# Adaptive numerical designs for the calibration of computer codes

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# Outline

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# Field data

- Field data provided by physical experiments:

$$\mathbf{y}^F = y^F(\mathbf{x}_1), \ldots, y^F(\mathbf{x}_n)\,,$$

- n is small, $\mathbf{x}_1, \ldots \mathbf{x}_n \in \mathbb{X}$ hard to set, sometimes uncontrollable, included in a small domain...

- Model:

$$y^F(\mathbf{x}_i) = \zeta(\mathbf{x}_i) + \epsilon(\mathbf{x}_i)\,,$$

where

  - $\zeta(\cdot)$ real physical process (unknown),
  - $\epsilon(\mathbf{x}_i)$ often assumed i.i.d. $\mathcal{N}(0, \sigma^2)$,
  - $\sigma^2$ sometimes treated as known...

# Computer model / simulator

**Computer experiments:**

Computer model (simulator) $(\mathbf{x}^*, \boldsymbol{\theta}) \mapsto f(\mathbf{x}^*, \boldsymbol{\theta}) \in \mathbb{R}^s$ where

- **physical parameters**: $\mathbf{x}^* \in \mathbb{X} \subset \mathbb{R}^m$ observable and often controllable inputs
  - $\mathbf{x}^*$ same meaning as in field data,
  - extrapolation if $\mathbf{x}^* > \max(\mathbf{x}_i)$ or $\mathbf{x}^* < \min(\mathbf{x}_i)$.
- **simulator parameters**: $\boldsymbol{\theta} \in \Theta \subset \mathbb{R}^d$ non-observable parameters, required to run the simulator.

  2 types:
  - "calibration parameters": physical meaning but unknown, necessary to make the code mimic the reality,
  - "tuning parameters": no physical interpretation.

$f$ designed to mimic the unknown physical process $\zeta(\cdot)$ for a value of $\boldsymbol{\theta}$.
The simulator is often an **expensive black-box function**.
$\Rightarrow$ limited number $N_{run}$ of runs of the simulator.

# Relationship between the simulator and the data

for $i = 1, \ldots, n$,

- if the simulator sufficiently represents the physical system:

$$y_i^F = f(\mathbf{x}_i, \boldsymbol{\theta}^*) + \epsilon(\mathbf{x}_i)\,,$$

  i.e. for the unknown value $\boldsymbol{\theta} = \boldsymbol{\theta}^*$ : $f(\mathbf{x}, \theta^*) = \zeta(\mathbf{x})$ for any $\mathbf{x} \in \mathbb{X}$,

- if the field observations are inconsistent with the simulations (irreducible model discrepancy):

$$y_i^F = f(\mathbf{x}_i, \boldsymbol{\theta}^*) + \delta(\mathbf{x}_i) + \epsilon(\mathbf{x}_i)\,.$$

  $\delta(\cdot)$ models the difference between the simulator and the physical system:

$$\delta(\mathbf{x}) = \zeta(\mathbf{x}) - f(\mathbf{x}, \theta^*)\,,$$

  but
  - What does $\theta^*$ mean ?
  - A best fitting ?
  - identifiability issues ?
  - usually assumed to be smoother than the real physical process $\zeta(\cdot)$

Ref.: Kennedy and O'Hagan (2001), Hidgon et al. (2005)...

# Outline

# A calibration example

**Hypotheses:**

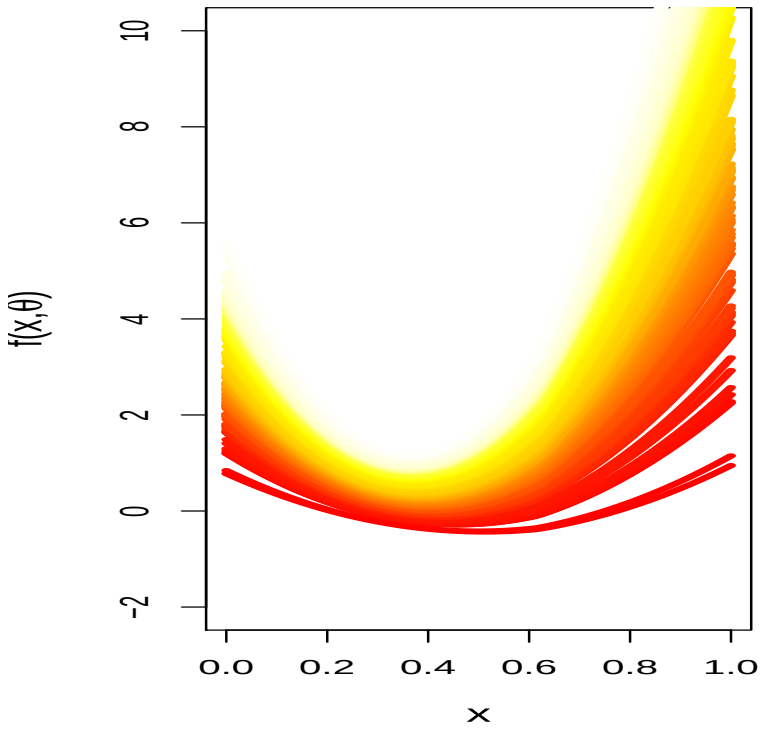
- The simulator represents sufficiently well the physical system:

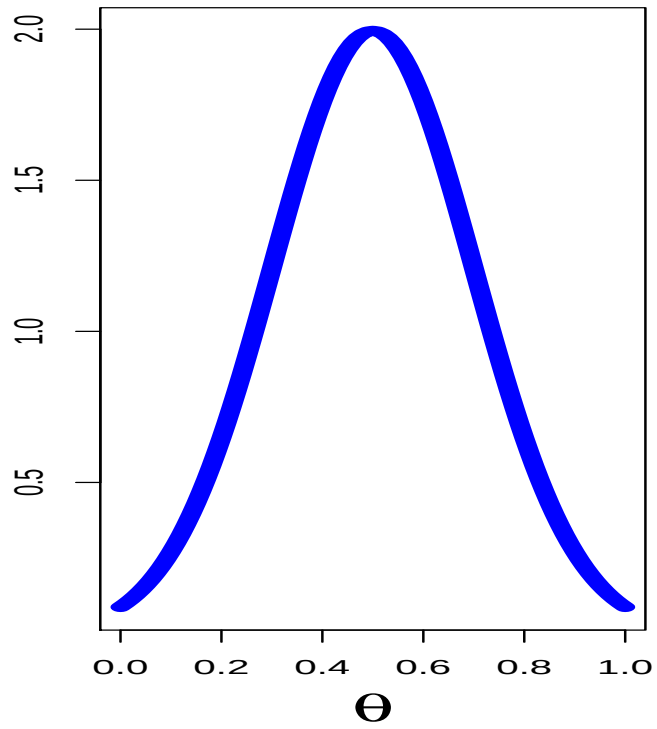
$$y^F(\mathbf{x}_i) = f(\mathbf{x}_i, \boldsymbol{\theta}^*) + \epsilon_i, \quad i = 1, \ldots, n.$$

- But unknown $\boldsymbol{\theta}^*$.
- $\epsilon_i \sim \mathcal{N}(0, \sigma^2)$ i.i.d. with known $\sigma^2$.
- $\sigma^2 = 0.3$
- $n = 6$,
- $\theta^* = 0.6$

# A calibration example

**Prior:**
prior distribution on unknown $\boldsymbol{\theta}$: $\pi(\cdot)$
from expert judgment, past experiments...
Possible choice $\pi(\boldsymbol{\theta}) = \mathcal{N}(\theta_0, \sigma_0^2) = \mathcal{N}(0.5, 0.04)$.



# A calibration example

**Data:**
Couples $(\mathbf{x}_1, y_1^F), \ldots, (\mathbf{x}_n, y_n^F)$ from physical experiments.

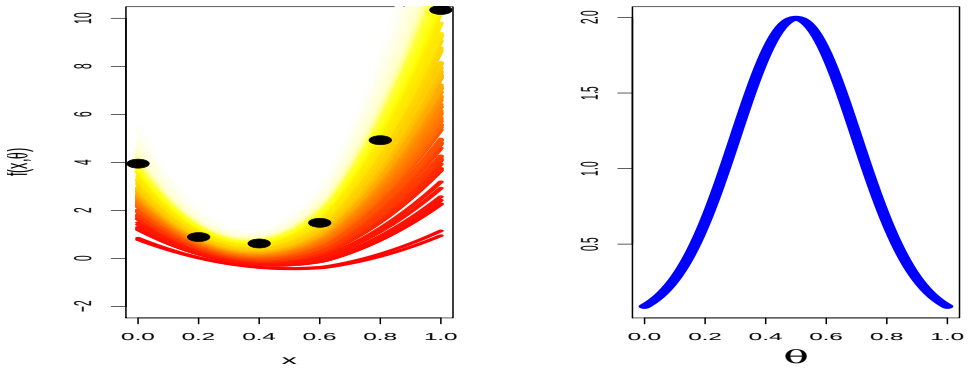
**Posterior distribution:**

$$\begin{aligned}\pi(\boldsymbol{\theta}|\mathbf{y}^F) &\propto l(\boldsymbol{\theta}|\mathbf{y}^F) \cdot \pi(\boldsymbol{\theta}) \\ &\propto \exp\left(-\frac{1}{2\sigma^2}\sum_{i=1}^{n}(y^F(\mathbf{x}_i) - f(\mathbf{x}_i, \boldsymbol{\theta}))^2 - \frac{1}{2\sigma_0^2}(\boldsymbol{\theta} - \boldsymbol{\theta}_0)^2\right)\end{aligned}$$

- Analytical posterior if $\boldsymbol{\theta} \mapsto f(\mathbf{x}, \boldsymbol{\theta})$ is a linear map,
- Otherwise MH sampling to simulate according to the posterior distribution.

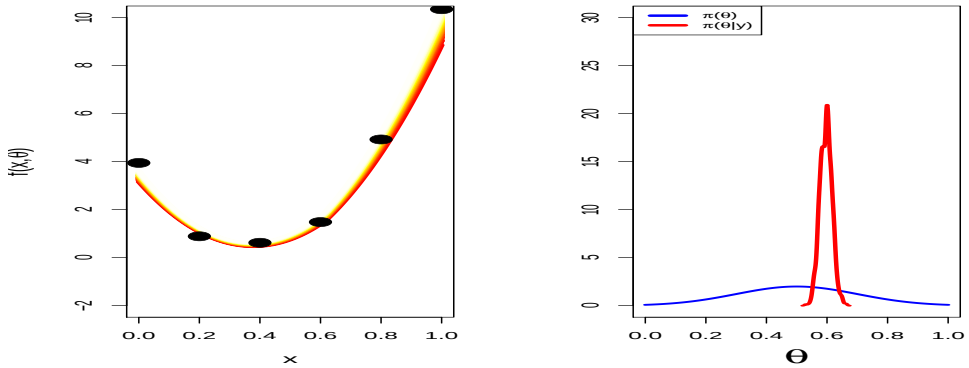
# A calibration example

**Prior with data:**




⇓ Metropolis-Hastings algorithm ⇓

**Posterior on $\theta$:**



# More details on the MH algorithm

**Initialisation:**
$\theta^0$ chosen.

**Update:**
iterations $t = 1, \ldots,$

1. Proposal: $\tilde{\theta}^{t+1} = \theta^t + \mathcal{N}(0, \tau^2)$.
2. Compute
$$\alpha(\theta^t, \tilde{\theta}^{t+1}) = \frac{\pi(\tilde{\theta}^{t+1}|\mathbf{y}^F)}{\pi(\theta^t|\mathbf{y}^F)}$$
3. Acceptation:
$$\theta^{t+1} = \begin{cases} \tilde{\theta}^{t+1} & \text{with probability } \alpha(\theta^t, \tilde{\theta}^{t+1}) \\ \theta^t & \text{otherwise.} \end{cases}$$

Note that the ratio $\alpha(\theta^t, \tilde{\theta}^{t+1})$ needs several computations of $f(\mathbf{x}, \boldsymbol{\theta})$ at each step since

$$\pi(\theta|\mathbf{y}^F) \propto \exp\left(-\frac{1}{2\sigma^2}\sum_{i=1}^{n}(y^F(\mathbf{x}_i) - f(\mathbf{x}_i, \boldsymbol{\theta}))^2 - \frac{1}{2\sigma_0^2}(\boldsymbol{\theta} - \boldsymbol{\theta}_0)^2\right).$$

# Outline

# Expensive black-box computer code

- Run the simulator for a given $(\mathbf{x}^*, \boldsymbol{\theta})$ is time-consuming / expensive.
- The simulator is a black-box, no intrusive methods are possible.

$\Rightarrow$ Only few runs of the simulator are possible then we cannot apply algorithms (as in Bayesian calibration) which make a massive use of simulator runs.

Using an emulator / metamodel / coarse model / approximation of the simulator which is fast to compute, but:

- loss on precision of prediction,
- new uncertainty source: accuracy of the model approximation,
- taken into account.

# Choosing a design of experiments

Choose $N_{run}$ couples

$$(\mathbf{x}_j^*, \theta_j)$$

- space filling for $x$,
- with respect to the prior distribution on $\theta$,
- $\mathbf{x}_j^* = \mathbf{x}_i$ ?

where the simulator is called.

# Emulator using Gaussian Process:

- Very popular in computer experiments.
- integrated in a Bayesian framework: appears in the likelihood function and a prior on the parameters of the Gaussian process are chosen.
- model uncertainty coming from approximation of $f$.
- After the calibration step, used in prediction for a new point $\mathbf{x}$.

# Meta-modeling: prior distribution on $f$

Sacks et al. (1989).
$f$ realization of a Gaussian process $F$:
$\forall (\mathbf{x}^*, \boldsymbol{\theta}) \in E$,

$$F((\mathbf{x}^*, \boldsymbol{\theta})) = \sum_{k=1}^{Q} \beta_k h_k((\mathbf{x}^*, \boldsymbol{\theta})) + Z((\mathbf{x}^*, \boldsymbol{\theta})) = H((\mathbf{x}^*, \boldsymbol{\theta}))^T \boldsymbol{\beta} + Z((\mathbf{x}^*, \boldsymbol{\theta}))\,,$$

où

- $h_1, \ldots, h_Q$ regression functions and $\boldsymbol{\beta}$ parameters vector,
- $Z$ centered Gaussians process with covariance function:

$$\mathrm{Cov}(Z((\mathbf{x}_1^*, \boldsymbol{\theta}_1)), Z((\mathbf{x}_2^*, \boldsymbol{\theta}_2))) = \sigma^2 K((\mathbf{x}_1^*, \boldsymbol{\theta}_1), (\mathbf{x}_2^*, \boldsymbol{\theta}_2))\,,$$

  where $K$ is correlation kernel.

## Hypotheses

- $K((\mathbf{x}_1^*, \boldsymbol{\theta}_1), (\mathbf{x}_2^*, \boldsymbol{\theta}_2)) = \sigma_K^2 \exp(-\xi_{\mathbf{x}^*} \sum |\mathbf{x}_1^* - \mathbf{x}_2^*|^\alpha - \xi_{\boldsymbol{\theta}} \sum |\boldsymbol{\theta}_1 - \boldsymbol{\theta}_2|^\alpha)$
- parameters $\phi = (\boldsymbol{\beta}, \sigma^2, K$ parameters) assumed fixed (in practice, maximum likelihood estimators);

# Meta-modeling: posterior

- $v_1 = f((\mathbf{x}^*, \boldsymbol{\theta})_1), \ldots, v_{N_{run}} = f((\mathbf{x}^*, \boldsymbol{\theta})_{N_{run}})$ evaluations of $f$ on a design $D_{N_{run}}$
- Process $F^{D_{N_{run}}}$ : Conditioning $F$ to $F((\mathbf{x}_1^*, \boldsymbol{\theta}_1)) = v_1, \ldots, F(\mathbf{x}_{N_{run}}^*, \boldsymbol{\theta}_{N_{run}})) = v_{N_{run}}$.
  Gaussian Process with mean $m((\mathbf{x}^*, \boldsymbol{\theta}))$ and covariance $C((\mathbf{x}^*, \boldsymbol{\theta}), (\mathbf{x}^*, \boldsymbol{\theta})') \; \forall (\mathbf{x}^*, \boldsymbol{\theta}), (\mathbf{x}^*, \boldsymbol{\theta})'$.

  For all $(\mathbf{x}^*, \boldsymbol{\theta}) \in E$,
  - $m((\mathbf{x}^*, \boldsymbol{\theta}))$ approximates $f((\mathbf{x}^*, \boldsymbol{\theta}))$,
  - $C((\mathbf{x}^*, \boldsymbol{\theta}), (\mathbf{x}^*, \boldsymbol{\theta}))$ uncertainty on this approximation.

For all $(\mathbf{x}_i^*, \boldsymbol{\theta}_i) \in D_{N_{run}}$,
- $m(\mathbf{x}_i^*, \boldsymbol{\theta}_i) = f(\mathbf{x}_i^*, \boldsymbol{\theta}_i)$,
- $C((\mathbf{x}_i^*, \boldsymbol{\theta}_i), (\mathbf{x}_i^*, \boldsymbol{\theta}_i)) = 0$.

# Gaussian process emulator: illustration

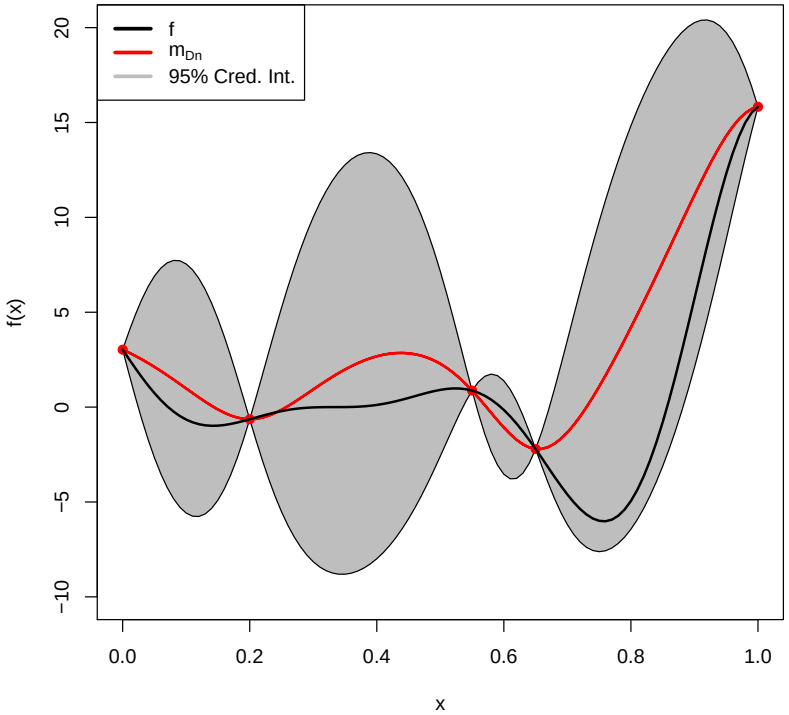



Figure : Posterior mean and pointwise credible interval

# Outline

# Likelihood with a Gaussian process hypothesis on $f$

- $\mathbf{z} = (\mathbf{y}_1^F, \ldots, \mathbf{y}_n^F, f(\mathbf{x}_1^*, \boldsymbol{\theta}_1), \ldots, f(\mathbf{x}_{N_{run}}^*, \boldsymbol{\theta}_{N_{run}}))$
- likelihood on $\mathbf{z}$

$$l(\boldsymbol{\theta}, \sigma^2 | \mathbf{z}) \propto |\Sigma_{\mathbf{z}}|^{-1/2} \exp\left(-\frac{1}{2}(\mathbf{z}-\mu)^T \Sigma_{\mathbf{z}}^{-1}(\mathbf{z}-\mu)\right)$$

where

- $\mu$ is the mean of the Gaussian process,
- $$\Sigma_{\mathbf{z}} = \Sigma_f + \begin{pmatrix} \Sigma_y & 0 \\ 0 & 0 \end{pmatrix}$$

  with $\Sigma_y = \sigma^2 I_n$ and $\Sigma_f$ is obtained as the covariance matrix corresponding to the points: $(\mathbf{x}_1, \boldsymbol{\theta}), \ldots, (\mathbf{x}_n, \boldsymbol{\theta}), (\mathbf{x}_1^*, \boldsymbol{\theta}_1), \ldots, (\mathbf{x}_{N_{run}}^*, \boldsymbol{\theta}_{N_{run}})$.

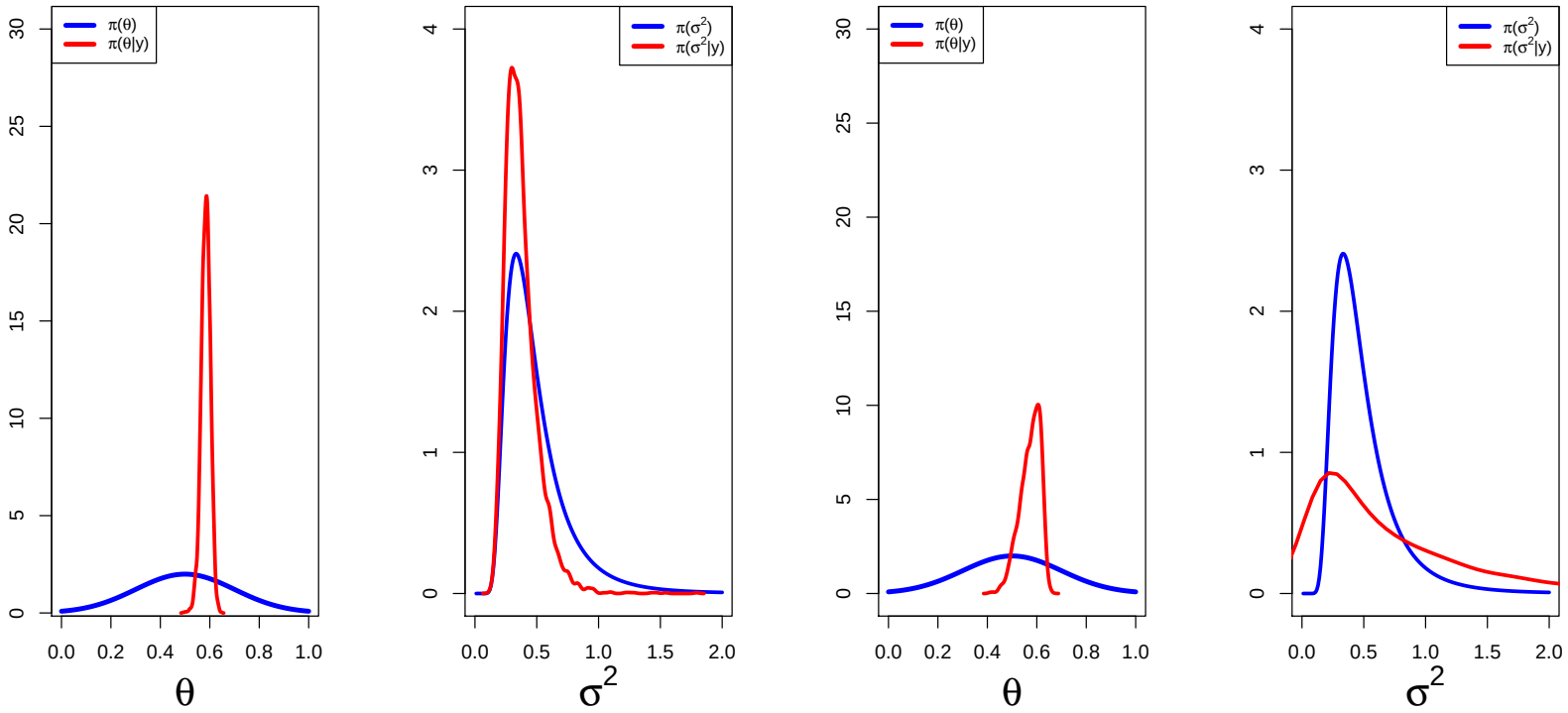
# Dealing with GP parameters

- prior distribution on $\mu$ and covariance parameters Hidgon et al. (2005)
  $\Rightarrow$ MCMC inference

- MLE estimators Kennedy and O'Hagan (2001)
  - treated as fixed,
  - only computer data $f(\mathbf{x}_1^*, \theta_1), \ldots, f(\mathbf{x}_{N_{run}}^*, \theta_{N_{run}})$ are used ($n < N_{run}$) for MLE
  - likelihood $l(\boldsymbol{\theta}, \sigma^2|\mathbf{z})$:

$$l(\boldsymbol{\theta}, \sigma^2|\mathbf{z}) \propto |\tilde{\Sigma}_{\mathbf{y}^F}|^{-1/2} \exp\left(-\frac{1}{2}(\mathbf{y}^F - m(\mathbf{x}, \boldsymbol{\theta}))^T \tilde{\Sigma}_{\mathbf{y}^F}^{-1}(\mathbf{y}^F - m(\mathbf{x}, \boldsymbol{\theta}))\right)$$

  where
  - $m(\cdot)$ is the mean of the GP conditioned to simulator data,
  - $\tilde{\Sigma}_{\mathbf{y}^F} = \Sigma_{\mathbf{y}^F} + \tilde{\Sigma}_f = \sigma^2 I_n + \tilde{\Sigma}_f$ where $\tilde{\Sigma}_f$ is constructed with the covariance function $C$ of the conditioned GP.




unlimited runs versus $N_{run} = 12$

# Posterior consistency

## Proposition

*Under the following assumptions:*

- $\pi(\boldsymbol{\theta})$ *has a bounded support* $\mathcal{T}$,
- *the code output* $f(\boldsymbol{x}, \boldsymbol{\theta})$ *is uniformly bounded on* $\mathcal{X} \times \mathcal{T}$,
- *the correlation function (kernel) is a classical radial basis function*
- $f$ *lies in the associated Reproducing Kernel Hilbert Space,*
- *the covering distances associated with the sequence of designs* $(\boldsymbol{D}_M)_M$ *tends to* 0 *with* $M \to \infty$,

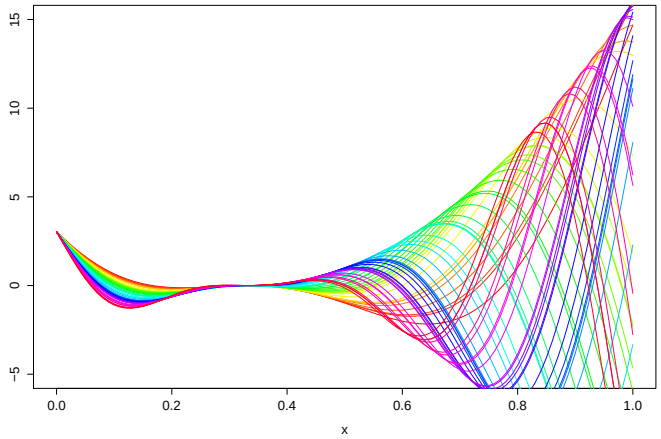
*then, we have:*

$$\lim_{M \to \infty} KL\left(\pi(\boldsymbol{\theta}|\boldsymbol{y}^F) || \pi^C(\boldsymbol{\theta}|\boldsymbol{y}^F, f(\boldsymbol{D}_M))\right) = 0\,. \tag{1}$$

# Motivation for adaptive designs in calibration

Example:

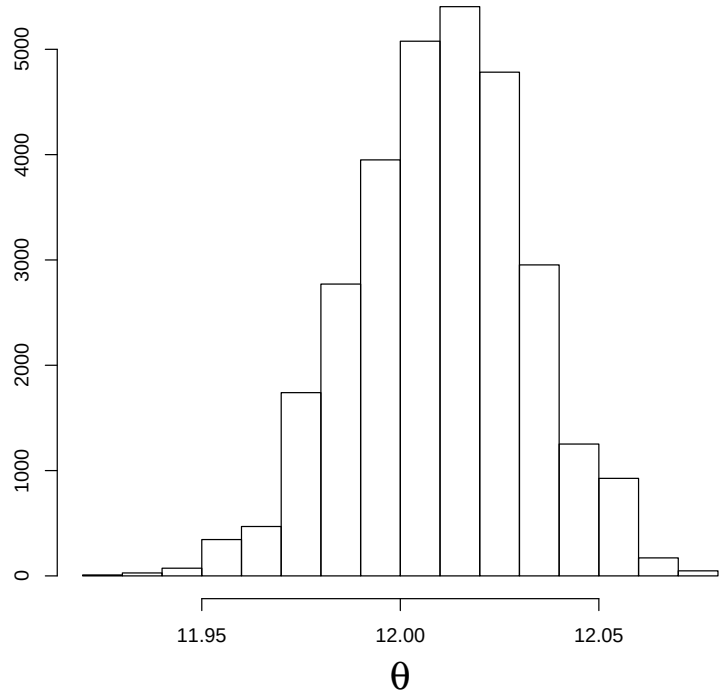
- $\theta = 12$,
- $(x_1, x_2, x_3) = (0.1, 0.3, 0.8)$,
- $f(x, \theta) = (6 \cdot x - 2)^2 \cdot \sin(\theta \cdot x - 4) + \epsilon$,
- $\epsilon_i \sim \mathcal{N}(0, 0.1^2)$ i.i.d.,
- prior $\theta \sim \mathcal{U}[5, 15]$,
- $y_i = f(x_i, \theta) + \epsilon_i$.



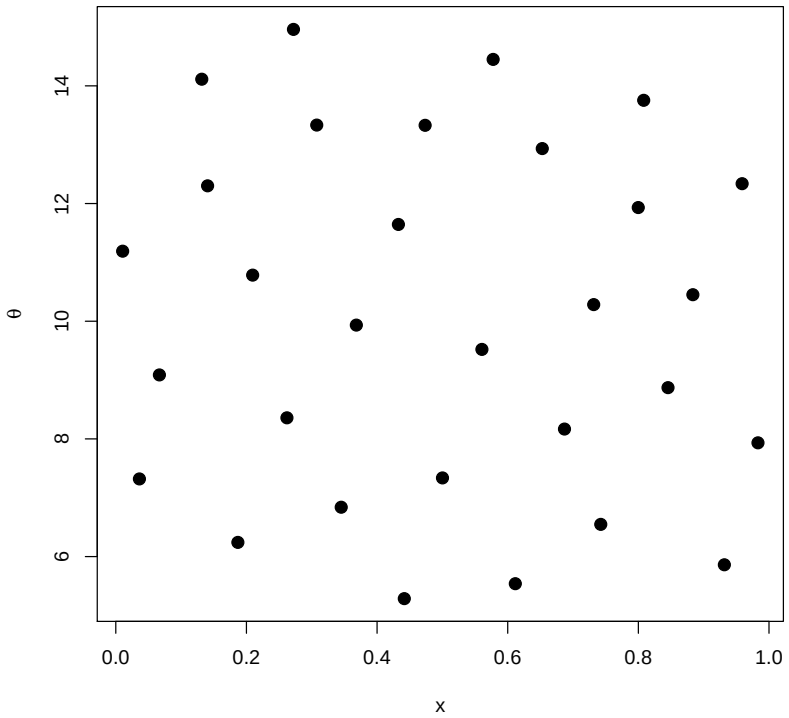
# Motivation for adaptive designs in calibration

Quality of calibration (Bayesian or ML) is affected by choice in the numerical design.

- Calibration with unlimited runs of $f$

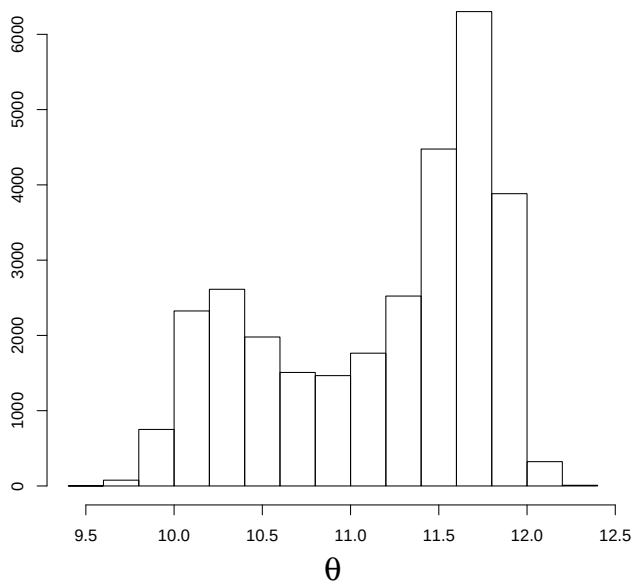


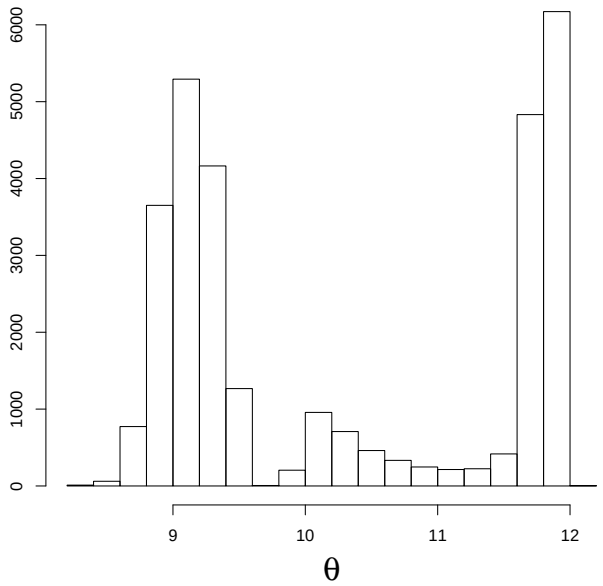
# LHS maximin design



# Motivation for adaptive designs in calibration

- Calibration with emulator built from a design with $N = 30$ calls to $f$



# Outline

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# Reducing the variance

- At step k, by conditioning we obtain a posterior distribution on $\theta$: $\pi(\cdot|\mathbf{y}^F, f(D_k))$.
- Adding a computer experiments at location $(x_{k+1}, \theta_{k+1})$ such that the uncertainty on $\theta$ is reduced.
- New location could aim at choosing $z_{k+1} = (x_{k+1}, \theta_{k+1})$ which minimises

$$\mathbb{E}(Var(\theta|f(D_{k+1}))|Z^s_{k+1} = z_{k+1}, f(D_k)) = \mathbb{E}_k(Var_{k+1}(\theta)|Z^s_{k+1} = z^s)\,.$$

By omitting the conditioning $Z^s_{k+1} = z^s$,

$$\begin{aligned}
\mathbb{E}_k(Var_{k+1}(\theta)) &= \mathbb{E}_k\left(\mathbb{E}_{k+1}((\theta - \mathbb{E}_{k+1}(\theta))^2)\right) \\
&= \mathbb{E}_k(\mathbb{E}_{k+1}(\theta^2) - \mathbb{E}_k(\mathbb{E}_{k+1}(\theta)^2) \\
&= \mathbb{E}_k(\theta^2) - \mathbb{E}_k(\mathbb{E}_{k+1}(\theta)^2)
\end{aligned}$$

Hence, since $\mathbb{E}_k(\theta^2)$ does not depend on $z^s_{k+1}$, the minimisation problem is equivalent to maximise $\mathbb{E}_k(\mathbb{E}_{k+1}(\theta)^2)$.

# Outline

# Efficient Global Optimization

- Goal: Find the global extremum (here minimum e.g.) of $f$,
- Expected improvement criterion proposed by Jones et al. (1998):

$$EI_n(\mathbf{x}) = \mathbb{E}((min_n - F(\mathbf{x}))^+ | F(D_n)),$$

where $min_n$ is the current minimum value:

$$min_n = \min_{1,\dots,n} f(\mathbf{x}_i)$$

- Closed-form computation:

$$EI_n(\mathbf{x}) = (min_n - m_{D_n}(\mathbf{x}))\Phi\left(\frac{min_n - m_{D_n}(\mathbf{x})}{\sqrt{C_{D_n}(\mathbf{x},\mathbf{x})}}\right) + \sqrt{C_{D_n}(\mathbf{x},\mathbf{x})}\phi\left(\frac{min_n - m_{D_n}(\mathbf{x})}{\sqrt{C_{D_n}(\mathbf{x},\mathbf{x})}}\right)$$

where $\Phi$ and $\phi$ are respectively the cdf and the pdf of $\mathcal{N}(0,1)$.

# EI for calibration

**Optimization goal :** maximize the likelihood $\Rightarrow$ Expected Improvement for calibration.

Maximize the likelihood $l(\boldsymbol{\theta}|\mathbf{z})$ over $\boldsymbol{\theta} \Leftrightarrow$ Minimize $SS(\boldsymbol{\theta}) = \|\mathbf{y}^F - F(\mathbf{x}, \boldsymbol{\theta})\|^2$ over $\boldsymbol{\theta}$.

For given:

- field experiments $\mathbf{y}^F = y^F(\mathbf{x}_1), \ldots, y^F(\mathbf{x}_n)$,
- $D_k$ numerical design on $\mathbb{X} \times \Theta$ with $M$ points,
- $m_k$ current minimal value of $SS(\boldsymbol{\theta})$.

EI criterion:

$$EI_{D_k}(\boldsymbol{\theta}) = \mathbb{E}_{D_k}\left((m_k - SS(\boldsymbol{\theta}))^+\right) ,$$

to be minimised.

*EI criterion is applied to a function of $f$.*

# EI computation

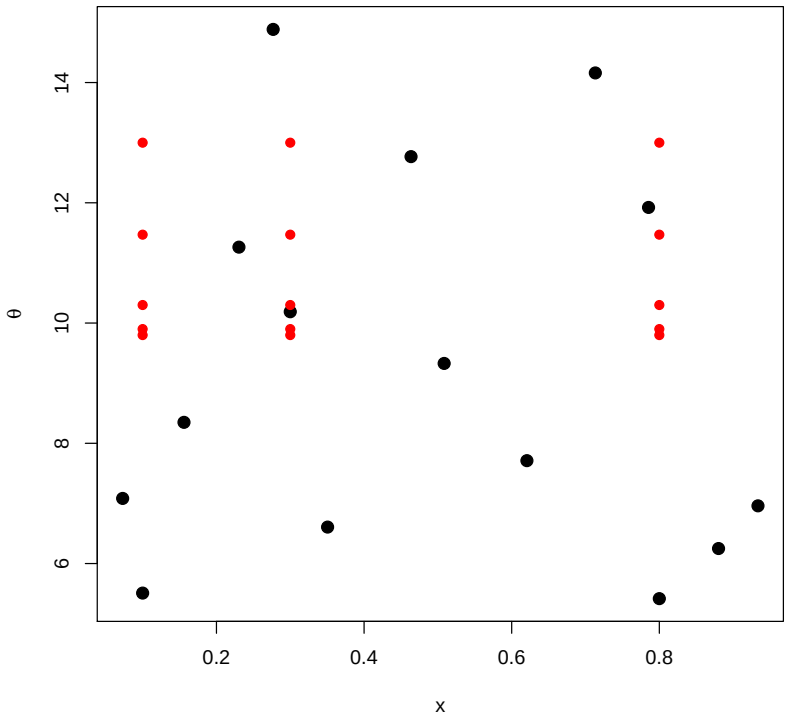
$$\begin{aligned} EI_{D_k}(\boldsymbol{\theta}) &= \int_{B(0,\sqrt{m_k})} (m_k - SS(\boldsymbol{\theta}))\, dF_{D_M} \\ &= m_k \cdot \mathbb{P}_{D_M}(SS(\boldsymbol{\theta}) \leq m_k) - \mathbb{E}_{D_M}\left(SS(\boldsymbol{\theta}) \mathbb{I}_{SS(\boldsymbol{\theta}) \leq m_k}\right) \end{aligned}$$

- no close form computation,
- $\mathbb{P}_{D_M}(SS(\boldsymbol{\theta}) \leq m_k)$ is an upper bound and easier to compute,
- importance sampling may be used for the second term.

# Algorithm

1. Build a first space-filling design $D_0$ on $\mathbb{X} \times \Theta$,
2. Find the maximum: $\tilde{\boldsymbol{\theta}}_0$ of $l(\boldsymbol{\theta}|\mathbf{z})$,
3. Evaluate $f(\mathbf{x}_1, \tilde{\boldsymbol{\theta}}_0), \ldots, f(\mathbf{x}_n, \tilde{\boldsymbol{\theta}}_0)$.
4. Set $m_0 = SS(\tilde{\boldsymbol{\theta}}_0)$,
5. for k=1..., repeat
    1. Compute $EI_{D_k}$ on a grid on $\Theta$,
    2. $\tilde{\boldsymbol{\theta}}_k = \arg\max_{\Theta} EI_{D_k}(\boldsymbol{\theta})$,
    3. Evaluate $f(\mathbf{x}_1, \tilde{\boldsymbol{\theta}}_k), \ldots, f(\mathbf{x}_n, \tilde{\boldsymbol{\theta}}_k)$

# Adapted design



# Bayesian calibration based on the adapted design

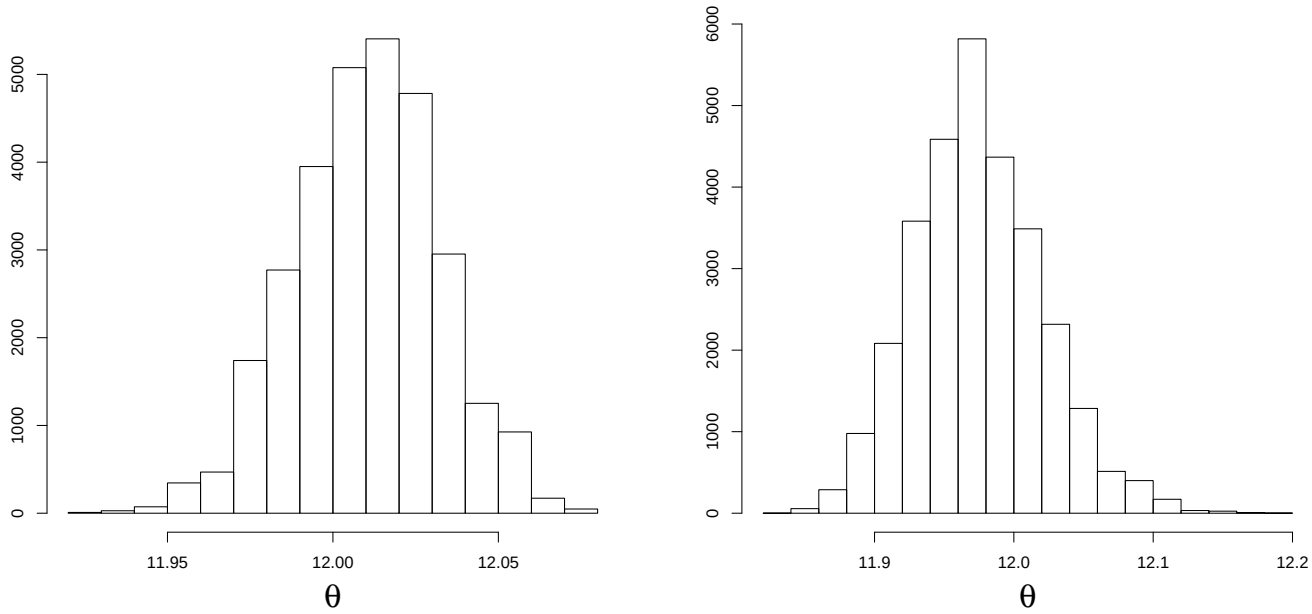



Figure : Bayesian calibration with unlimited runs vs Bayesian calibration with $N = 30$ chosen by EGO

# Algorithm one at a time

**Algorithm (step k $\longrightarrow$ step k + 1) :**

- $\theta_{k+1} = \underset{\theta}{\operatorname{argmax}}\, EI_k(\theta)$,
- $\mathbf{D}_{k+1} = \mathbf{D}_k \cup (\mathbf{x}^\star, \theta_{k+1})$ where $\mathbf{x}^\star \in \mathbf{X}^f = \left[\mathbf{x}_1^f, \cdots, \mathbf{x}_n^f\right]^T$,
- $f(\mathbf{D}_{k+1}) = f(\mathbf{D}_k) \cup \{f(\mathbf{x}^\star, \theta_{k+1})\}$,
- $F^{D_{k+1}} = F|f(\mathbf{D}_{k+1})$,
- $m_{k+1} := \min\{\mathbb{E}[SS_{k+1}(\theta_1)], \cdots, \mathbb{E}[SS_{k+1}(\theta_k)], \mathbb{E}[SS_{k+1}(\theta_{k+1})]\}$.

**Only 1 simulation to compute $m_{k+1}$!**

# Algorithm one at a time

Two criteria to choose $\mathbf{x}^{\star}_{k+1}$ :

1. $\mathbf{x}^{\star} = \underset{\mathbf{x}_i^f}{\operatorname{argmax}} \operatorname{Var}(F^{D_k}(\mathbf{x}_i^f, \boldsymbol{\theta}_{k+1}))$,
2. $\mathbf{x}^{\star} = \underset{\mathbf{x}_i^f}{\operatorname{argmax}} \left( \frac{\operatorname{Var}\left(F^{D_k}(\mathbf{x}_i^f, \boldsymbol{\theta}_{k+1})\right)}{\max\limits_{i=1,\cdots,n} \operatorname{Var}\left(F^{D_k}(\mathbf{x}_i^f, \boldsymbol{\theta}_{k+1})\right)} \times \frac{\operatorname{Var}(\mu^k_{\boldsymbol{\beta},\boldsymbol{\Psi}}(\mathbf{x}_i^f, \boldsymbol{\theta}))}{\max\limits_{i=1,\cdots,n} \operatorname{Var}(\mu^k_{\boldsymbol{\beta},\boldsymbol{\Psi}}(\mathbf{x}_i^f, \boldsymbol{\theta}))} \right)$

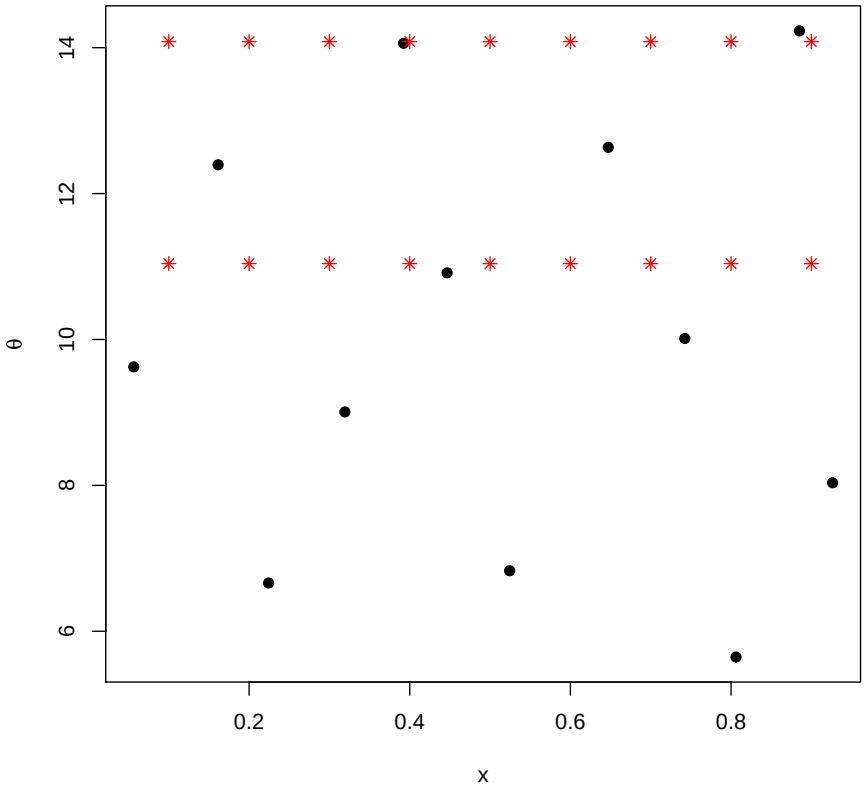
where $\operatorname{Var}(\mu^k_{\boldsymbol{\beta},\boldsymbol{\Psi}}(\mathbf{x}_i^f, \boldsymbol{\theta}))$ is computed with respect to $\pi(\boldsymbol{\theta})$.

**criterion 2 = trade-off between uncertainty on $\mathbf{F^{D_k}}$ for $(\mathbf{x_i^f}, \boldsymbol{\theta}_{\mathbf{k+1}})$ and sensibility of $\mathbf{F^{D_k}}(\mathbf{x_i^f}, \boldsymbol{\theta})$ to $\boldsymbol{\theta}$**

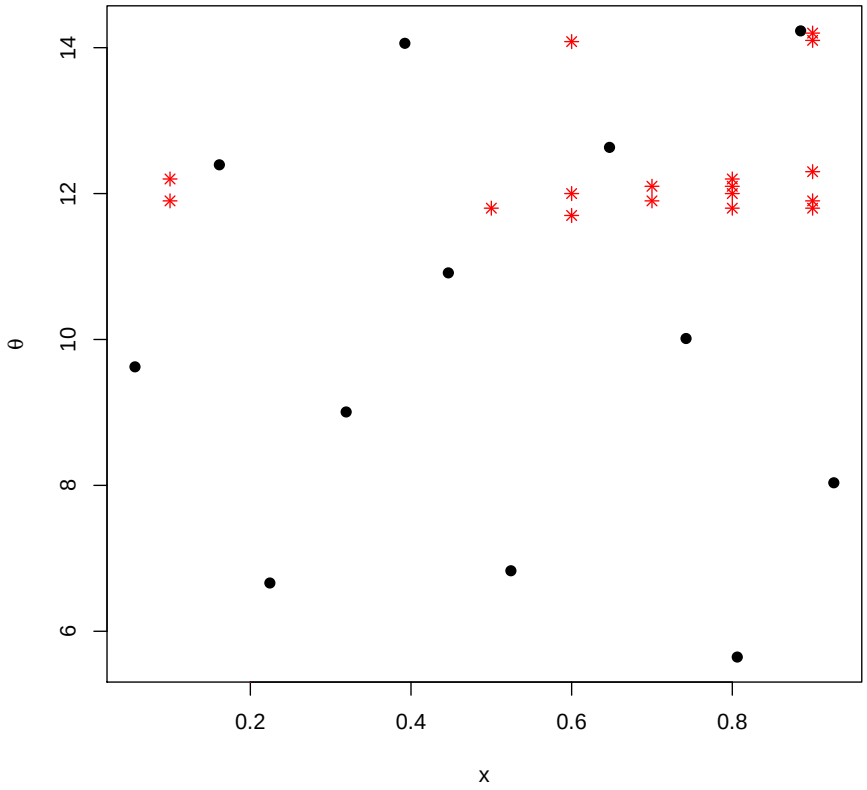
# Comparison full EI / EI one at a time

$\mathbf{X}^f = (0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9)$, $\Theta = [5, 15]$

Figure : ***full EI***




**EI *OAT***



# Outline

# Conclusion

- Designs of numerical experiments adapted to calibration purpose,
- Robustness in calibration.
- Higher dimension questions, number of field experiments, dimension of $\theta$...
- New field experiments ?
- discrepancy issues ?

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