

Introduction générale au machine learning

L'IA, un outil de compétitivité pour les entreprises

Formation DIRECCTE

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Contenu de la séance

Nous allons voir

- en quoi consiste l'approche " machine learning" pour la résolution de problèmes
- quels sont les grands types de problème résolubles par apprentissage
- quel type de données les algorithmes d'apprentissage savent traiter, et comment parfois s'y ramener
- quels sont les grandes familles d'algorithmes d'apprentissage et quels outils permettent de les mettre en œuvre

Nous n'allons pas voir :

- l'ensemble des techniques de ML
- la théorie du ML et les problèmes de recherche les plus récents
- comment utiliser des GPU et autres architectures massivement parallèles

Bibliographie - Ressources



Pattern Classification (2001) - Wiley Interscience, *R. Duda, P. Hart, D. Stork*



The Elements of Statistical Learning (2001) - Springer, *T. Hastie, R. Tibshirani, J. Friedman*

Disponible en ligne : <http://www-stat.stanford.edu/~tibs/ElemStatLearn/>



Data Mining - Technip, *S. Tufféry*



Cours en ligne de Andrew Ng (Stanford):
<https://www.coursera.org/course/ml>



<http://wikistat.fr/>

dont sont issus certains de ces slides !



<http://scikit-learn.org>



Base de données de benchmarking:
<http://archive.ics.uci.edu/ml/>

Outline

Machine Learning: when Artificial Intelligence meets Big Data

The Learning Models

Machine Learning Data and Methodology

Machine Learning Algorithms: a flavor

Unsupervised learning

Supervised learning

Artificial Intelligence (AI): Definition

Intelligence exhibited by machines

- emulate cognitive capabilities of humans
(big data: humans learn from abundant and diverse sources of data).
- a machine mimics "cognitive" functions that humans associate with other human minds, such as "learning" and "problem solving".

Ideal "intelligent" machine =

flexible rational agent that perceives its environment and takes actions that maximize its chance of success at some goal.

Founded on the claim that human intelligence

"can be so precisely described that a machine can be made to simulate it."

Artificial Intelligence: Tension

Operational goals

- Autonomous robots for not-too-specialized tasks
- In particular, vision + understand and produce language

Tension between operational and philosophical goals

- As machines become increasingly capable, facilities once thought to require intelligence are removed from the definition. For example, optical character recognition is no longer perceived as an exemplar of "artificial intelligence"; having become a routine technology.
- Capabilities still classified as AI include advanced Chess and Go systems and self-driving cars.

Machine Learning (ML): Definition

Arthur Samuel (1959)

Field of study that gives computers the ability to learn without being explicitly programmed

Tom M. Mitchell (1997)

A computer program is said to learn from experience E with respect to some class of tasks T and performance measure P if its performance at tasks in T, as measured by P, improves with experience E.

ML: Learn from and make predictions on data

- Algorithms operate by building a model from **example** inputs in order to make data-driven **predictions or decisions**...
- ...rather than following strictly static program instructions: useful when designing and programming explicit algorithms is unfeasible or poorly efficient.

Within Data Analytics

- Machine Learning used to devise complex models and algorithms that lend themselves to **prediction** - in commercial use, this is known as *predictive analytics*.
- www.sas.com: "Produce reliable, repeatable decisions and results" and uncover "hidden insights" through learning from historical **relationships and trends** in the data.
- evolved from the study of pattern recognition and computational learning theory in artificial intelligence.

Machine Learning: Typical Problems

- spam filtering, text classification
- optical character recognition (OCR)
- search engines
- recommendation platforms
- speech recognition software
- computer vision
- bio-informatics, DNA analysis, medicine
- etc.

For each of this task, it is possible but very inefficient to write an explicit program reaching the prescribed goal.

It proves much more successful to have a machine infer what the good decision rules are.

Related Fields

- **Computational Statistics:** focuses in prediction-making through the use of computers together with statistical models (ex: Bayesian methods).
- **Statistical Learning:** ML by statistical methods, with statistical point of view (probabilistic guarantees: consistency, oracle inequalities, minimax)
→ more focused on *correlation*, less on *causality*
- **Data Mining** (unsupervised learning) focuses more on exploratory data analysis: discovery of (previously) unknown properties in the data. This is the analysis step of Knowledge Discovery in Databases.
- Importance of **probability-** and **statistics**-based methods → **Data Science** (Michael Jordan)
- Strong ties to **Mathematical Optimization**, which delivers methods, theory and application domains to the field

What is Data?

Data (here): digital content used for learning.

- images (objects, satellite, hand-written text, hyperspectral, etc.)
- times series (finance, earthquakes activity, ECG, etc.)
- internet traces (marketing, social network)
- texts
- audio (signal)
- behaviour: game, robot activity
- ...

Qu'est-ce qu'une (très grande) masse de données ?



VLDB
XLDB
Big Data
Very Big Data
Data Deluge
Massive Data
Data Masses

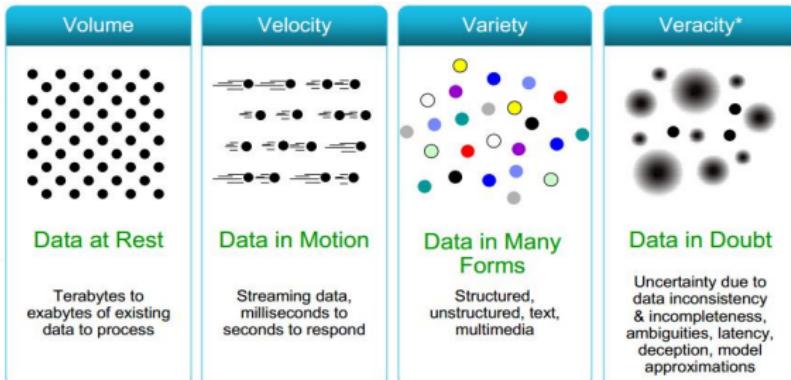
Data inflation

Unit	Size	What it means
Bit (b)	1 or 0	Short for "binary digit", after the binary code (1 or 0) computers use to store and process data
Byte (B)	8 bits	Enough information to create an English letter or number in computer code. It is the basic unit of computing
Kilobyte (KB)	1,000, or 2^{10} bytes	From "thousand" in Greek. One page of typed text is 2KB
Megabyte (MB)	1,000KB; 2^{20} bytes	From "large" in Greek. The complete works of Shakespeare total 5MB. A typical pop song is about 4MB
Gigabyte (GB)	1,000MB; 2^{30} bytes	From "giant" in Greek. A two-hour film can be compressed into 1-2GB
Terabyte (TB)	1,000GB; 2^{40} bytes	From "monster" in Greek. All the catalogued books in America's Library of Congress total 15TB
Petabyte (PB)	1,000TB; 2^{50} bytes	All letters delivered by America's postal service this year will amount to around 5PB. Google processes around 1PB every hour
Exabyte (EB)	1,000PB; 2^{60} bytes	Equivalent to 10 billion copies of <i>The Economist</i>
Zettabyte (ZB)	1,000EB; 2^{70} bytes	The total amount of information in existence this year is forecast to be around 1.22B
Yottabyte (YB)	1,000ZB; 2^{80} bytes	Currently too big to imagine

The prefixes are set by an intergovernmental group, the International Bureau of Weights and Measures. Yotta and Zetta were added in 1991; terms for larger amounts have yet to be established.

Grandes Conf du domaine: VLDB, XLDB, ICDE, EDBT, ...

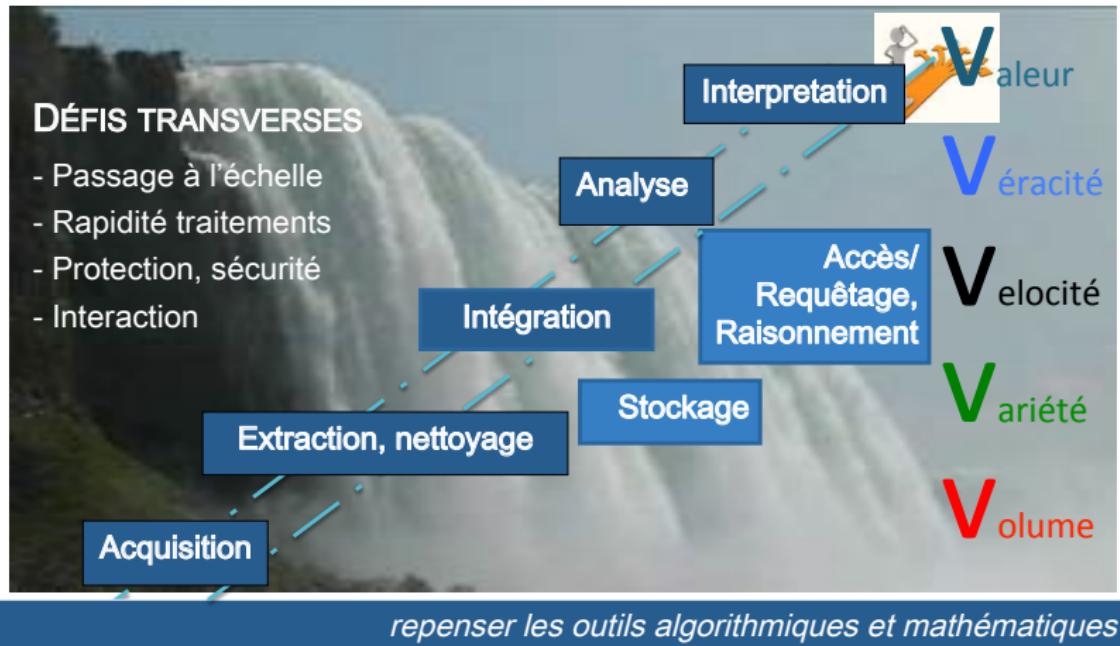
Complexité multidimensionnelle des Big Data



- Nouvelles archi. de stockage
- Défi pour les réseaux de communication
- Nettoyage et transformation
- Nouvelles modèles de qualité (données & processus de traitement)
- Nouvelles archi. d'interopérabilité
- Nouveaux modèles de calcul sur des flux
- Fusion de données

<http://www.datasciencecentral.com/profiles/blogs/data-veracity>

Défis accompagnant les chgts



inspired by "Big Data and Its Technical Challenges, Communications of the ACM, July 2014, vol 57, n°7", © H.V. Jagadish et all.

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Machine Learning and Statistics

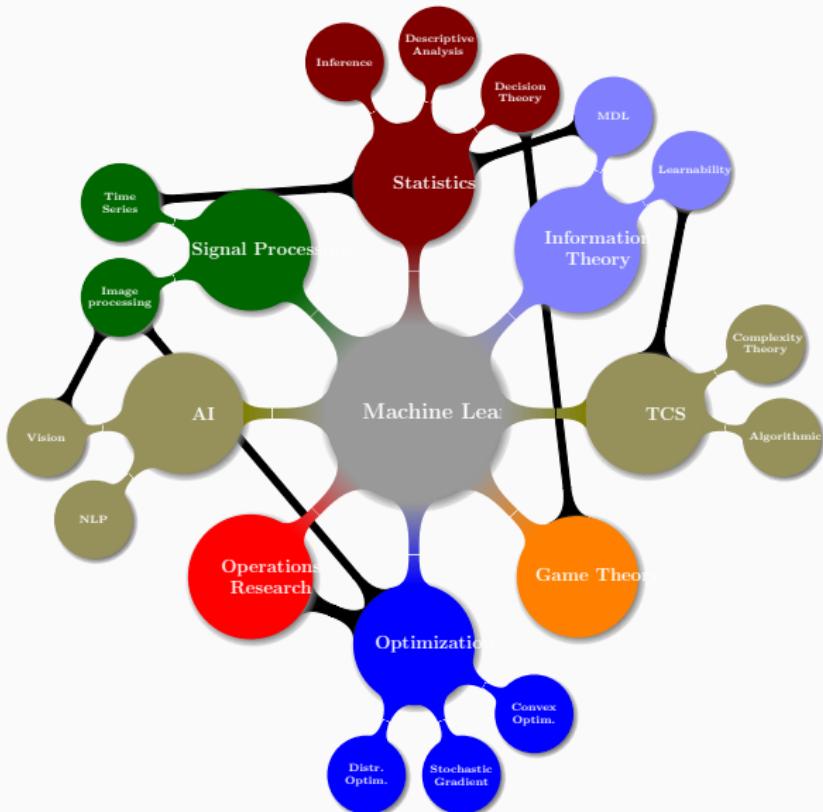
- Data analysis (inference, description) is the goal of statistics for long.
- Machine Learning has more **operational** goals (ex: consistency is important the statistics literature, but often makes little sense in ML).

Models (if any) are *instrumental*

Ex: linear model (nice mathematical theory) vs Random Forests.

- Machine Learning/big data: no separation between statistical modelling and optimization (in contrast to the statistics tradition).
- In ML, data is often here before (unfortunately)
- No clear separation (statistics evolves as well).

ML and its neighbors



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Unsupervised learning

Supervised learning

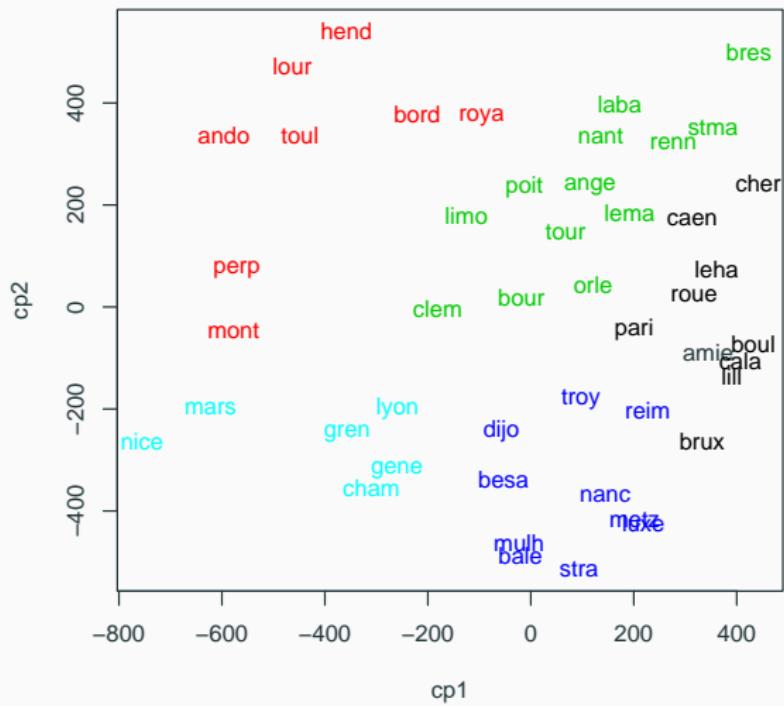
What ML is composed of



Unsupervised Learning

- (many) observations on (many) individuals
- need to have a simplified, structured overview of the data
- *taxonomy*: untargeted search for *homogeneous clusters* emerging from the data
- Examples:
 - customer segmentation
 - image analysis (recognizing different zones)
 - exploration of data

Example



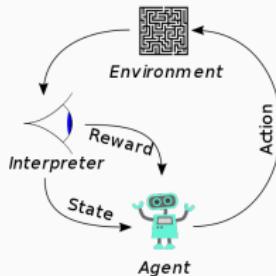
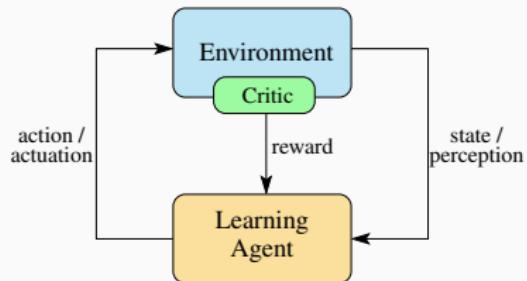
Supervised Learning

- observations = pairs (X_i, Y_i)
- goal = learn to *predict* Y_i given X_i
- regression (when Y is continuous)
- classification (when Y is discrete)
- statistical technique: linear models

Example: Character Recognition

Input space \mathcal{X}	64 × 64 images
Output space \mathcal{Y}	$\{0, 1, \dots, 9\}$
Joint distribution $P(x, y)$?
Prediction function $h \in \mathcal{H}$	
Risk $R(h) = P(h(X) \neq Y)$	
Sample $\{(x_i, y_i)\}_{i=1}^n$	MNIST dataset
Empirical risk $\hat{R}_n(h) = \frac{1}{n} \sum_{i=1}^n \mathbb{1}\{h(x_i) \neq y_i\}$	
Learning algorithm $\phi_n : (\mathcal{X} \times \mathcal{Y})^n \rightarrow \mathcal{H}$	NN, boosting...
Expected risk $R_n(\phi) = \mathbb{E}_n[R(\phi_n)]$	
Empirical risk minimizer $\hat{h}_n = \operatorname{argmin}_{h \in \mathcal{H}} \hat{R}_n(h)$	
Regularized empirical risk minimizer $\hat{h}_n = \operatorname{argmin}_{h \in \mathcal{H}} \hat{R}_n(h) + \lambda C(h)$	

Reinforcement Learning



[Src: https://en.wikipedia.org/wiki/Reinforcement_learning]

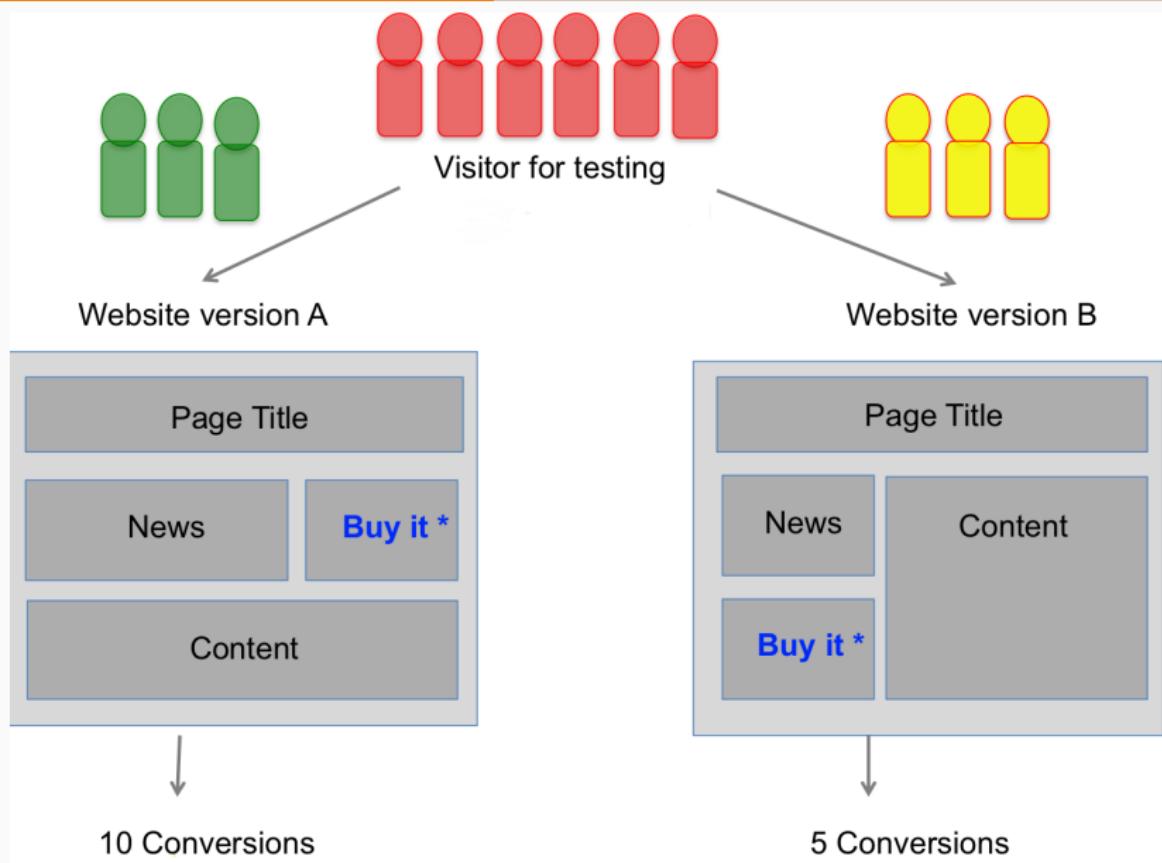
- area of machine learning inspired by behaviourist psychology
- how software agents ought to take actions in an environment so as to maximize some notion of cumulative reward.
- Model: random system (typically : Markov Decision Process)
 - agent
 - state
 - actions
 - rewards
- sometimes called approximate dynamic programming, or neuro-dynamic programming

Example: the Retail Store Management Problem

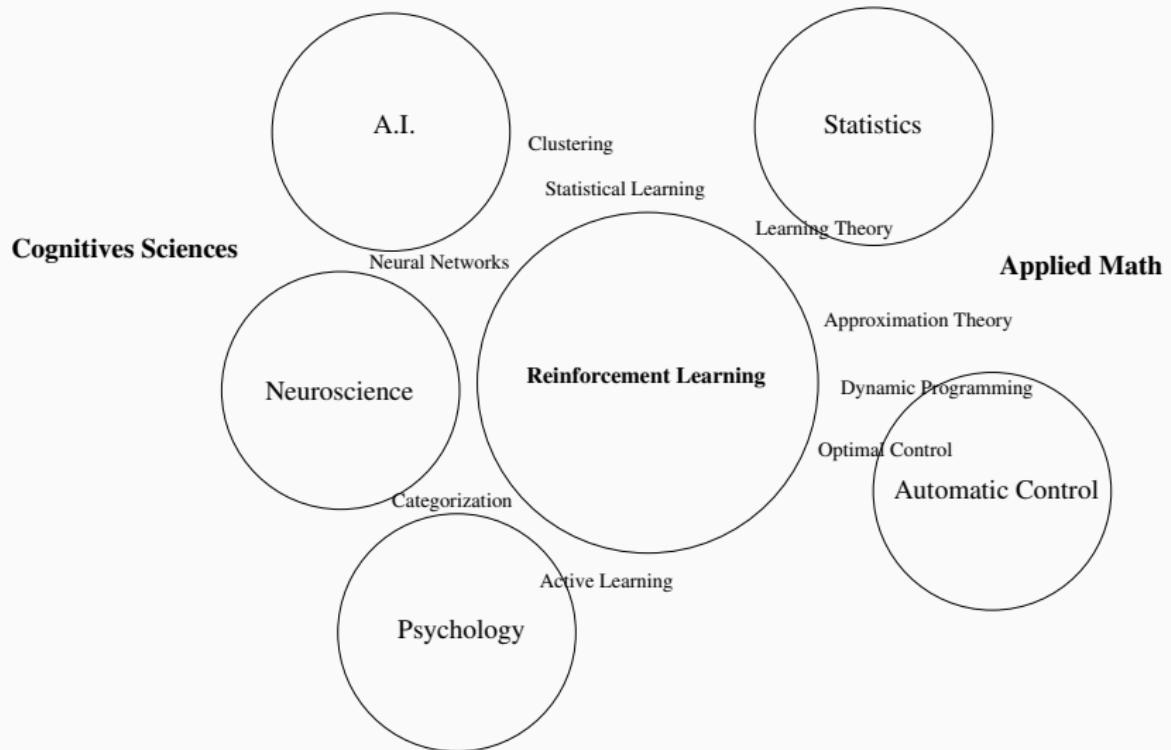
At each month t , a store contains x_t items of a specific goods and the demand for that goods is D_t . At the end of each month the manager of the store can order a_t more items from his supplier. Furthermore we know that:

- The **cost** of maintaining an inventory of x is $h(x)$.
- The **cost** to order a items is $C(a)$.
- The **income** for selling q items is $f(q)$.
- If the demand D is bigger than the available inventory x , customers that cannot be served leave.
- The **value of the remaining inventory** at the end of the year is $g(x)$.
- **Constraint:** the store has a maximum capacity M .

Example: A/B testing



Reinforcement Learning and the others



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n -by- p matrix X

- n examples = points of observations
- p features = characteristics measured for each example

Questions to consider:

- Are the features centered?
- Are the features normalized? bounded?

In scikitlearn, all methods expect a 2D array of shape (n, p) often called

`X (n_samples, n_features)`

Data repositories

- Inside R: package datasets
- Inside scikitlearn: package sklearn.datasets
- UCI Machine Learning Repository
- Challenges: Kaggle, etc.



The big steps of data analysis

1. Extracting the data to expected format
2. Exploring the data
 - detection of outliers, of inconsistencies
 - descriptive exploration of the distributions, of correlations
 - data transformations
3. Random partitioning of the data: (see also: cross-validation)
 - learning sample
 - validation sample
 - test sample
4. For each algorithm: parameter estimation using training and validation samples
5. Choice of final algorithm using testing sample, risk estimation

Machine Learning tools: R

File Edit Code View Plots Session Build Debug Tools Help

```
## R script ## Source on Save Run Source
```

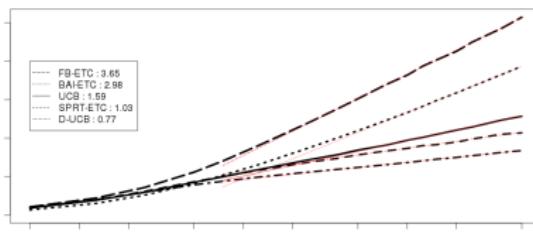
```
264 plot(TT, RR[,1], type="l", lwd=4, lty=ltys[1], log="x", xlim=c(min(TT), max(TT)), ylim = c(0, max(RR)) )
265 #arrrow(TT, RR[,1], lwd=4, lty=ltys[1])
266 #lines(TT, RR[,1], lwd=4, lty=ltys[1])
267 # lines(TT, RR[,1], lwd=4, lty=ltys[1])
268 r = lm(RR[,1], TT~1000)
269 lines(TT, predict(r), lwd=4, lty=ltys[1])
270 slopes[1] <- r$coefficients[1]+r$coefficients[2]*log(TT*500), col="red", lty=ltys[1])
271 #arrrow(TT, RR[,1], lwd=4, lty=ltys[1])
272 #lines(TT, RR[,1], lwd=4, lty=ltys[1])
273 #for (h in c(0.5, 1, 2, 4)){ lines(TT, h*log(TT^d/24)/d, col="red", lnd=2) }
274 #plot(TT, RR[,1], log(TT), type="l", lwd=4, lty=ltys[1], log="x", xlim=c(min(TT), max(TT)), ylim = c(0, max(RR)) )
275 #arrrow(TT, RR[,1], lwd=4, lty=ltys[1])
276 # lines(TT, RR[,1], lwd=4, lty=ltys[1])
277 #for (h in 2:6){strategize(h)}
278 #arrrow(TT, RR[,1], lwd=4, lty=ltys[1])
279 #lines(TT, RR[,1], lwd=4, lty=ltys[1])
280 #
```

```
281 names <- c("FB-ETC", "SPRT-ETC", "BAI-ETC", "D-UCB", "UCB")  
order <- c(1, 3, 5, 2, 4)  
legend(50, 80, supply.order, function(k) paste(names[k], ":", round(slopes[k], 2))), lty = ltys[order]
```

```
285  
# T <- c(2.5, 10, 20, 30, 50, 100, 150, 200, 300, 400, 500, 600, 800, 1000, 1500, 2000, 3000, 5000, 8000, 10000)/d^2; Ncc  
# T <- c(2.5, 10, 20, 30, 50, 100, 200, 300, 400, 500, 600, 800, 1000)/d^2; Ncc <- 10000; createData(T, Ncc);  
# T <- c(1000, 2000, 3000, 5000, 10000)/d^2; Ncc <- 10000; createData(T, Ncc);  
# T <- c(400, 800, 1500)/d^2; Ncc <- 10000; createData(T, Ncc);  
#
```

Console: /Users/luiz/Downloads/etcs.R

```
[1] 0.0119 0.00303 1.00000 0.00039 0.00618  
[1] 10000  
[1] 72.7240 34.6597 53.2542 27.32934 38.85430  
[1] 0.00208 0.00208 0.00023 0.00425  
[1] 12500  
[1] 77.4495 36.14412 56.56396 28.25963 40.41959  
[1] 0.00179 0.00177 1.00000 0.00021 0.03961  
[1] 15000  
[1] 80.5030 36.59257 59.43882 28.01284 41.89409  
[1] 0.00248 0.00148 1.00000 0.00013 0.02272  
[1] 17500  
[1] 85.1294 38.04091 63.31994 29.39995 44.16748  
[1] 0.00435 0.00107 1.00000 0.00016 0.02210  
[1] 20000  
[1] 89.9376 39.54227 66.87458 30.98849 45.81386  
[1] 0.00270 0.00083 1.00000 0.00012 0.01666  
[1] 22500  
[1] 94.80037 42.05158 72.35729 32.54084 49.32925  
[1] 0.00236 0.00055 1.00000 0.00008 0.00127  
[1] 25000  
[1] 102.77106 42.75523 77.29644 33.54590 51.36300  
[1] 0.00177 0.00050 1.00000 0.00008 0.00060
```



Machine Learning tools: python

The figure shows a screenshot of the Spyder Python IDE interface. The top menu bar includes 'Fichier', 'Edition', 'Recherche', 'Source', 'Exécution', 'Déboguer', 'Consoles', 'Outils', 'Affichage', and 'Aide'. The title bar indicates 'mer 19:17' and 'Spyder (Python 2.7)'. The main workspace contains several tabs: 'payment_renewal_study_mysoothhazard.py', 'survival.py', and 'kaplan_meier.py'. The 'kaplan_meier.py' tab is active, displaying the following code:

```

1 #!/usr/bin/python
2 # coding: utf-8
3
4 # Created on Tue Aug 16 03:05:03 2016
5 # Author: agrivrie
6
7 import numpy as np
8 from random import randint
9 import matplotlib.pyplot as plt
10
11
12 def kaplan_meier(tau, Delta, censorDate):
13     N = len(censorDate)
14     survival = np.ones(N)
15     survived = np.zeros(N)
16     for i in tau:
17         t = i
18         while 1 < i and x[i] + Delta[i] < censorDate and x[i] + Delta[i] < x[i+1]:
19             t = i + Delta[i]
20             if t + Delta[i+1] < x[i+1]:
21                 x[i+1] = t + Delta[i+1]
22             else:
23                 x[i+1] = t + Delta[i+1] - 1
24         if t == i:
25             S = np.concatenate((survived[:i], np.cumprod(survived[i:t])))
26             return S
27 # arbitrary: index shift of 17 in kaplan_meier_2 which conforms to package
28
29 plt.close('all')
30
31 n = 20000
32 x = np.random.uniform(0, 1, n)
33 now = 0
34 for i in range(n):
35     z1 = randint(0, 1)[0]*now*(1-x[i])
36     for z in x[i]:
37         if z >= z1:
38             break
39     now += 1
40 Delta = np.array([float(i)/N for i in range(N)])
41 S = kaplan_meier(x, Delta, now)
42 print(S)
43
44 # plt.step(Delta, S)
45 # plt.hold('off')
46 # plt.title('Kaplan-Meier estimate')
47 # plt.ylim(0, 1)
48 # plt.xlabel('time')
49 # plt.ylabel('NA_estimate')
50
51
52 def KM(tau, E):
53     I = np.arange(E)
54     t = [tau[k] for k in I]
55     x = [x[k] for k in I]
56     n = len(t)
57     S = np.ones(n)
58     for k in range(n-1):
59         S[k+1] = S[k] * (1 - Delta[k]/(n-k))
60     S[n-1] = S[n-1] * E/(n-k)

```

The bottom half of the screen displays a plot titled 'Figure 2' showing the Kaplan-Meier survival estimate. The x-axis is labeled 'time' and ranges from 0.0 to 1.0. The y-axis ranges from 0.0 to 1.4. A blue line represents the 'NA_estimate', which starts at (0, 1) and decreases towards 0 as time increases. A legend in the top right corner identifies the blue line as 'NA_estimate'.

The right side of the interface features a 'Console' window showing command-line history and output, and a 'Historique' (History) window.

The screenshot shows the official website for scikit-learn at <http://scikit-learn.org/stable/index.html>. The page features a blue header with the scikit-learn logo and navigation links for Home, Installation, Documentation, Examples, and Google Custom Search. A prominent orange "Feed me on GitHub" button is visible. The main content area has a blue background with a grid of colorful, abstract data visualizations. The title "scikit-learn" is displayed in large white font, followed by "Machine Learning in Python". Below the title is a bulleted list of features:

- Simple and efficient tools for data mining and data analysis
- Accessible to everybody, and reusable in various contexts
- Built on NumPy, SciPy, and matplotlib
- Open source, commercially usable - BSD license

The page is organized into several sections with sub-sections and examples:

- Classification**: Identifying to which category an object belongs to.
Applications: Spam detection, Image recognition.
Algorithms: SVM, nearest neighbors, random forest, ...
— Examples
- Regression**: Predicting a continuous-valued attribute associated with an object.
Applications: Drug response, Stock prices.
Algorithms: EVR, ridge regression, Lasso, ...
— Examples
- Clustering**: Automatic grouping of similar objects into sets.
Applications: Customer segmentation, Grouping experiment outcomes
Algorithms: k-Means, spectral clustering, mean-shift, ...
— Examples
- Dimensionality reduction**: Reducing the number of random variables to consider.
Applications: Visualization, Increased efficiency
Algorithms: PCA, feature selection, non-negative matrix factorization.
— Examples
- Model selection**: Comparing, validating and choosing parameters and models.
Goal: Improved accuracy via parameter tuning
Modules: grid search, cross validation, metrics.
— Examples
- Preprocessing**: Feature extraction and normalization.
Application: Transforming input data such as text for use with machine learning algorithms.
Modules: preprocessing, feature extraction.
— Examples
- News**: On-going development: What's new (Changelog)
- Community**: About us See authors and contributing
More Machine Learning Find related
- Who uses scikit-learn?**: AWeber

Knime, Weka and co: integrated environments

Weka Explorer

Preprocess Classify Cluster Associate Select attributes Visualize

Classifier

Choose J48 - C 0.25 - M 2

Test options

Use training set

Supplied test set Set...

Cross-validation Folds 10

Percentage split % 66

More options...

Classifier output

```
==== Stratified cross-validation ====
==== Summary ====
Correctly Classified Instances      144          96      %
Incorrectly Classified Instances     6           4      %
Kappa statistic                      0.94
Mean absolute error                  0.035
Root mean square error               0.035
```

Weka Classifier Tree Visualizer: 12:18:13 - trees.j48.J4

(Nom) class

Tree View

```
graph TD
    Root[petalwidth] --> L1_1[petalwidth]
    L1_1 --> L2_1[Iris-setosa 50.0]
    L1_1 --> L2_2[Iris-versicolor 48.0/1.0]
    L2_1 --> L3_1[petalwidth]
    L3_1 --> L4_1[Iris-virginica 46.0/1.0]
    L3_1 --> L4_2[Iris-versicolor 3.0/1.0]
    L2_2 --> L3_2[petallength]
    L3_2 --> L4_3[Iris-virginica 3.0]
    L3_2 --> L4_4[Iris-versicolor 3.0/1.0]
```

Y: petalwidth (Num)

Select Instance

ave Jitter

Visualize classifier

Visualize tree

Visualize margin

Visualize threshold

St

Pr

Visualize: 12:18:13 - trees.j48.J4

Y: petalwidth (Num)

Select Instance

ave Jitter

Visualize classifier

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Visualize margin

Visualize threshold

St

Pr

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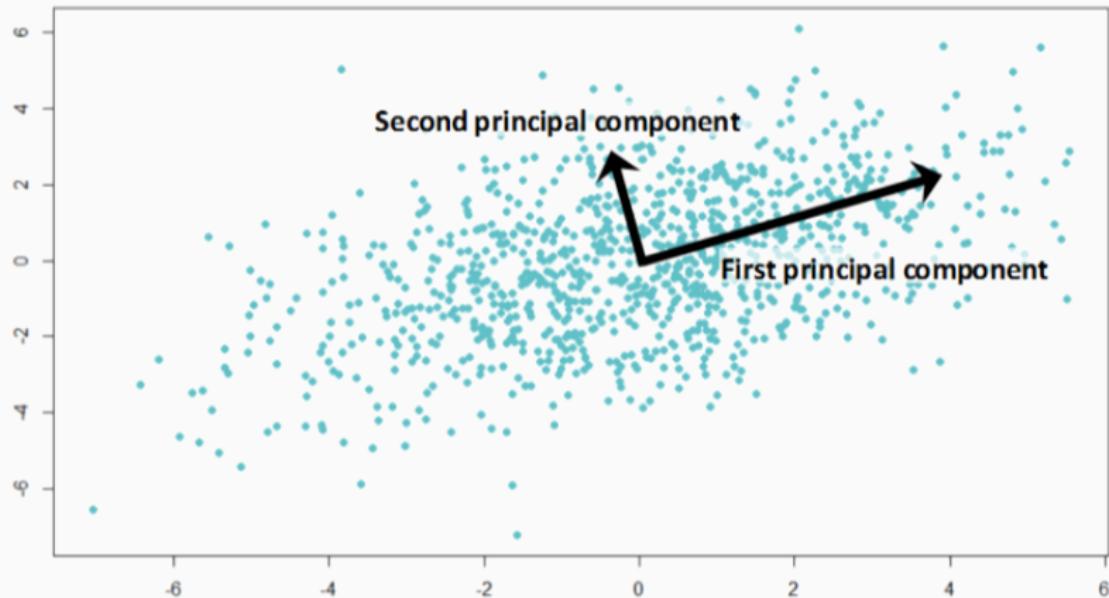
Unsupervised learning

Supervised learning

Unsupervised methods

- PCA
- K-means
- Hierarchical Methods
- Others:
 - Spectral clustering
 - t-SNE
 - ...

PCA: visualization



Src: [https://techannouncer.com/global-pca-unit-market-2017-adelte-airmak-industries-amss-ltd-cavotec-airport-division-ciati-effeti/]

PCA algorithm

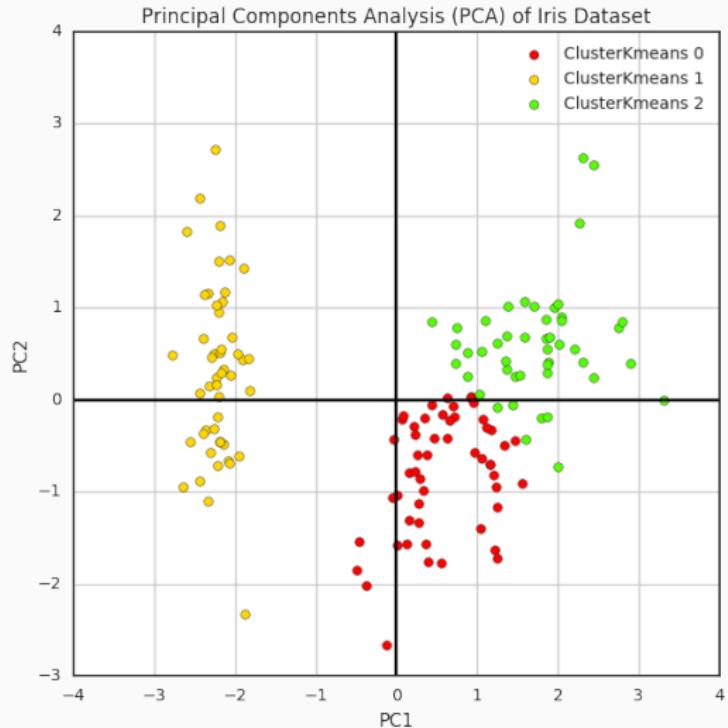
PCA

- Center all variables
- Compute the $p \times p$ empirical covariance matrix $X^T X$.
- Compute the components $W_d =$ the d first eigenvectors of $X^T X$ in decreasing order of the eigenvalues
- Return the projection of X onto the d first components $T_d = X W_d$.

Then:

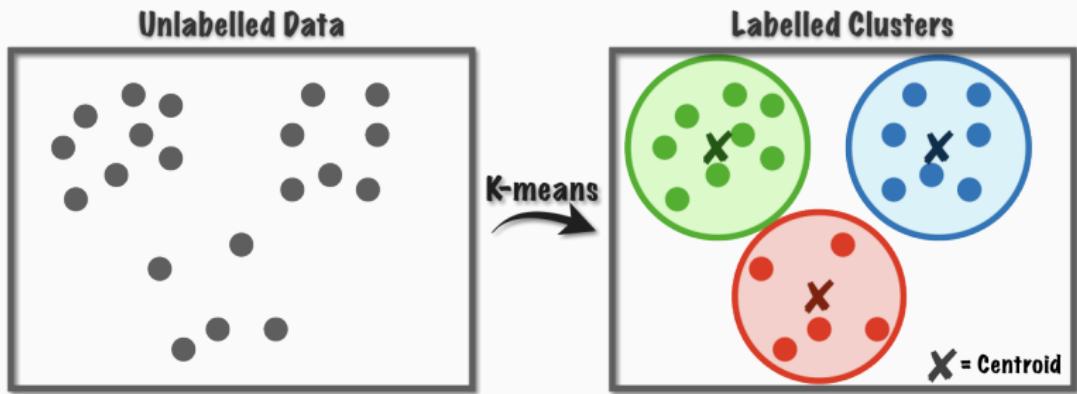
- either vizualize clusters (2d or 3d plots)
- or use another clustering algorithm on the lower-dimensionnal data T_d (*dimension reduction*)

Example: IRIS



[Src: <https://www.kaggle.com/bburns/iris-exploration-pca-k-means-and-gmm-clustering>]

Model-free clustering: K-means



Src: [towardsdatascience.com](https://towardsdatascience.com/k-means-clustering-explained-with-pictures-1000e3f3a2)

K-means target

- Observations X_1, \dots, X_n in \mathbb{R}^p ;
- Objective function: for candidate cluster centers $\mu = (\mu_1, \dots, \mu_K)$ and cluster assignations $z = (z_1, \dots, z_n)$:

$$L(\mu, z) = \sum_{k=1}^K \sum_{i:z_i=k} \|X_i - \mu_k\|^2 = \sum_{i=1}^n \sum_{k=1}^K \mathbb{1}\{z_i = k\} \|X_i - \mu_k\|^2$$

- If $S_k = \{i : z_i = k\}$,

$$L(\mu, z) = \sum_{k=1}^K |S_k| \text{Var}[S_k]$$

- Minimizing L is equivalent to minimizing pairwise deviations in the clusters:

$$\operatorname{argmin}_{\mu, z} L(\mu, z) = \operatorname{argmin}_{\mu, z} \sum_{k=1}^K \frac{1}{|S_k|} \sum_{i, j \in S_k} \|X_i - X_j\|^2$$

Lloyd's algorithm

- For a fixed μ , optimizing in z is easy: choose $z_i = \operatorname{argmin}_k \|X_i - \mu_k\|$
- BUT optimizing in μ is NP-hard!

k-means

- randomly initialize θ_0
- compute Lloyd's iterations until convergence:
 - *membership variables* $z_i^j = \operatorname{argmin}_k \|X_i - \mu_k^j\|$
 - *updated cluster weights* $N_k^j = \sum_{i=1}^n \mathbb{1}\{z_i^j = k\}$
 - *updated cluster means* $\mu_k^{j+1} = \frac{\sum_{i:z_i^j=k} X_i}{N_k^j}$
- start again (a few times) to look for a better local optimum

(Agglomerative) Hierarchical Cluster Analysis

- greedy bottom-up algorithm
- requires a distance (dissimilarity) between observations $\|x - x'\|$
- choice of *distance between clusters*:
 - complete linkage: $d(A, B) = \max \{ \|x - x'\| : x \in A, x' \in B \}$
 - single linkage: $d(A, B) = \min \{ \|x - x'\| : x \in A, x' \in B \}$
 - average linkage distance: $d(A, B) = \frac{1}{|A||B|} \sum_{x \in A} \sum_{x' \in B} \|x - x'\|$
 - Ward distance for Euclidian mean: $d(A, B) = \frac{|A||B|}{n(|A| + |B|)} \|\bar{A} - \bar{B}\|$
 - sum of intra-cluster variance
 - etc.

HCA algorithm

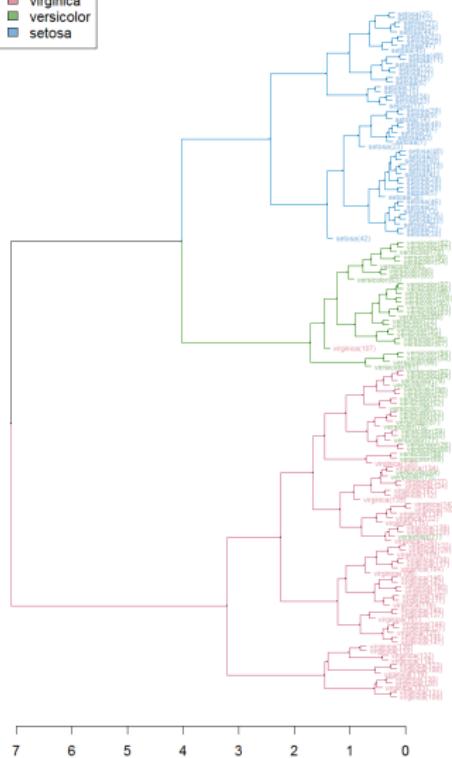
HCA

- Initialization: all observations are clusters $\{X_1\}, \dots, \{X_n\}$
- As long as there are at least two clusters:
 - add a link between two clusters with smallest distance
 - merge them for the next iterations
- Return the *dendrogram* = hierarchy of clusters

HCA: Dendrogram

Clustered Iris data set
(the labels give the true flower species)

virginica
versicolor
setosa



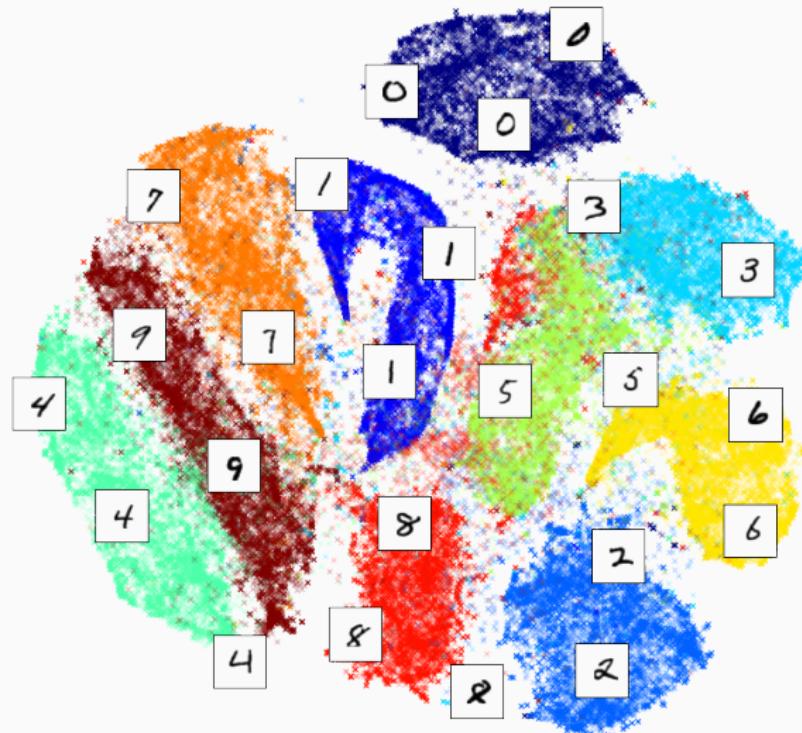
Author: Talgalili

[https://commons.wikimedia.org/wiki/File:
Iris.dendrogram.png](https://commons.wikimedia.org/wiki/File:Iris.dendrogram.png)

t-SNE, and the others...

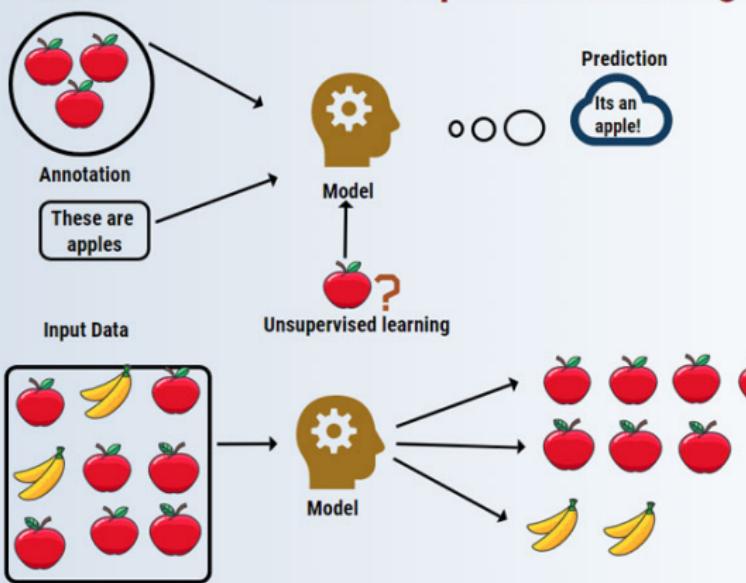
MNIST dataset

Two-dimensional embedding of 70,000 handwritten digits with t-SNE



Supervised learning

What is Supervised Learning?



Qualities of a supervised learning algorithm

- Efficiency (consistency / minimax)
- Tuning: parameter-free
- Result-critic
- Interpretability / Explicability
- Complexity
- Sequentiality
- Privacy

La méthodes des k plus proches voisins

Règle des k plus proches voisins : pour tout $x \in \mathcal{X}$, trouver ses plus proches voisins $x_{(1)}, \dots, x_{(k)}$



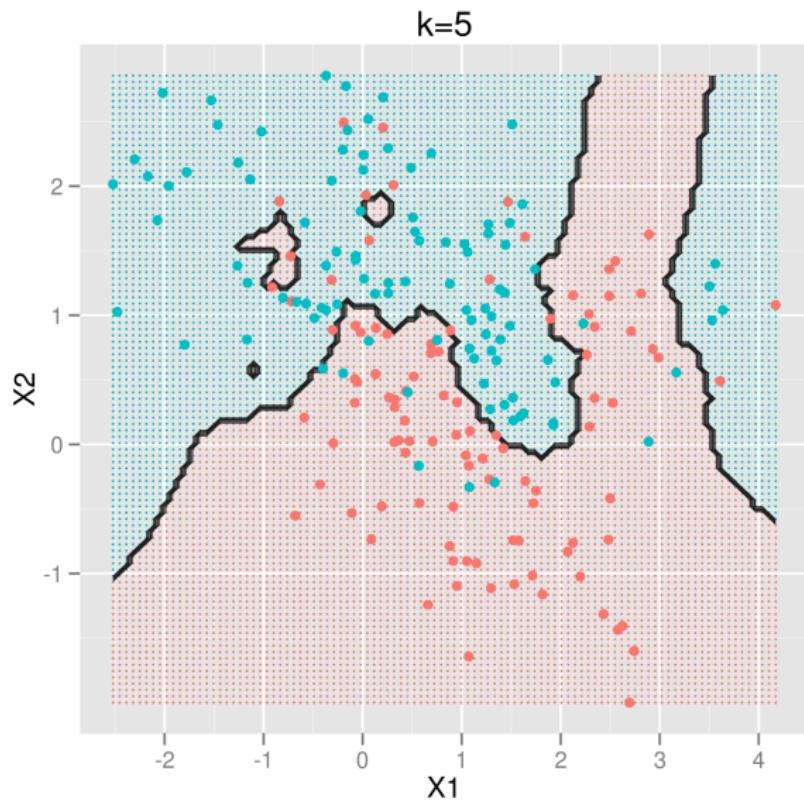
- classification:

$$f_n(x) = \operatorname{argmax}_{y \in \mathcal{Y}} \sum_{j=1}^k \mathbb{1}\{y_{(j)} = y\}$$

- régression:

$$f_n(x) = \frac{1}{k} \sum_{j=1}^k y_{(j)}$$

Visualisation d'une règle k-NN



Propriétés de k-NN

Qualités :

- simplicité
- interprétabilité (?)
- pas de consistance (quel que soit k)
- MAIS asymptotiquement erreur au plus 2x supérieure à la règle de Bayes.
- possibilité de faire croître k avec n , consistance (théorique) par exemple pour $k = \log(n)$

Paramétrage:

- quelle métrique sur \mathcal{X} ?
⇒ au minimum, *normaliser* les variables (pb pour les qualitatives)
- Quelle valeur de k choisir ?

Qualités de kNN

Interprétabilité: OUI et NON

Critique: OUI mais pas très fiable

Consistance: NON mais possible si $k = \log(n)$ (par exemple)

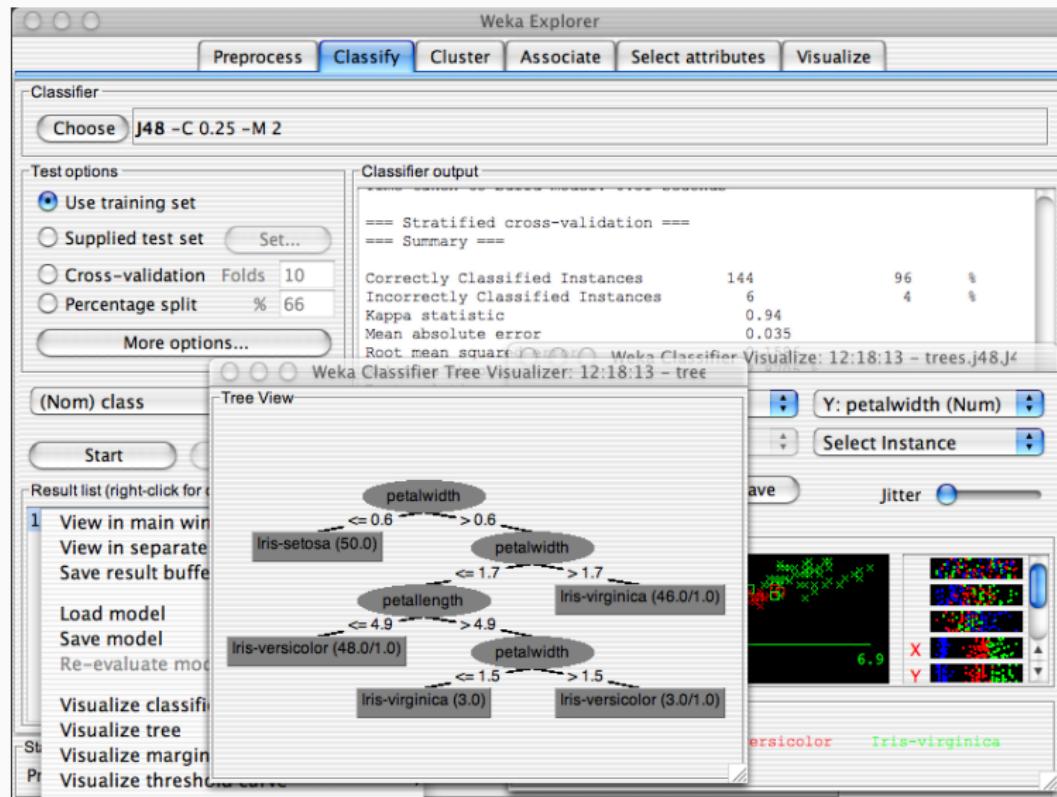
Minimax: NON

Parameter-free: NON

Vitesse: OUI et NON, implémentation possible en $O(n \log n)$

Online: OUI

Arbres de décision



La segmentation par arbre est une approche non-paramétrique de l'analyse discriminante.

But : expliquer une variable réponse (qualitative ou quantitative) à l'aide d'autres variables.

Principe : construire un arbre à l'aide de divisions successives des individus d'un ensemble E en deux segments (appelés aussi noeuds) homogènes par rapport à une variable Y (binaire, nominale, ordinale ou quantitative) en utilisant l'information de p variables X^1, \dots, X^p (binaires, nominales, ordinaires ou quantitatives).

L'arbre obtenu est sous forme d'un arbre inversé comportant à la racine l'échantillon total E à segmenter et les autres segments sont

- soit des segments intermédiaires (encore divisibles),
- soit des segments terminaux.

L'ensemble des segments terminaux constitue une partition de l'ensemble E en classes homogènes et distinctes, relativement à la variable Y .

Il s'agit d'**arbre de classement** si Y est qualitative et d'**arbre de régression** si Y est quantitative.

Avantages / Inconvénients

- La méthode CART (**Classification And Regression Tree**) fournit des solutions sous formes graphiques simples à interpréter.
- Elle est complémentaire des méthodes statistiques classiques, très calculatoire et efficace à condition d'avoir de grandes tailles d'échantillon.
- Elle est capable de gérer à la fois les variables quantitatives ET qualitatives simultanément.
- Peu d'hypothèses requises !
- Algorithme étant basé sur une stratégie pas à pas hiérarchisée, il peut passer à côté d'un optimum global.

Soient p variables quantitatives ou qualitatives explicatives X^j et une variable à expliquer Y qualitative à m modalités $\{\tau_l, l = 1, \dots, m\}$ ou quantitative réelle, observée sur un échantillon de n individus.

La construction d'un arbre de discrimination binaire consiste à déterminer une séquence de **noeuds**.

- Un noeud est défini par le choix conjoint d'une variable parmi les explicatives et d'une **division** qui induit une partition en deux classes.
- Une division est elle-même définie par une valeur seuil de la variable quantitative sélectionnée ou un partage en deux groupes des modalités si la variable est qualitative.
- À la racine ou au noeud initial correspond l'ensemble de l'échantillon. La procédure est ensuite itérée sur chacun des sous-ensembles.

L'algorithme considéré nécessite

- ① la définition d'un critère permettant de sélectionner la "meilleure" division parmi toutes celles **admissibles** pour les différentes variables ;
- ② une règle permettant de décider qu'un noeud est terminal : il devient alors **feuille** ;
- ③ l'affectation de chaque feuille à l'une des classes ou à une valeur de la variable à expliquer.

Le point 2. correspond encore à la recherche d'un modèle parcimonieux. Un arbre trop détaillé, associé à une sur-paramétrisation, est instable et donc probablement plus défaillant pour la prévision d'autres observations.

Breiman et al. ont mise en place une stratégie de recherche de l'arbre optimal.

- ① Construire l'arbre maximal A_{max} .
- ② Ordonner les sous-arbres selon une séquence emboîtée suivant la décroissance d'un critère pénalisé de déviance ou de taux de mal-classés.
- ③ Sélectionner le sous-arbre optimal : c'est la procédure d'**élagage**.

Objectif : Rechercher le meilleur compromis entre

- un arbre très détaillé, fortement dépendant des observations qui ont permis son estimation, qui fournira un modèle de prévision très instable
- un arbre trop robuste mais grossier qui donne des prédictions trop approximatives.

Principe

- Construire une suite emboîtée de sous-arbres de l'arbre maximum par élagage successif.
- Choisir, parmi cette suite, l'arbre optimal au sens d'un critère.

La solution obtenue par algorithme pas à pas n'est pas nécessairement, globalement optimale mais l'efficacité et la fiabilité sont préférées à l'optimalité.

Qualités du classifieur CART

Interprétabilité: OUI !

Critique: OUI mais pas très précis

Consistance: OUI (sous certaines réserves) MAIS instable!

Minimax: NON !

Parameter-free: NON

Vitesse: OUI

Online: NON