) Pendules couplés $\quad f_0' = (\quad \pm \quad)$ mHz (1/4)

$\omega_0' = \sqrt{\frac{-mgl+C}{I}} \quad C = (\quad \pm \quad)$ J (1/4)

$C = I(\omega_0^2 - \omega_0'^2)$

3) Effet sensible du couplage
→ ω_0 proches

4) Modes $\omega_S = \omega_0 \quad f_S = (\quad \pm \quad)$ mHz $\quad f_{S\,exp} = (\quad \pm \quad)$ mHz

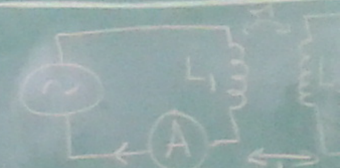
$\det = 0 \quad \omega_A = \sqrt{\omega_0^2 + 2\omega_0'^2} \quad f_A = (\quad \pm \quad)$ mHz $\quad f_{A\,exp} = (\quad \pm \quad)$ mHz

5) Etude énergétique

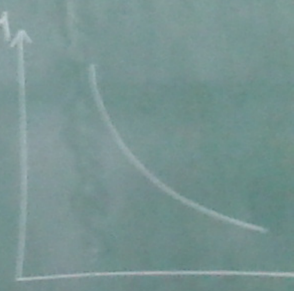
$i = 1,2 \quad E_i = \frac{1}{2}I\dot{\theta}_i^2 + mgl(1 - \cos\theta_i)$

$E_{12} = \frac{1}{2}C(\theta_1 - \theta_2)^2$

II Couplage par mol



1) Etalonnage M(

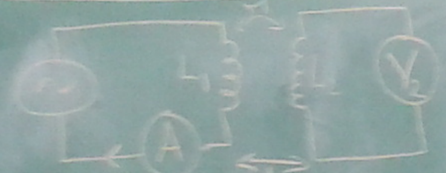


$M = \frac{V_2}{I_1(\omega)}$

$R_1 =$

$R_2 =$

II Couplage par mutuelle induction



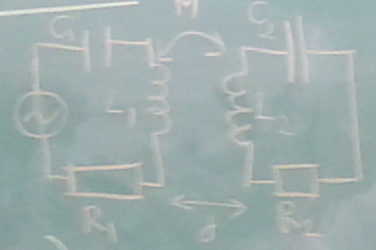
1) Etalonnage $M(\omega)$

M_0

$M = \frac{V_2}{I_1 \omega}$

$R_1 =$ $L_1 =$ $C_1 =$

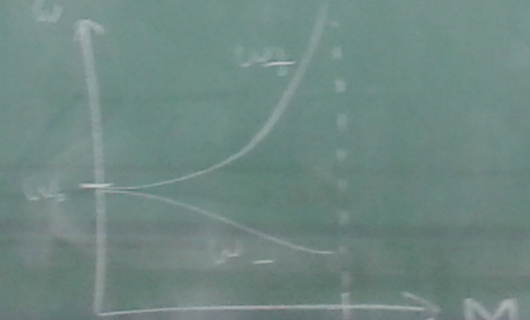
$R_2 =$ $L_2 =$ $C_2 =$



2) Dépendance des modes

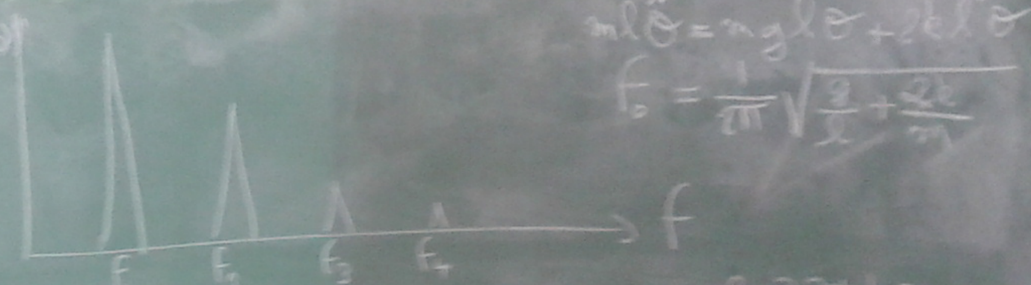
$\omega_0 \rightarrow \omega_+ = \frac{1}{\sqrt{C(L+T)}}$

$\omega_- = \frac{1}{\sqrt{C(L-T)}}$



III Couplage de 4 oscillateurs

1) Mesure des fréquences propres



$ml^2\ddot{\theta} = -mgl\theta + kl^2\theta$

$f_0 = \frac{1}{2\pi}\sqrt{\frac{g}{l} + \frac{2k}{m}}$

$l = 32 \pm 1$ cm

$k = 5 \pm 1$ N m^{-1}

2) Relation de dispersion

$\omega^2 = \omega_0^2 + \frac{4k}{m}\sin^2\left(\frac{pT}{2(N+1)}\right)$

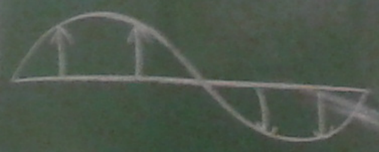
$\frac{4k}{m} = \pm$

$\rightarrow f_1 =$

$f_{1_{th}} = 3{,}0 \pm 0{,}1$ Hz

3) Observation des modes

$F_2 \rightarrow$



Conclusion

Vers $N \rightarrow \infty$