High Performance Parallel Tucker Decomposition of Sparse Tensors

Oguz Kaya

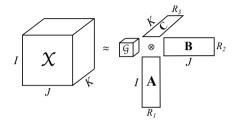
INRIA and LIP, ENS Lyon, France

SIAM PP'16, April 14, 2016, Paris, France

Joint work with:

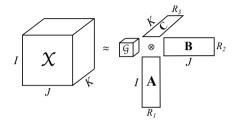
Bora Uçar, CNRS and LIP, ENS Lyon, France





- Tucker decomposition

 - provides a rank-(R₁,..., R_N) approximation of a tensor.
 consists of a core tensor G∈ R^{R₁×···×R_N} and N matrices having R₁,..., R_N columns.
- We are interested in the case when \mathcal{X} is **big. sparse**, and is of **low rank**.
 - Example: Google web queries, Netflix movie ratings, Amazon product reviews, etc.



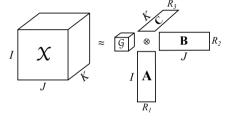
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- Related Work:
 - Matlab Tensor Toolbox by Kolda et al.
 - Efficient and scalable computations with sparse tensors (Baskaran et al., '12)
 - Parallel Tensor Compression for Large-Scale Scientific Data (Austin et al., '15)
 - Haten2: Billion-scale tensor decompositions (Jung et al., '15)
- Applications (in data mining):
 - CubeSVD: A Novel Approach to Personalized Web Search (Sun et al., '05)
 - Tag Recommendations Based on Tensor Dimensionality Reduction (Symeonidis et al., '08)
 - Extended feature combination model for recommendations in location-based mobile services (Sattari et al. '15)
- Goal: To compute sparse Tucker decomposition in parallel (shared/distributed memory).

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Algorithm: HOOI for 3rd order tensors

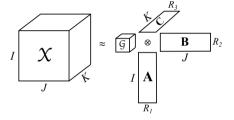
```
repeat

1  \hat{A} \leftarrow [X \times_2 B \times_3 C]_{(1)}
```

until no more improvement or maximum iterations reached

- 7 $\mathcal{G} \leftarrow \mathcal{X} \times_1 \mathbf{A} \times_2 \mathbf{B} \times_3 \mathbf{C}$
- 8 <u>return [*G*</u>; A, B, C]
- We discuss the case where $R_1 = R_2 = \cdots = R_N = R$ and N = 3.
- $\mathbf{A} \in \mathbb{R}^{I \times R}$, $\mathbf{B} \in \mathbb{R}^{J \times R}$, and $\mathbf{C} \in \mathbb{R}^{K \times R}$ are dense.
- $\mathbf{A} \leftarrow [\mathcal{X} \times_2 \mathbf{B} \times_3 \mathbf{C}]_{(1)} \in \mathbb{R}^{I \times R^*}$ is called tensor-times-matrix multiply (TTM).
- $\mathbf{A} \in \mathbb{R}^{I \times R^{n-1}}, \mathbf{B} \in \mathbb{R}^{J \times R^{n-1}}$, and $\mathbf{C} \in \mathbb{R}^{K \times R^{n-1}}$ are dense. $(R^2 \text{ columns for } N = 3)$





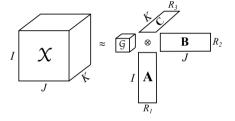
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```
repeat
```

```
\hat{\mathbf{A}} \leftarrow [\mathbf{\mathcal{X}} \times_2 \mathbf{B} \times_3 \mathbf{C}]_{(1)}
2 \mathbf{A} \leftarrow \text{TRSVD}(\hat{\mathbf{A}}, R_1) / R_1 leading left singular vectors
\hat{\mathbf{B}} \leftarrow [\mathbf{\mathcal{X}} \times_1 \mathbf{A} \times_3 \mathbf{C}]_{(2)}
4 \mathbf{B} \leftarrow \text{TRSVD}(\hat{\mathbf{B}}, R_2)
           \hat{\mathbf{C}} \leftarrow [\mathbf{X} \times_1 \mathbf{A} \times_2 \mathbf{B}]_{(2)}
              \mathbf{C} \leftarrow \text{TRSVD}(\hat{\mathbf{C}}, R_3)
     until no more improvement or maximum iterations reached
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Algorithm: HOOI for 3rd order tensors

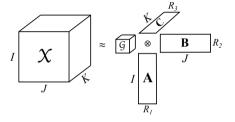
```
repeat \mathbf{\hat{A}} \leftarrow [\mathcal{X} \times_2 \mathbf{B} \times_3 \mathbf{C}]_{(1)}
```

```
    Â ← [X ×<sub>2</sub> B ×<sub>3</sub> C]<sub>(1)</sub>
    A ← TRSVD(Â, R<sub>1</sub>) //R<sub>1</sub> leading left singular vectors
    Â ← [X ×<sub>1</sub> A ×<sub>5</sub> C]<sub>(2)</sub>
    B ← TRSVD(Â, R<sub>2</sub>)
    Ĉ ← [X ×<sub>1</sub> A ×<sub>2</sub> B]<sub>(2)</sub>
    C ← TRSVD(Â, R<sub>3</sub>)
    until no more improvement or maximum iterations reached
```

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- 7 $\mathcal{G} \leftarrow \mathcal{X} \times_1 \mathbf{A} \times_2 \mathbf{B} \times_3 \mathbf{C}$
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3 | \hat{\mathbf{B}} \leftarrow [\mathcal{X} \times_1 \mathbf{A} \times_3 \mathbf{C}]_{(2)}

4 | \mathbf{B} \leftarrow \text{TRSVD}(\hat{\mathbf{B}}, R_2)

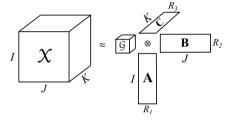
5 | \hat{\mathbf{C}} \leftarrow [\mathcal{X} \times_1 \mathbf{A} \times_2 \mathbf{B}]_{(2)}

6 | \mathbf{C} \leftarrow \text{TRSVD}(\hat{\mathbf{C}}, R_3)
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Tensor-Times-Matrix Multiply

•
$$\hat{\mathbf{A}} \leftarrow [\mathbf{\mathcal{X}} \times_2 \mathbf{B} \times_3 \mathbf{C}]_{(1)}, \ \hat{\mathbf{A}} \in \mathbb{R}^{I \times R^2}$$

- $\mathbf{B}(j,:)\otimes\mathbf{C}(k,:)\in\mathbb{R}^{R^2}$ is a Kronecker product.
- For each nonzero $x_{i,j,k}$; $\hat{\mathbf{A}}(i, \cdot)$ receives the undate

$$\hat{\mathbf{A}}(i,:)$$
 receives the update $x_{i,j,k}[\mathbf{B}(j,:)\otimes\mathbf{C}(k,:)]$.

Algorithm:
$$\hat{\mathbf{A}} \leftarrow [\mathbf{\mathcal{X}} \times_2 \mathbf{B} \times_3 \mathbf{C}]_{(1)}$$

1 $\hat{\mathbf{A}} \leftarrow zeros(I, R^2)$ foreach $x_{i,j,k} \in \mathcal{X}$ do

$$\mathbf{2} \quad \begin{bmatrix} \hat{\mathbf{A}}(i,:) \leftarrow \hat{\mathbf{A}}(i,:) + x_{i,j,k} [\mathbf{B}(j,:) \otimes \mathbf{C}(k,:)] \end{bmatrix}$$

Outline

- Introduction
- Parallel HOOI
- Results
- 4 Conclusion

(Bad) Fine-Grain Parallel TTM within Tucker-ALS

- A and are rowwise distributed
 - Process p owns and computes $\mathbf{A}(I_p,:)$ and $\hat{\mathbf{A}}(I_p,:)$.
- Tensor nonzeros are partitioned (arbitrarily)
 - Process p owns the subset of nonzeros \mathcal{X}_n
 - Performing $x_{i,j,k}[\mathbf{B}(j,:)\otimes\mathbf{C}(k,:)]$ and generating a partial result for $\hat{\mathbf{A}}(i,:)$ is a fine-grain task
 - We use post-communication scheme at each iteration: \rightarrow at the beginning, rows of **A**. **B**. and **C** are available.
 - → at the end, only **A** is updated and communicated.
- Partial row results of are sent/received (fold).
- Rows of $A(I_p,:)$ are sent/received (expand).

Algorithm: Computing A in fine-grain HOOI at process p

foreach $x_{i,i,k} \in \mathcal{X}_p$ do

$$\mathbf{1} \quad | \quad \hat{\mathbf{A}}(i,:) \leftarrow \hat{\mathbf{A}}(i,:) + x_{i,j,k}[\mathbf{B}(j,:) \otimes \mathbf{C}(k,:)]$$

- 2 Send/Receive and sum up "partial" rows of Â
- 3 $\mathbf{A}(I_n,:) \leftarrow \text{TRSVD}(\hat{\mathbf{A}}, R)$
- 4 Send/Receive rows of A

(Bad) Fine-Grain Parallel TTM within Tucker-ALS

- Number of rows sent/received in fold/expand are equal.
 - Each communication unit of expand has size R.
 - Each communication unit of fold has size R^{N-1} .
- We want to avoid assembling in fold communication.
- We need to compute $TRSVD(\hat{\mathbf{A}}, R)$.

Algorithm: Computing A in fine-grain HOOI at process p

foreach $x_{i,i,k} \in \mathcal{X}_p$ do 1 $\hat{\mathbf{A}}(i,:) \leftarrow \hat{\mathbf{A}}(i,:) + x_{i,j,k}[\mathbf{B}(j,:) \otimes \mathbf{C}(k,:)]$

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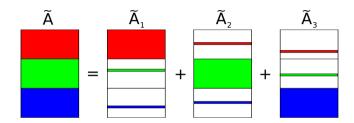
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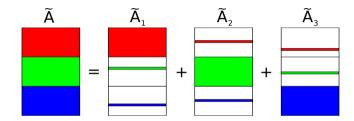
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Computing TRSVD



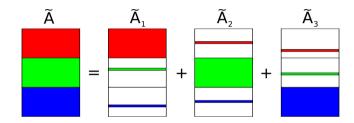
- Gram matrix $\hat{\mathbf{A}}\hat{\mathbf{A}}^T$?
- Iterative solvers?
 - Need to perform $\hat{\mathbf{A}}x$ and $\hat{\mathbf{A}}^Tx$ efficiently.

Computing TRSVD



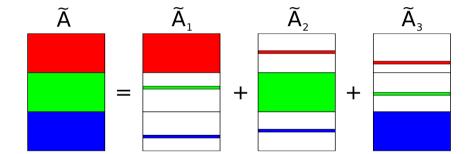
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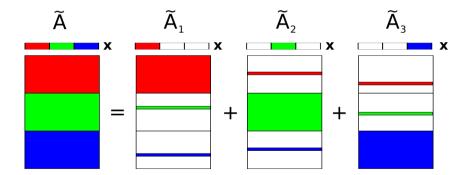


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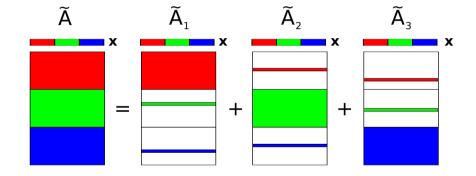
Computing $y \leftarrow \hat{\mathbf{A}}x$



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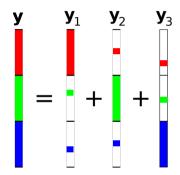


Computing $y \leftarrow \mathbf{\hat{A}}x$



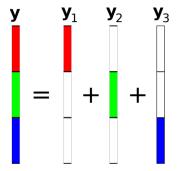
• For each unit of communication, we perform extra work in MxV.

Computing $y \leftarrow \hat{\mathbf{A}}x$



• Instead of communicating R^{N-1} entries, we communicate 1! (per SVD iteration)

Computing $y \leftarrow \mathbf{\hat{A}}x$



- $y \leftarrow \mathbf{\hat{A}}^T x$ works in reverse with the same communication cost.
- ullet Row distribution of ullet and left-singular vectors are the same as $\hat{oldsymbol{A}}$
 - A gets the same row distribution as Â.



(Good) Fine-Grain Parallel TTM within Tucker-ALS

Algorithm: Computing **A** in fine-grain HOOI at process p

foreach
$$x_{i,j,k} \in \mathcal{X}_p$$
 do
1 $\hat{\mathbf{A}}(i,:) \leftarrow \hat{\mathbf{A}}(i,:) + x_{i,j,k}[\mathbf{B}(j,:) \otimes \mathbf{C}(k,:)]$

- 2 $\mathbf{A}(I_p,:) \leftarrow \text{TRSVD}(\mathbf{\hat{A}}, R, \mathsf{M} \times \mathsf{V}(\dots), \mathsf{M} \mathsf{T} \times \mathsf{V}(\dots))$
- 3 Send/Receive rows of A

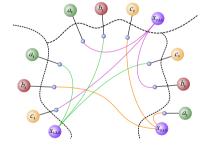
Hypergraph Model for Parallel HOOI

- Multi-constraint hypergraph partitioning
 - We balance computation and memory costs.
- we minimize:

• By minimizing the **cutsize** of the hypergraph,

- the total communication volume of MtV/MTxV,
- the total extra MxV/MTxV work,
- and the total volume of communication for TTM.
- Ideally, should minimize the maximum, not total

$$\mathcal{X} = \{(1,2,3), (2,3,1), (3,1,2)\}$$



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Experimental Setup

- HyperTensor
 - Hybrid OpenMP/MPI code in C++
 - Dependencies to BLAS, LAPACK, and C++11 STL
 - SLEPc/PETSc for distributed memory TRSVD computations
- IBM BlueGene/Q Machine
 - 16GB memory and 16 cores (at 1.6GHz) per node
 - Experiments using up to 4096 cores (256 nodes)
- R_i is set to 5/10 for 4/3-dimensional tensors.

Tensor sizes

Tensor	I_1	I_2	<i>I</i> ₃	<i>I</i> ₄	#nonzeros
Netflix	480K	17K	2K	-	100M
NELL	3.2M	301	638K	-	78M
Delicious	1K	530K	17M	2.4M	140M
Flickr	713	319K	28M	1.6M	112M

Results - Flickr/Delicious

Per iteration runtime of the parallel HOOI (in seconds)

$\# nodes \times \# cores$	Delicious			Flickr				
	fine-hp	fine-rd	coarse-hp	coarse-bl	fine-hp	fine-rd	coarse-hp	coarse-bl
1 × 16	-	-	-	-	-	-	-	-
2×16	-	-	-	-	-	-	-	-
4×16	-	-	-	-	-	-	-	-
8 × 16	164.9	-	235.3	400.5	206.2	-	287.5	308.5
16 imes 16	85.2	162.0	197.5	302.4	115.6	221.8	210.5	230.1
32 × 16	47.6	96.2	155.6	206.5	64.6	124.5	166.3	190.1
64×16	27.2	57.8	98.9	159.6	36.8	69.9	124.1	129.0
128 × 16	18.2	34.7	80.8	96.4	22.6	42.9	87.9	102.3
256 imes 16	12.2	22.1	65.1	77.1	20.0	29.2	73.8	86.3

- Coarse-grain kernel is slow due to load imbalance and communication.
- On Delicious, fine-hp is 1.8x/5.4x/6.4x faster than fine-rd/coarse-hp/coarse-bl.
- On Flickr, fine-hp is 1.5x/3.7x/4.3x faster than fine-rd/coarse-hp/coarse-bl.
- All instances achieve scalability to 4096 cores.



Results - NELL/Netflix

Per iteration runtime of the parallel HOOI (in seconds)

#nodes×#cores	NELL				Netflix			
	fine-hp	fine-rd	coarse-hp	coarse-bl	fine-hp	fine-rd	coarse-hp	coarse-bl
1×16	222.1	222.1	240.1	240.1	-	-	-	-
2 × 16	151.6	137.6	198.5	164.4	-	-	-	-
4 × 16	87.7	75.9	180.6	131.4	33.7	39.2	46.0	42.8
8 imes 16	67.8	46.9	172.5	109.7	18.6	26.1	30.6	33.4
16 imes 16	54.9	28.3	112.4	94.1	10.3	18.3	32.2	27.8
32×16	43.9	17.2	73.8	68.2	5.7	13.9	26.2	26.7
64×16	35.4	11.9	67.1	54.5	3.9	10.9	26.2	21.7
128 × 16	26.7	8.4	50.3	48.5	2.9	8.7	19.8	18.7
256 imes 16	14.8	7.7	48.1	44.9	3.8	8.3	14.7	16.1

- On Netflix, fine-hp is 2.8x/5x/5.5x faster than fine-rd/coarse-hp/coarse-bl.
- On NELL, fine-rd is faster than fine-hp (5x less total comm. but 2x more max comm.)

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- We provide
 - the first high performance shared/distributed memory parallel algorithm/implementation for the Tucker decomposition of sparse tensors
 - hypergraph partitioning models of these computations for better scalability.
- We achieve scalability up to 4096 cores even with random partitioning.
- We enable Tucker-based tensor analysis of very big sparse data.

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References

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