Applied maths PhD topic: Game theory and artificial intelligence

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Practical details This 3-year PhD project is funded by the new Artificial and Natural Intelligence Toulouse Institute (ANITI). It will be part of the chair *Game Theory and Artificial Intelligence*. We are looking for an excellent candidate with a strong background in mathematics (e.g. in game theory, optimization, analysis, probability and statistics) or theoretical computer science. The PhD will start around September 2020, with a monthly net salary of 2097 euros and 64 hours of teaching every year. To apply, send any relevant information to fabien.gensbittel@ut-capitole.fr and jerome.renault@ut-capitole.fr

Research topic This PhD will investigate several links between Game Theory and Artificial Intelligence. The topic may be adapted, depending on the background and the interests of the PhD student.

1) Adversarial and multi-agent models, such as Generative Adversarial Networks, are receiving a lot of attention in the AI community, and there is a recent growing literature on the behavior of gradient or second-order algorithms (see e.g. Letcher et al. (2019),Daskalakis and Panageas (2018), Mazumdar et al. (2019)) in general n-players games with differentiable payoff functions. This theoretical framework corresponds for instance to the case where each agent is updating parameters of a neural network which outputs his action. In such cases we need to consider algorithms that are implementable, and cannot assume convexity properties on the payoff functions.

A research direction would be to analyze the behavior of dynamics induced by recent algorithms in several specific classes of n-players games, where the equilibrium strategies are possibly very complex. When do they converge to (ε-, local) Nash equilibrium outcomes ?

A related question is to understand the outcomes naturally emerging in AI contexts, when the players are restricted to certain classes of algorithms or have limited computational power, such as neural networks or finite automata. (for example, no-regret algorithms in standard strategic games typically converge to the set of correlated equilibria, see Hart and Mas-Colell (2013))

In both cases, possible classes of games include stochastic games, large games (large number of players, large actions sets, large state spaces,...), games with incomplete information or imperfect monitoring, etc. A particular attention should be paid to the existence, characterization and computation of the emerging outcomes.

2) It is also interesting to study hybrid games where some players are rational agents while the other players are restricted to use fixed algorithms. The analysis of equilibria in such games, the comparison with equilibria with only rational players, and the analysis of models with possibly incomplete information about the type of the agents can be envisaged.

References

Constantinos Daskalakis and Ioannis Panageas. The limit points of (optimistic) gradient descent in min-max optimization. In S. Bengio, H. Wallach, H. Larochelle, K. Grauman, N. Cesa-Bianchi, and R. Garnett, editors, *Advances in Neural Information Processing Systems 31*, pages 9236–9246. Curran Associates, Inc., 2018.

Fabien Gensbittel and Jérôme Renault. The value of Markov chain games with incomplete information on both sides. *Math. Oper. Res.*, 40(4):820–841, 2015. ISSN 0364-765X; 1526-5471/e.

Sergiu Hart and Andreu Mas-Colell. *Simple adaptive strategies: from regret-matching to uncoupled dynamics*, volume 4. World Scientific, 2013.

Rida Laraki, Jérôme Renault, and Sylvain Sorin. *Bases mathématiques de la Théorie des Jeux (in French)*. Editions de l'Ecole Polytechnique, 2013.

Alistair Letcher, David Balduzzi, Sébastien Racanière, James Martens, Jakob N Foerster, Karl Tuyls, and Thore Graepel. Differentiable game mechanics. *Journal of Machine Learning Research*, 20(84):1–40, 2019.

Michael Maschler, Eilon Solan, and Shmuel Zamir. *Game Theory*. Cambridge University Press, 2013. doi: 10.1017/CBO9780511794216.

Eric V Mazumdar, Michael I Jordan, and S Shankar Sastry. On finding local nash equilibria (and only local nash equilibria) in zero-sum games. *arXiv preprint arXiv:1901.00838*, 2019.

Sylvain Sorin. *A first course on zero-sum repeated games*, volume 37. Springer Science & Business Media, 2002.