HOMEWORK — Saturday 13th march 23h

We are interested in solving $\hat{x} = \underset{x \in \mathbb{R}^N}{\operatorname{argmin}} \frac{1}{2} \| x - z \|^2 + \lambda \| Dx \|_1$

where $\begin{cases} z \in \mathbb{R}^K \\ D \in \mathbb{R}^{K \times N} \\ \lambda > 0 \end{cases}$

1/ Prove the existence of a unique minimizer

2/ Is there a closed form expression for $\hat{x}$? If yes under which condition(s).

3/ Let D associated with a finite difference filter $d = [-\frac{1}{2} \ \frac{1}{2}]$.

$$Dx = d \underbrace{*}_{\text{convolution}} x$$

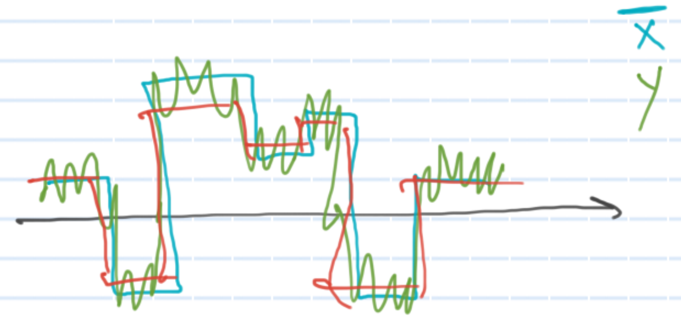
Does $\hat{x}$ has a closed form expression in this context?

4/ Derive the closed form expression of $\operatorname{prox}_{\lambda \|\cdot\|_1}$.

5/ Provide the conjugate function of $f = \lambda \|\cdot\|_1$.

6/ Give the relation between $P_C$ when $C = \{ x \in \mathbb{R}^K \mid \|x\|_\infty \leq \lambda \}$ and $\operatorname{prox}_{\lambda \|\cdot\|_1}$

$$\hat{x} \in \underset{x}{\operatorname{argmin}} \; \|x-y\|_2^2 + \lambda \|Dx\|_1$$




D matrix
associated with
a filter which promotes piece wise
constant behaviour.

Filter =  $\quad d = \left[\frac{1}{2} \;\; -\frac{1}{2}\right]$

$$\underline{x} * d = Dx$$

$$\begin{bmatrix} \frac{1}{2} & -\frac{1}{2} & 0 & \cdots & \cdots & 0 \\ 0 & \frac{1}{2} & -\frac{1}{2} & 0 & \cdots & 0 \\ 0 & 0 & \frac{1}{2} & -\frac{1}{2} & 0 \cdots & 0 \\ & & & \ddots & \ddots & \\ & & & & & \end{bmatrix}$$