gfun[minimizediffeq] minimize differential equation given initial conditions

Calling Sequence

`minimizediffeq`(deq, y(z), options)

Parameters

`deq` - a set containing a linear differential equation with polynomial coefficients and initial conditions specifying a unique solution of it;

`y(z)` - the unknown function and its variable;

`homogeneous` - (optional) boolean telling whether one is interested in the minimal homogeneous or inhomogeneous linear differential equation (default is true).

Description

- Given a linear differential equation together with initial conditions specifying a unique solution of it, the `minimizediffeq` command finds the linear differential equation with polynomial coefficients of minimal order having this solution. If the option homogeneous is set to false, then it returns a non-homogeneous one of minimal order, if one exists, otherwise the minimal homogeneous equation is returned.
- This is an analogue for differential equations of the command MinimalRecurrence of the package LREtools.
- This command is part of the `gfun` package, so it can be used in the form `minimizediffeq(..)` only after executing the command `with(gfun)`. However, it can always be accessed through the long form of the command by using `gfun[minimizediffeq](..)`.

Examples

```
> with(gfun):
```

```
> deq:={(20*z^6+12*z^5)*y(z)+(4*z^7+z^2+3*z-9)*diff(y(z),z)+(z^3-3*
  z^2)*diff(diff(y(z)
  ,z),z), y(0) = 1};
```

$$deq := \left\{\left(20\,z^6+12\,z^5\right)y(z)+\left(4\,z^7+z^2+3\,z-9\right)\left(\frac{d}{dz}\,y(z)\right)+\left(z^3-3\,z^2\right)\left(\frac{d^2}{dz^2}\,y(z)\right),\ y(0)=1\right\}$$

```
> minimizediffeq(deq,y(z));
```

$$\left\{4\,y(z)\,z^5 + (z-3)\left(\frac{d}{dz}\,y(z)\right), y(0) = 1\right\}$$

```
> deq:={z^2*(1+z)*diff(diff(diff(y(z),z),z),z)-z*(2*z^2+2*z-1)*diff
  (diff(y(z),z),z)+(-
  z^2-4*z-1)*diff(y(z),z), y(0) = 0, (D@@2)(y)(0) = 1/4};
```

$$deq := \left\{z^2\,(1+z)\left(\frac{d^3}{dz^3}\,y(z)\right) - z\left(2\,z^2+2\,z-1\right)\left(\frac{d^2}{dz^2}\,y(z)\right) + \left(-z^2-4\,z-1\right)\left(\frac{d}{dz}\,y(z)\right), y(0) = 0, D^{(2)}(y)(0) = \frac{1}{4}\right\}$$

```
> minimizediffeq(deq,y(z));
```

$$\left\{\left(-z^2-4\,z-1\right)\left(\frac{d}{dz}\,y(z)\right) + \left(-2\,z^3-2\,z^2+z\right)\left(\frac{d^2}{dz^2}\,y(z)\right) + \left(z^3+z^2\right)\left(\frac{d^3}{dz^3}\,y(z)\right), y(0) = 0, D^{(2)}(y)(0) = \frac{1}{4}\right\}$$

So this equation is already minimal. However, the solution also satisfies a non-homogeneous equation of smaller order:

```
> minimizediffeq(deq,y(z),false);
```

$$\left\{y(z) - \frac{\left(2\,z^3-3\,z^2+2\,z-1\right)\left(\frac{d}{dz}\,y(z)\right)}{z\,(1+z)} + \frac{\left(z^2-2\,z+1\right)\left(z^3+z^2\right)\left(\frac{d^2}{dz^2}\,y(z)\right)}{z^2\,(1+z)^2} - \frac{1}{2}, y(0) = 0, D^{(2)}(y)(0) = \frac{1}{4}\right\}$$

See Also

LREtools[MinimalRecurrence], gfun