gfun[minimizediffeq] minimize differential equation given initial conditions

Calling Sequence

minimizediffeq(deq, y(z), options)

Parameters

deq - a set containing a linear differential equation with polynomial coefficients and initial conditions specifying a unique solution of it;
y(z) - the unknown function and its variable;
homogeneous - (optional) boolean telling whether one is interested in the minimal homogeneous or inhomogeneous linear differential equation (default is true).

Description

- Given a linear differential equation together with initial conditions specifying a unique solution of it, the minimizediffeq command finds the linear differential equation with polynomial coefficients of minimal order having this solution. If the option homogeneous is set to false, then it returns a non-homogeneous one of minimal order, if one exists, otherwise the minimal homogeneous equation is returned.
- This is an analogue for differential equations of the command MinimalRecurrence of the package LREtools.
- This command is part of the gfun package, so it can be used in the form minimizediffeq(..) only after executing the command with(gfun). However, it can always be accessed through the long form of the command by using gfun [minimizediffeq](..).

Examples

```maple
> with(gfun):
> deq:={(20*z^6+12*z^5)*y(z)+(4*z^7+z^2+3*z-9)*diff(y(z),z)+(z^3-3*z^2)*diff(diff(y(z),z),z), y(0) = 1};
```

```maple
deq := 
\[
\begin{align*}
(20 \, z^6 + 12 \, z^5) \, y(z) + (4 \, z^7 + z^2 + 3 \, z - 9) \, \frac{d}{dz} \, y(z) + (z^3 - 3 \, z^2) \, \frac{d^2}{dz^2} \, y(z), \\
y(0) = 1
\end{align*}
\]
```
> minimizediffeq(deq,y(z));
\[
\begin{align*}
4y(z)z^5 + (z - 3) \left( \frac{d}{dz} y(z) \right), y(0) &= 1
\end{align*}
\]

> deq:={z^2*(1+z)*diff(diff(diff(y(z),z),z),z)-z*(2*z^2+2*z-1)*diff(diff(y(z),z),z)+(-z^2-4*z-1)*diff(y(z),z), y(0) = 0, (D@@2)(y)(0) = 1/4};
\[
deq := \left\{ z^2 (1 + z) \left( \frac{d^3}{dz^3} y(z) \right) - z \left( 2 z^2 + 2 z - 1 \right) \left( \frac{d^2}{dz^2} y(z) \right) + (-z^2 - 4 z - 1) \left( \frac{d}{dz} y(z) \right), y(0) = 0, D^{(2)}(y)(0) = \frac{1}{4} \right\}
\]

> minimizediffeq(deq,y(z));
\[
\begin{align*}
\left\{ (-z^2 - 4 z - 1) \left( \frac{d}{dz} y(z) \right) + (-2 z^3 - 2 z^2 + z) \left( \frac{d^2}{dz^2} y(z) \right) + (z^3 + z^2) \left( \frac{d^3}{dz^3} y(z) \right), y(0) = 0, D^{(2)}(y)(0) = \frac{1}{4} \right\}
\end{align*}
\]

So this equation is already minimal. However, the solution also satisfies a non-homogeneous equation of smaller order:

> minimizediffeq(deq,y(z),false);
\[
\begin{align*}
\left\{ y(z) - \frac{(2 z^3 - 3 z^2 + 2 z - 1)}{z (1 + z)} \left( \frac{d}{dz} y(z) \right) + \frac{(z^2 - 2 z + 1) \left( z^3 + z^2 \right)}{z^2 (1 + z)^2} \left( \frac{d^2}{dz^2} y(z) \right) - \frac{1}{2}, y(0) = 0, D^{(2)}(y)(0) = \frac{1}{4} \right\}
\end{align*}
\]

See Also

LREtools[MinimalRecurrence], gfun