```
> restart;
```
Load module:
```
> read
  "/Users/salvy/Distants/Git/positivityproofs/PositivityProofs.
  mpl";
```
and some data for examples
```
> read `recGRZini.m`;
```

Straub-Zudilin example

```
> S:=Sum((-27)^(n-k)*2^(2*k-n)*(3*k)!/k!^3*binomial(k,n-k),k=0..n)
  ;
```
$$S := \sum_{k=0}^{n} \frac{(-27)^{n-k}\, 2^{2k-n}\, (3k)!\, \binom{k}{n-k}}{k!^3} \tag{1.1}$$
```
> SumTools[Hypergeometric][ZeilbergerRecurrence](op(1,S),n,k,s,0..
  n);
```
$$(-729 n^2 - 1458 n - 648)\, s(n) + (-2 n^2 - 8 n - 8)\, s(n+2) + (81 n^2 + 243 n + 186)\, s(n+1) = 0 \tag{1.2}$$
```
> rec:={%,seq(s(i)=value(eval(S,n=i)),i=0..1)};
```
$$rec := \{(-729 n^2 - 1458 n - 648)\, s(n) + (-2 n^2 - 8 n - 8)\, s(n+2) + (81 n^2 + 243 n + 186)\, s(n+1) = 0, s(0) = 1, s(1) = 12\} \tag{1.3}$$
```
> Positivity(rec,s,n);
```
$$true, 1, \begin{bmatrix} \frac{1}{2} & 1 \\ \frac{27}{2} & \frac{27}{2} \end{bmatrix} \tag{1.4}$$
the second part of the answer is the "index of contraction" and the matrix gives the cone which gives a proof by induction of the positivity.

An example of order 3

```
> rec:={(n+1)*u(n+3)=(77/30*n+2)*u(n+2)-(13/6*n-3)*u(n+1)+(3/5*
  n+2)*u(n),u(0)=1,u(1)=15/14,u(2)=8/7};
```
$$rec := \left\{(n+1)\, u(3+n) = \left(\frac{77 n}{30} + 2\right) u(n+2) - \left(\frac{13 n}{6} - 3\right) u(n+1) + \left(\frac{3 n}{5} + 2\right) u(n), u(0) = 1, u(1) = \frac{15}{14}, u(2) = \frac{8}{7}\right\} \tag{2.1}$$
```
> Positivity(rec,u,n);
```
(2.2)

$$\mathit{true},\ 1236,\ \begin{bmatrix} \frac{1129}{900} & 1 & 1 \\ \frac{1129}{900} & \frac{2}{3} & \frac{9}{10} \\ \frac{1129}{900} & \frac{4}{9} & \frac{81}{100} \end{bmatrix} \quad \textbf{(2.2)}$$

Positivity of the first Gillis-Reznick-Zeilberger sequences

```
> recgrz[4];
```

$$\{(10616832\, n^6 + 138018816\, n^5 + 701374464\, n^4 + 1765380096\, n^3 + 2308829184\, n^2 + 1500622848\, n + 383201280)\, u(n) + (1769472\, n^6 + 25657344\, n^5 + 150073344\, n^4 + 453150720\, n^3 + 746896896\, n^2 + 640811520\, n + 224985600)\, u(n+1) + (110592\, n^6 + 1769472\, n^5 + 11582208\, n^4 + 39696768\, n^3 + 75175488\, n^2 + 74657088\, n + 30421440)\, u(n+2) + (-5120\, n^6 - 89600\, n^5 - 647744\, n^4 - 2473952\, n^3 - 5258744\, n^2 - 5889032\, n - 2708160)\, u(3+n) + (32\, n^6 + 608\, n^5 + 4738\, n^4 + 19353\, n^3 + 43628\, n^2 + 51376\, n + 24640)\, u(n+4) = 0, u(0) = 1, u(1) = 0, u(2) = 216, u(3) = 18816\} \quad \textbf{(3.1)}$$

```
> Positivity(%,u,n);
```

$$\mathit{true},\ 2,\ \begin{bmatrix} \frac{2369}{68952} & 1 & \dots \\ \frac{1054406365}{236091648} & \frac{414767}{9866} & \dots \\ \frac{469300456966025}{808377802752} & \frac{172031664289}{97337956} & \frac{14}{1} \dots \\ \frac{20887859388872323 7125}{2767885596622848} & \frac{713530573021556 63}{960336273896} & \frac{2635}{121} \dots \end{bmatrix} \quad \textbf{(3.2)}$$

Change one initial condition:

```
> subs(18816=14,recgrz[4]);
```

$$\{(10616832\, n^6 + 138018816\, n^5 + 701374464\, n^4 + 1765380096\, n^3 + 2308829184\, n^2 + 1500622848\, n + 383201280)\, u(n) + (1769472\, n^6 + 25657344\, n^5 + 150073344\, n^4 + 453150720\, n^3 + 746896896\, n^2 + 640811520\, n + 224985600)\, u(n+1) + (110592\, n^6 + 1769472\, n^5 + 11582208\, n^4 + 39696768\, n^3 + 75175488\, n^2 + 74657088\, n + 30421440)\, u(n+2) + (-5120\, n^6 - 89600\, n^5 - 647744\, n^4 - 2473952\, n^3 - 5258744\, n^2 - 5889032\, n - 2708160)\, u(3+n) + (32\, n^6 + 608\, n^5 + 4738\, n^4 + 19353\, n^3 + 43628\, n^2 + 51376\, n + 24640)\, u(n+4) = 0, u(0) = 1, u(1) \quad \textbf{(3.3)}$$

$= 0, u(2) = 216, u(3) = 14\}$

```
> Positivity(%,u,n);
```

$$\mathit{false}, 4 \qquad (3.4)$$

the sequence is no longer positive and the first index where this can be seen is 4.

Example for the case k=10, where the recurrence is significantly larger:

```
> recgrz[10];
```

$$\{(96665[...84\ digits...]00000\, n^{45} + 26389[...87\ digits...]00000\, n^{44} \qquad (3.5)$$
$$+ 35020[...89\ digits...]00000\, n^{43} + 30099[...91\ digits...]00000\, n^{42}$$
$$+ 18836[...93\ digits...]00000\, n^{41} + 91481[...94\ digits...]00000\, n^{40}$$
$$+ 35886[...96\ digits...]00000\, n^{39} + 11686[...98\ digits...]00000\, n^{38}$$
$$+ 32218[...99\ digits...]00000\, n^{37} + 76327[...100\ digits...]00000\, n^{36}$$
$$+ 15716[...102\ digits...]00000\, n^{35} + 28381[...103\ digits...]00000\, n^{34}$$
$$+ 45273[...104\ digits...]00000\, n^{33} + 64163[...105\ digits...]00000\, n^{32}$$
$$+ 81173[...106\ digits...]00000\, n^{31} + 92014[...107\ digits...]00000\, n^{30}$$
$$+ 93742[...108\ digits...]00000\, n^{29} + 86039[...109\ digits...]00000\, n^{28}$$
$$+ 71275[...110\ digits...]00000\, n^{27} + 53364[...111\ digits...]00000\, n^{26}$$
$$+ 36144[...112\ digits...]00000\, n^{25} + 22157[...113\ digits...]00000\, n^{24}$$
$$+ 12295[...114\ digits...]00000\, n^{23} + 61741[...114\ digits...]00000\, n^{22}$$
$$+ 28040[...115\ digits...]00000\, n^{21} + 11505[...116\ digits...]00000\, n^{20}$$
$$+ 42595[...116\ digits...]00000\, n^{19} + 14203[...117\ digits...]00000\, n^{18}$$
$$+ 42561[...117\ digits...]00000\, n^{17} + 11431[...118\ digits...]00000\, n^{16}$$
$$+ 27436[...118\ digits...]00000\, n^{15} + 58617[...118\ digits...]00000\, n^{14}$$
$$+ 11100[...119\ digits...]00000\, n^{13} + 18540[...119\ digits...]00000\, n^{12}$$
$$+ 27146[...119\ digits...]00000\, n^{11} + 34602[...119\ digits...]00000\, n^{10}$$
$$+ 38077[...119\ digits...]00000\, n^{9} + 35811[...119\ digits...]00000\, n^{8}$$
$$+ 28428[...119\ digits...]00000\, n^{7} + 18753[...119\ digits...]00000\, n^{6}$$
$$+ 10070[...119\ digits...]00000\, n^{5} + 42815[...118\ digits...]00000\, n^{4}$$
$$+ 13839[...118\ digits...]00000\, n^{3} + 31900[...117\ digits...]00000\, n^{2}$$
$$+ 46637[...116\ digits...]00000\, n + 32456[...115\ digits...]00000)\, u(n)$$
$$+ (26638[...79\ digits...]00000\, n^{45} + 73921[...81\ digits...]00000\, n^{44}$$
$$+ 99795[...83\ digits...]00000\, n^{43} + 87335[...85\ digits...]00000\, n^{42}$$
$$+ 55702[...87\ digits...]00000\, n^{41} + 27598[...89\ digits...]00000\, n^{40}$$
$$+ 11057[...91\ digits...]00000\, n^{39} + 36815[...92\ digits...]00000\, n^{38}$$
$$+ 10391[...94\ digits...]00000\, n^{37} + 25237[...95\ digits...]00000\, n^{36}$$
$$+ 53352[...96\ digits...]00000\, n^{35} + 99071[...97\ digits...]00000\, n^{34}$$
$$+ 16278[...99\ digits...]00000\, n^{33} + 23809[...100\ digits...]00000\, n^{32}$$
$$+ 31147[...101\ digits...]00000\, n^{31} + 36590[...102\ digits...]00000\, n^{30}$$

$$+ 38725[...103 \text{ digits}...]00000\, n^{29} + 37020[...104 \text{ digits}...]00000\, n^{28}$$
$$+ 32032[...105 \text{ digits}...]00000\, n^{27} + 25129[...106 \text{ digits}...]00000\, n^{26}$$
$$+ 17894[...107 \text{ digits}...]00000\, n^{25} + 11575[...108 \text{ digits}...]00000\, n^{24}$$
$$+ 68060[...108 \text{ digits}...]00000\, n^{23} + 36376[...109 \text{ digits}...]00000\, n^{22}$$
$$+ 17669[...110 \text{ digits}...]00000\, n^{21} + 77963[...110 \text{ digits}...]00000\, n^{20}$$
$$+ 31221[...111 \text{ digits}...]00000\, n^{19} + 11334[...112 \text{ digits}...]00000\, n^{18}$$
$$+ 37239[...112 \text{ digits}...]00000\, n^{17} + 11052[...113 \text{ digits}...]00000\, n^{16}$$
$$+ 29561[...113 \text{ digits}...]00000\, n^{15} + 71037[...113 \text{ digits}...]00000\, n^{14}$$
$$+ 15284[...114 \text{ digits}...]00000\, n^{13} + 29319[...114 \text{ digits}...]00000\, n^{12}$$
$$+ 49889[...114 \text{ digits}...]00000\, n^{11} + 74839[...114 \text{ digits}...]00000\, n^{10}$$
$$+ 98234[...114 \text{ digits}...]00000\, n^{9} + 11177[...115 \text{ digits}...]00000\, n^{8}$$
$$+ 10898[...115 \text{ digits}...]00000\, n^{7} + 89678[...114 \text{ digits}...]00000\, n^{6}$$
$$+ 61053[...114 \text{ digits}...]00000\, n^{5} + 33455[...114 \text{ digits}...]00000\, n^{4}$$
$$+ 14173[...114 \text{ digits}...]00000\, n^{3} + 43552[...113 \text{ digits}...]00000\, n^{2}$$
$$+ 86320[...112 \text{ digits}...]00000\, n + 82813[...111 \text{ digits}...]00000 \big)\, u(n+1)$$
$$+ \big(33033[...73 \text{ digits}...]00000\, n^{45} + 93154[...75 \text{ digits}...]00000\, n^{44}$$
$$+ 12788[...78 \text{ digits}...]00000\, n^{43} + 11389[...80 \text{ digits}...]00000\, n^{42}$$
$$+ 73982[...81 \text{ digits}...]00000\, n^{41} + 37361[...83 \text{ digits}...]00000\, n^{40}$$
$$+ 15270[...85 \text{ digits}...]00000\, n^{39} + 51915[...86 \text{ digits}...]00000\, n^{38}$$
$$+ 14977[...88 \text{ digits}...]00000\, n^{37} + 37215[...89 \text{ digits}...]00000\, n^{36}$$
$$+ 80580[...90 \text{ digits}...]00000\, n^{35} + 15343[...92 \text{ digits}...]00000\, n^{34}$$
$$+ 25884[...93 \text{ digits}...]00000\, n^{33} + 38920[...94 \text{ digits}...]00000\, n^{32}$$
$$+ 52415[...95 \text{ digits}...]00000\, n^{31} + 63482[...96 \text{ digits}...]00000\, n^{30}$$
$$+ 69374[...97 \text{ digits}...]00000\, n^{29} + 68593[...98 \text{ digits}...]00000\, n^{28}$$
$$+ 61495[...99 \text{ digits}...]00000\, n^{27} + 50076[...100 \text{ digits}...]00000\, n^{26}$$
$$+ 37087[...101 \text{ digits}...]00000\, n^{25} + 25005[...102 \text{ digits}...]00000\, n^{24}$$
$$+ 15357[...103 \text{ digits}...]00000\, n^{23} + 85932[...103 \text{ digits}...]00000\, n^{22}$$
$$+ 43808[...104 \text{ digits}...]00000\, n^{21} + 20339[...105 \text{ digits}...]00000\, n^{20}$$
$$+ 85937[...105 \text{ digits}...]00000\, n^{19} + 33008[...106 \text{ digits}...]00000\, n^{18}$$
$$+ 11509[...107 \text{ digits}...]00000\, n^{17} + 36360[...107 \text{ digits}...]00000\, n^{16}$$
$$+ 10384[...108 \text{ digits}...]00000\, n^{15} + 26736[...108 \text{ digits}...]00000\, n^{14}$$
$$+ 61841[...108 \text{ digits}...]00000\, n^{13} + 12797[...109 \text{ digits}...]00000\, n^{12}$$
$$+ 23576[...109 \text{ digits}...]00000\, n^{11} + 38432[...109 \text{ digits}...]00000\, n^{10}$$
$$+ 55021[...109 \text{ digits}...]00000\, n^{9} + 68544[...109 \text{ digits}...]00000\, n^{8}$$
$$+ 73445[...109 \text{ digits}...]00000\, n^{7} + 66679[...109 \text{ digits}...]00000\, n^{6}$$
$$+ 50280[...109 \text{ digits}...]00000\, n^{5} + 30637[...109 \text{ digits}...]00000\, n^{4}$$

$$
\begin{aligned}
&+ 14491[...109\text{ digits}...]00000\, n^{3} + 49908[...108\text{ digits}...]00000\, n^{2} \\
&+ 11132[...108\text{ digits}...]00000\, n + 12067[...107\text{ digits}...]00000 \big) u(n+2) \\
&+ \big(24275[...67\text{ digits}...]00000\, n^{45} + 69548[...69\text{ digits}...]00000\, n^{44} \\
&+ 97058[...71\text{ digits}...]00000\, n^{43} + 87921[...73\text{ digits}...]00000\, n^{42} \\
&+ 58124[...75\text{ digits}...]00000\, n^{41} + 29894[...77\text{ digits}...]00000\, n^{40} \\
&+ 12451[...79\text{ digits}...]00000\, n^{39} + 43174[...80\text{ digits}...]00000\, n^{38} \\
&+ 12711[...82\text{ digits}...]00000\, n^{37} + 32262[...83\text{ digits}...]00000\, n^{36} \\
&+ 71409[...84\text{ digits}...]00000\, n^{35} + 13911[...86\text{ digits}...]00000\, n^{34} \\
&+ 24031[...87\text{ digits}...]00000\, n^{33} + 37034[...88\text{ digits}...]00000\, n^{32} \\
&+ 51167[...89\text{ digits}...]00000\, n^{31} + 63641[...90\text{ digits}...]00000\, n^{30} \\
&+ 71496[...91\text{ digits}...]00000\, n^{29} + 72749[...92\text{ digits}...]00000\, n^{28} \\
&+ 67196[...93\text{ digits}...]00000\, n^{27} + 56440[...94\text{ digits}...]00000\, n^{26} \\
&+ 43168[...95\text{ digits}...]00000\, n^{25} + 30094[...96\text{ digits}...]00000\, n^{24} \\
&+ 19135[...97\text{ digits}...]00000\, n^{23} + 11100[...98\text{ digits}...]00000\, n^{22} \\
&+ 58745[...98\text{ digits}...]00000\, n^{21} + 28352[...99\text{ digits}...]00000\, n^{20} \\
&+ 12470[...100\text{ digits}...]00000\, n^{19} + 49936[...100\text{ digits}...]00000\, n^{18} \\
&+ 18178[...101\text{ digits}...]00000\, n^{17} + 60047[...101\text{ digits}...]00000\, n^{16} \\
&+ 17958[...102\text{ digits}...]00000\, n^{15} + 48488[...102\text{ digits}...]00000\, n^{14} \\
&+ 11779[...103\text{ digits}...]00000\, n^{13} + 25642[...103\text{ digits}...]00000\, n^{12} \\
&+ 49764[...103\text{ digits}...]00000\, n^{11} + 85581[...103\text{ digits}...]00000\, n^{10} \\
&+ 12944[...104\text{ digits}...]00000\, n^{9} + 17062[...104\text{ digits}...]00000\, n^{8} \\
&+ 19369[...104\text{ digits}...]00000\, n^{7} + 18656[...104\text{ digits}...]00000\, n^{6} \\
&+ 14945[...104\text{ digits}...]00000\, n^{5} + 96861[...103\text{ digits}...]00000\, n^{4} \\
&+ 48789[...103\text{ digits}...]00000\, n^{3} + 17915[...103\text{ digits}...]00000\, n^{2} \\
&+ 42652[...102\text{ digits}...]00000\, n + 49403[...101\text{ digits}...]00000 \big) u(3+n) \\
&+ \big(11706[...61\text{ digits}...]00000\, n^{45} + 34066[...63\text{ digits}...]00000\, n^{44} \\
&+ 48310[...65\text{ digits}...]00000\, n^{43} + 44491[...67\text{ digits}...]00000\, n^{42} \\
&+ 29918[...69\text{ digits}...]00000\, n^{41} + 15659[...71\text{ digits}...]00000\, n^{40} \\
&+ 66412[...72\text{ digits}...]00000\, n^{39} + 23459[...74\text{ digits}...]00000\, n^{38} \\
&+ 70405[...75\text{ digits}...]00000\, n^{37} + 18224[...77\text{ digits}...]00000\, n^{36} \\
&+ 41165[...78\text{ digits}...]00000\, n^{35} + 81891[...79\text{ digits}...]00000\, n^{34} \\
&+ 14454[...81\text{ digits}...]00000\, n^{33} + 22775[...82\text{ digits}...]00000\, n^{32} \\
&+ 32194[...83\text{ digits}...]00000\, n^{31} + 40996[...84\text{ digits}...]00000\, n^{30} \\
&+ 47186[...85\text{ digits}...]00000\, n^{29} + 49226[...86\text{ digits}...]00000\, n^{28} \\
&+ 46651[...87\text{ digits}...]00000\, n^{27} + 40233[...88\text{ digits}...]00000\, n^{26} \\
&+ 31619[...89\text{ digits}...]00000\, n^{25} + 22667[...90\text{ digits}...]00000\, n^{24}
\end{aligned}
$$

$$+ 14832[...91 \text{ digits}...]00000\, n^{23} + 88617[...91 \text{ digits}...]00000\, n^{22}$$
$$+ 48339[...92 \text{ digits}...]00000\, n^{21} + 24066[...93 \text{ digits}...]00000\, n^{20}$$
$$+ 10927[...94 \text{ digits}...]00000\, n^{19} + 45209[...94 \text{ digits}...]00000\, n^{18}$$
$$+ 17016[...95 \text{ digits}...]00000\, n^{17} + 58166[...95 \text{ digits}...]00000\, n^{16}$$
$$+ 18014[...96 \text{ digits}...]00000\, n^{15} + 50408[...96 \text{ digits}...]00000\, n^{14}$$
$$+ 12700[...97 \text{ digits}...]00000\, n^{13} + 28692[...97 \text{ digits}...]00000\, n^{12}$$
$$+ 57831[...97 \text{ digits}...]00000\, n^{11} + 10335[...98 \text{ digits}...]00000\, n^{10}$$
$$+ 16257[...98 \text{ digits}...]00000\, n^{9} + 22295[...98 \text{ digits}...]00000\, n^{8}$$
$$+ 26351[...98 \text{ digits}...]00000\, n^{7} + 26437[...98 \text{ digits}...]00000\, n^{6}$$
$$+ 22069[...98 \text{ digits}...]00000\, n^{5} + 14912[...98 \text{ digits}...]00000\, n^{4}$$
$$+ 78338[...97 \text{ digits}...]00000\, n^{3} + 30010[...97 \text{ digits}...]00000\, n^{2}$$
$$+ 74560[...96 \text{ digits}...]00000\, n + 90140[...95 \text{ digits}...]00000\big)\, u(n+4)$$
$$+ \big(38712[...54 \text{ digits}...]00000\, n^{45} + 11439[...57 \text{ digits}...]00000\, n^{44}$$
$$+ 16479[...59 \text{ digits}...]00000\, n^{43} + 15422[...61 \text{ digits}...]00000\, n^{42}$$
$$+ 10542[...63 \text{ digits}...]00000\, n^{41} + 56119[...64 \text{ digits}...]00000\, n^{40}$$
$$+ 24214[...66 \text{ digits}...]00000\, n^{39} + 87053[...67 \text{ digits}...]00000\, n^{38}$$
$$+ 26602[...69 \text{ digits}...]00000\, n^{37} + 70142[...70 \text{ digits}...]00000\, n^{36}$$
$$+ 16145[...72 \text{ digits}...]00000\, n^{35} + 32744[...73 \text{ digits}...]00000\, n^{34}$$
$$+ 58950[...74 \text{ digits}...]00000\, n^{33} + 94780[...75 \text{ digits}...]00000\, n^{32}$$
$$+ 13677[...77 \text{ digits}...]00000\, n^{31} + 17788[...78 \text{ digits}...]00000\, n^{30}$$
$$+ 20920[...79 \text{ digits}...]00000\, n^{29} + 22310[...80 \text{ digits}...]00000\, n^{28}$$
$$+ 21624[...81 \text{ digits}...]00000\, n^{27} + 19082[...82 \text{ digits}...]00000\, n^{26}$$
$$+ 15352[...83 \text{ digits}...]00000\, n^{25} + 11272[...84 \text{ digits}...]00000\, n^{24}$$
$$+ 75578[...84 \text{ digits}...]00000\, n^{23} + 46289[...85 \text{ digits}...]00000\, n^{22}$$
$$+ 25896[...86 \text{ digits}...]00000\, n^{21} + 13228[...87 \text{ digits}...]00000\, n^{20}$$
$$+ 61662[...87 \text{ digits}...]00000\, n^{19} + 26198[...88 \text{ digits}...]00000\, n^{18}$$
$$+ 10131[...89 \text{ digits}...]00000\, n^{17} + 35594[...89 \text{ digits}...]00000\, n^{16}$$
$$+ 11335[...90 \text{ digits}...]00000\, n^{15} + 32626[...90 \text{ digits}...]00000\, n^{14}$$
$$+ 84585[...90 \text{ digits}...]00000\, n^{13} + 19669[...91 \text{ digits}...]00000\, n^{12}$$
$$+ 40821[...91 \text{ digits}...]00000\, n^{11} + 75143[...91 \text{ digits}...]00000\, n^{10}$$
$$+ 12176[...92 \text{ digits}...]00000\, n^{9} + 17208[...92 \text{ digits}...]00000\, n^{8}$$
$$+ 20962[...92 \text{ digits}...]00000\, n^{7} + 21680[...92 \text{ digits}...]00000\, n^{6}$$
$$+ 18659[...92 \text{ digits}...]00000\, n^{5} + 12999[...92 \text{ digits}...]00000\, n^{4}$$
$$+ 70420[...91 \text{ digits}...]00000\, n^{3} + 27818[...91 \text{ digits}...]00000\, n^{2}$$
$$+ 71267[...90 \text{ digits}...]00000\, n + 88838[...89 \text{ digits}...]00000\big)\, u(n+5)$$
$$+ \big(88901[...47 \text{ digits}...]00000\, n^{45} + 26670[...50 \text{ digits}...]00000\, n^{44}$$

$+ 39016[...52 digits...]00000\, n^{43} + 37090[...54 digits...]00000\, n^{42}$
$+ 25761[...56 digits...]00000\, n^{41} + 13936[...58 digits...]00000\, n^{40}$
$+ 61133[...59 digits...]00000\, n^{39} + 22350[...61 digits...]00000\, n^{38}$
$+ 69473[...62 digits...]00000\, n^{37} + 18638[...64 digits...]00000\, n^{36}$
$+ 43666[...65 digits...]00000\, n^{35} + 90162[...66 digits...]00000\, n^{34}$
$+ 16530[...68 digits...]00000\, n^{33} + 27074[...69 digits...]00000\, n^{32}$
$+ 39811[...70 digits...]00000\, n^{31} + 52776[...71 digits...]00000\, n^{30}$
$+ 63284[...72 digits...]00000\, n^{29} + 68831[...73 digits...]00000\, n^{28}$
$+ 68060[...74 digits...]00000\, n^{27} + 61289[...75 digits...]00000\, n^{26}$
$+ 50332[...76 digits...]00000\, n^{25} + 37733[...77 digits...]00000\, n^{24}$
$+ 25838[...78 digits...]00000\, n^{23} + 16167[...79 digits...]00000\, n^{22}$
$+ 92423[...79 digits...]00000\, n^{21} + 48256[...80 digits...]00000\, n^{20}$
$+ 22996[...81 digits...]00000\, n^{19} + 99909[...81 digits...]00000\, n^{18}$
$+ 39517[...82 digits...]00000\, n^{17} + 14203[...83 digits...]00000\, n^{16}$
$+ 46279[...83 digits...]00000\, n^{15} + 13632[...84 digits...]00000\, n^{14}$
$+ 36174[...84 digits...]00000\, n^{13} + 86115[...84 digits...]00000\, n^{12}$
$+ 18297[...85 digits...]00000\, n^{11} + 34487[...85 digits...]00000\, n^{10}$
$+ 57226[...85 digits...]00000\, n^{9} + 82822[...85 digits...]00000\, n^{8}$
$+ 10332[...86 digits...]00000\, n^{7} + 10943[...86 digits...]00000\, n^{6}$
$+ 96457[...85 digits...]00000\, n^{5} + 68818[...85 digits...]00000\, n^{4}$
$+ 38173[...85 digits...]00000\, n^{3} + 15440[...85 digits...]00000\, n^{2}$
$+ 40494[...84 digits...]00000\, n + 51669[...83 digits...]00000\big)\, u(n+6)$
$+ \big(13999[...41 digits...]00000\, n^{45} + 42628[...43 digits...]00000\, n^{44}$
$+ 63308[...45 digits...]00000\, n^{43} + 61107[...47 digits...]00000\, n^{42}$
$+ 43104[...49 digits...]00000\, n^{41} + 23686[...51 digits...]00000\, n^{40}$
$+ 10555[...53 digits...]00000\, n^{39} + 39212[...54 digits...]00000\, n^{38}$
$+ 12387[...56 digits...]00000\, n^{37} + 33783[...57 digits...]00000\, n^{36}$
$+ 80467[...58 digits...]00000\, n^{35} + 16895[...60 digits...]00000\, n^{34}$
$+ 31503[...61 digits...]00000\, n^{33} + 52488[...62 digits...]00000\, n^{32}$
$+ 78526[...63 digits...]00000\, n^{31} + 10592[...65 digits...]00000\, n^{30}$
$+ 12927[...66 digits...]00000\, n^{29} + 14313[...67 digits...]00000\, n^{28}$
$+ 14409[...68 digits...]00000\, n^{27} + 13212[...69 digits...]00000\, n^{26}$
$+ 11050[...70 digits...]00000\, n^{25} + 84380[...70 digits...]00000\, n^{24}$
$+ 58863[...71 digits...]00000\, n^{23} + 37524[...72 digits...]00000\, n^{22}$
$+ 21858[...73 digits...]00000\, n^{21} + 11631[...74 digits...]00000\, n^{20}$
$+ 56491[...74 digits...]00000\, n^{19} + 25017[...75 digits...]00000\, n^{18}$

$+ 10087[...76 \text{ digits...}]00000\, n^{17} + 36961[...76 \text{ digits...}]00000\, n^{16}$
$+ 12279[...77 \text{ digits...}]00000\, n^{15} + 36880[...77 \text{ digits...}]00000\, n^{14}$
$+ 99790[...77 \text{ digits...}]00000\, n^{13} + 24223[...78 \text{ digits...}]00000\, n^{12}$
$+ 52484[...78 \text{ digits...}]00000\, n^{11} + 10087[...79 \text{ digits...}]00000\, n^{10}$
$+ 17068[...79 \text{ digits...}]00000\, n^{9} + 25188[...79 \text{ digits...}]00000\, n^{8}$
$+ 32039[...79 \text{ digits...}]00000\, n^{7} + 34599[...79 \text{ digits...}]00000\, n^{6}$
$+ 31089[...79 \text{ digits...}]00000\, n^{5} + 22610[...79 \text{ digits...}]00000\, n^{4}$
$+ 12783[...79 \text{ digits...}]00000\, n^{3} + 52691[...78 \text{ digits...}]00000\, n^{2}$
$+ 14080[...78 \text{ digits...}]00000\, n + 18302[...77 \text{ digits...}]00000)\, u(7 + n)$
$+ (14467[...34 \text{ digits...}]00000\, n^{45} + 44703[...36 \text{ digits...}]00000\, n^{44}$
$+ 67377[...38 \text{ digits...}]00000\, n^{43} + 66011[...40 \text{ digits...}]00000\, n^{42}$
$+ 47266[...42 \text{ digits...}]00000\, n^{41} + 26368[...44 \text{ digits...}]00000\, n^{40}$
$+ 11930[...46 \text{ digits...}]00000\, n^{39} + 45004[...47 \text{ digits...}]00000\, n^{38}$
$+ 14438[...49 \text{ digits...}]00000\, n^{37} + 39989[...50 \text{ digits...}]00000\, n^{36}$
$+ 96746[...51 \text{ digits...}]00000\, n^{35} + 20634[...53 \text{ digits...}]00000\, n^{34}$
$+ 39088[...54 \text{ digits...}]00000\, n^{33} + 66165[...55 \text{ digits...}]00000\, n^{32}$
$+ 10058[...57 \text{ digits...}]00000\, n^{31} + 13787[...58 \text{ digits...}]00000\, n^{30}$
$+ 17099[...59 \text{ digits...}]00000\, n^{29} + 19241[...60 \text{ digits...}]00000\, n^{28}$
$+ 19687[...61 \text{ digits...}]00000\, n^{27} + 18349[...62 \text{ digits...}]00000\, n^{26}$
$+ 15600[...63 \text{ digits...}]00000\, n^{25} + 12110[...64 \text{ digits...}]00000\, n^{24}$
$+ 85884[...64 \text{ digits...}]00000\, n^{23} + 55663[...65 \text{ digits...}]00000\, n^{22}$
$+ 32967[...66 \text{ digits...}]00000\, n^{21} + 17836[...67 \text{ digits...}]00000\, n^{20}$
$+ 88082[...67 \text{ digits...}]00000\, n^{19} + 39663[...68 \text{ digits...}]00000\, n^{18}$
$+ 16261[...69 \text{ digits...}]00000\, n^{17} + 60588[...69 \text{ digits...}]00000\, n^{16}$
$+ 20466[...70 \text{ digits...}]00000\, n^{15} + 62503[...70 \text{ digits...}]00000\, n^{14}$
$+ 17196[...71 \text{ digits...}]00000\, n^{13} + 42441[...71 \text{ digits...}]80000\, n^{12}$
$+ 93491[...71 \text{ digits...}]20000\, n^{11} + 18267[...72 \text{ digits...}]20000\, n^{10}$
$+ 31422[...72 \text{ digits...}]00000\, n^{9} + 47136[...72 \text{ digits...}]20000\, n^{8}$
$+ 60941[...72 \text{ digits...}]80000\, n^{7} + 66882[...72 \text{ digits...}]40000\, n^{6}$
$+ 61070[...72 \text{ digits...}]20000\, n^{5} + 45128[...72 \text{ digits...}]00000\, n^{4}$
$+ 25919[...72 \text{ digits...}]20000\, n^{3} + 10851[...72 \text{ digits...}]20000\, n^{2}$
$+ 29449[...71 \text{ digits...}]00000\, n + 38865[...70 \text{ digits...}]80000)\, u(8 + n) + ($
$- 24325[...29 \text{ digits...}]00000\, n^{45} - 76260[...31 \text{ digits...}]00000\, n^{44}$
$- 11663[...34 \text{ digits...}]00000\, n^{43} - 11598[...36 \text{ digits...}]00000\, n^{42}$
$- 84304[...37 \text{ digits...}]00000\, n^{41} - 47751[...39 \text{ digits...}]00000\, n^{40}$
$- 21941[...41 \text{ digits...}]00000\, n^{39} - 84066[...42 \text{ digits...}]00000\, n^{38}$

$$
\begin{aligned}
&- 27398[\text{...44 digits...}]00000\, n^{37} - 77105[\text{...45 digits...}]00000\, n^{36} \\
&- 18956[\text{...47 digits...}]00000\, n^{35} - 41095[\text{...48 digits...}]00000\, n^{34} \\
&- 79138[\text{...49 digits...}]00000\, n^{33} - 13620[\text{...51 digits...}]00000\, n^{32} \\
&- 21054[\text{...52 digits...}]00000\, n^{31} - 29353[\text{...53 digits...}]00000\, n^{30} \\
&- 37030[\text{...54 digits...}]00000\, n^{29} - 42391[\text{...55 digits...}]00000\, n^{28} \\
&- 44132[\text{...56 digits...}]00000\, n^{27} - 41858[\text{...57 digits...}]00000\, n^{26} \\
&- 36218[\text{...58 digits...}]00000\, n^{25} - 28616[\text{...59 digits...}]00000\, n^{24} \\
&- 20659[\text{...60 digits...}]00000\, n^{23} - 13631[\text{...61 digits...}]00000\, n^{22} \\
&- 82203[\text{...61 digits...}]00000\, n^{21} - 45285[\text{...62 digits...}]00000\, n^{20} \\
&- 22773[\text{...63 digits...}]00000\, n^{19} - 10443[\text{...64 digits...}]00000\, n^{18} \\
&- 43608[\text{...64 digits...}]00000\, n^{17} - 16548[\text{...65 digits...}]00000\, n^{16} \\
&- 56939[\text{...65 digits...}]00000\, n^{15} - 17711[\text{...66 digits...}]60000\, n^{14} \\
&- 49634[\text{...66 digits...}]16000\, n^{13} - 12478[\text{...67 digits...}]45600\, n^{12} \\
&- 27998[\text{...67 digits...}]17600\, n^{11} - 55724[\text{...67 digits...}]82400\, n^{10} \\
&- 97628[\text{...67 digits...}]40000\, n^{9} - 14915[\text{...68 digits...}]06400\, n^{8} \\
&- 19639[\text{...68 digits...}]36000\, n^{7} - 21949[\text{...68 digits...}]86400\, n^{6} \\
&- 20407[\text{...68 digits...}]61600\, n^{5} - 15353[\text{...68 digits...}]32000\, n^{4} \\
&- 89769[\text{...67 digits...}]28800\, n^{3} - 38254[\text{...67 digits...}]36000\, n^{2} \\
&- 10564[\text{...67 digits...}]87200\, n - 14186[\text{...66 digits...}]04000\big)\, u(n+9) \\
&+ \big(24414062500000000000000000\, n^{45} + 77636[\text{...21 digits...}]00000\, n^{44} \\
&+ 12042[\text{...24 digits...}]00000\, n^{43} + 12142[\text{...26 digits...}]00000\, n^{42} \\
&+ 89475[\text{...27 digits...}]00000\, n^{41} + 51370[\text{...29 digits...}]00000\, n^{40} \\
&+ 23920[\text{...31 digits...}]00000\, n^{39} + 92863[\text{...32 digits...}]00000\, n^{38} \\
&+ 30660[\text{...34 digits...}]00000\, n^{37} + 87395[\text{...35 digits...}]00000\, n^{36} \\
&+ 21759[\text{...37 digits...}]00000\, n^{35} + 47760[\text{...38 digits...}]00000\, n^{34} \\
&+ 93106[\text{...39 digits...}]00000\, n^{33} + 16218[\text{...41 digits...}]00000\, n^{32} \\
&+ 25371[\text{...42 digits...}]00000\, n^{31} + 35787[\text{...43 digits...}]00000\, n^{30} \\
&+ 45671[\text{...44 digits...}]50000\, n^{29} + 52879[\text{...45 digits...}]00000\, n^{28} \\
&+ 55670[\text{...46 digits...}]75000\, n^{27} + 53385[\text{...47 digits...}]00000\, n^{26} \\
&+ 46695[\text{...48 digits...}]90625\, n^{25} + 37289[\text{...49 digits...}]81250\, n^{24} \\
&+ 27204[\text{...50 digits...}]87500\, n^{23} + 18135[\text{...51 digits...}]00000\, n^{22} \\
&+ 11047[\text{...52 digits...}]68750\, n^{21} + 61469[\text{...52 digits...}]62500\, n^{20} \\
&+ 31216[\text{...53 digits...}]37500\, n^{19} + 14453[\text{...54 digits...}]00000\, n^{18} \\
&+ 60922[\text{...54 digits...}]25625\, n^{17} + 23334[\text{...55 digits...}]78250\, n^{16} \\
&+ 81018[\text{...55 digits...}]94200\, n^{15} + 25427[\text{...56 digits...}]22480\, n^{14} \\
&+ 71882[\text{...56 digits...}]18896\, n^{13} + 18226[\text{...57 digits...}]09728\, n^{12}
\end{aligned}
$$

$+ 41242[...57\ \text{digits}...]28704\, n^{11} + 82760[...57\ \text{digits}...]32832\, n^{10}$
$+ 14616[...58\ \text{digits}...]80320\, n^{9} + 22508[...58\ \text{digits}...]44800\, n^{8}$
$+ 29865[...58\ \text{digits}...]72000\, n^{7} + 33630[...58\ \text{digits}...]80000\, n^{6}$
$+ 31499[...58\ \text{digits}...]00000\, n^{5} + 23869[...58\ \text{digits}...]00000\, n^{4}$
$+ 14053[...58\ \text{digits}...]00000\, n^{3} + 60299[...57\ \text{digits}...]00000\, n^{2}$
$+ 16764[...57\ \text{digits}...]00000\, n + 22656[...56\ \text{digits}...]00000)\, u(n+10) = 0, u(0) = 1,$
$u(1) = 0, u(2) = 2244198972960000, u(3) = 420884981714444083200000, u(4)$
$= 12382[...25\ \text{digits}...]00000, u(5) = 47234[...34\ \text{digits}...]28800, u(6)$
$= 21330[...44\ \text{digits}...]00000, u(7) = 10843[...54\ \text{digits}...]00000, u(8)$
$= 60162[...63\ \text{digits}...]00000, u(9) = 35708[...73\ \text{digits}...]00000\}$

```
> st:=time(): Positivity(recgrz[10],u,n)[1..2];time()-st;
```

true, 8

18.482 **(3.6)**