gfun[istranscendental] test transcendence of a solution to a linear differential equation with initial conditions

Calling Sequence

istranscendental(deq, y(z))

Parameters

deq - a set containing a linear differential equation with polynomial coefficients and initial conditions specifying a unique solution of it;

y(z) - the unknown function and its variable.

Description

- Given a linear differential equation together with initial conditions specifying a unique solution of it, the `istranscendent` command determines whether that solution is transcendent or not. It returns either true, followed by an explanation, in which case the solution is transcendental, or FAIL, meaning that it was not able to prove that the solution is transcendental.
- This command relies on the command minimizediffeq of the gfun package.
- This command is part of the `gfun` package, so it can be used in the form `istranscendental(..)` only after executing the command `with(gfun)`. However, it can always be accessed through the long form of the command by using `gfun[istranscendental](..)`.

Examples

```
> with(gfun):
> f:=exp(z):
> deq:=holexprtodiffeq(f,y(z));
```

$$deq := \left\{ \frac{d}{dz} y(z) - y(z), y(0) = 1 \right\} \quad (4.1)$$

```
> istranscendental(deq,y(z));
```

true, "irregular singularity at infinity" (4.2)

```
> deq:={(20*z^6+12*z^5)*y(z)+(4*z^7+z^2+3*z-9)*diff(y(z),z)+(z^3
  -3*z^2)*diff(diff(y(z)
  ,z),z), y(0) = 1};
```

$$deq := \left\{ \left(20 z^6 + 12 z^5\right) y(z) + \left(4 z^7 + z^2 + 3 z - 9\right) \left(\frac{d}{dz} y(z)\right) + \left(z^3 - 3 z^2\right) \left(\frac{d^2}{dz^2} \right. \right. \quad (4.3)$$

$$y(z)\Big), y(0) = 1\Big\}$$

```
> istranscendental(deq,y(z));
```

$$true, \text{"irregular singularity at infinity"} \quad (4.4)$$

```
> deq:={z^2*(1+z)*diff(diff(diff(y(z),z),z),z)-z*(2*z^2+2*z-1)*
  diff(diff(y(z),z),z)+(-
  z^2-4*z-1)*diff(y(z),z), y(0) = 0, (D@@2)(y)(0) = 1/4};
```

$$deq := \left\{z^2 (1+z) \left(\frac{d^3}{dz^3} y(z)\right) - z (2z^2 + 2z - 1) \left(\frac{d^2}{dz^2} y(z)\right) + (-z^2 - 4z - 1) \left(\frac{d}{dz} y(z)\right), y(0) = 0, D^{(2)}(y)(0) = \frac{1}{4}\right\} \quad (4.5)$$

```
> istranscendental(deq,y(z));
```

$$true, \text{"multiple root of the indicial equation ==> ln at 0"} \quad (4.6)$$

```
> f:=sqrt(1-z);
```

$$f := \sqrt{1-z} \quad (4.7)$$

```
> deq:=holexprtodiffeq(f,y(z));
```

$$deq := \left\{-y(z) + (2z-2)\left(\frac{d}{dz} y(z)\right), y(0) = 1\right\} \quad (4.8)$$

```
> istranscendental(deq,y(z));
```

$$FAIL \quad (4.9)$$

A tricky hypergeometric function:

```
> f:=hypergeom([1/6,5/6],[7/6],z):
> deq:=holexprtodiffeq(f,y(z));
```

$$deq := \left\{5 y(z) + (72z - 42)\left(\frac{d}{dz} y(z)\right) + (36z^2 - 36z)\left(\frac{d^2}{dz^2} y(z)\right), y(0) = 1\right\} \quad (4.10)$$

```
> istranscendental(deq,y(z));
```

$$FAIL \quad (4.11)$$

In this case the function is transcendental, but this was not proved by the algorithm used in `istranscendental`.

See Also

gfun[minimizediffeq], gfun