

# Inequalities - JNCF 26

## Demo Part I

> restart

### AMM 11501

The goal is to show that the Taylor coefficient of the following rational function are nonnegative for  $a \geq \sqrt{3}$ :

>  $g := 1 - 3 / (1 / (1 - a * z) + 1 / (1 - I * z) + 1 / (1 + I * z));$

$$g := 1 - \frac{3}{\frac{1}{-az + 1} + \frac{1}{1 - Iz} + \frac{1}{1 + Iz}} \quad (1.1)$$

Necessary condition:

>  $\text{map}(\text{expand}, \text{series}(g, z, 4));$

$$\frac{a}{3} z + \left( \frac{2a^2}{9} - \frac{2}{3} \right) z^2 + \left( \frac{4}{27} a^3 + \frac{4}{9} a \right) z^3 + O(z^4) \quad (1.2)$$

The first coefficient forces  $a \geq 0$  and the second one forces  $a^2 \geq 3$ .

Linear recurrence:

>  $\text{gfun}:-\text{diffqtoec}(\text{gfun}:-\text{holexprtodiffeq}(g, y(z)), y(z), u(n));$

$$\left\{ -u(n) + 2a u(n+1) - 3u(n+2), u(0) = 0, u(1) = \frac{a}{3}, u(2) = \frac{2a^2}{9} - \frac{2}{3}, u(3) = \frac{4}{27} a^3 + \frac{4}{9} a \right\} \quad (1.3)$$

There are 4 initial conditions for this recurrence of order 2 meaning that the recurrence applies only for  $n \geq 2$ .

>  $\text{gfun}:-\text{ratpolytcoeff}(g, z, n);$

$$\sum_{\alpha = \text{RootOf}(\_Z^2 - 2\_Za + 3)} \left( -\frac{(-6\_a a^3 + 6\_a a + 9a^2 - 3)\_a^{-n}}{(a - \_a)\_a} \right) + \left( \begin{array}{ll} 3a & n=1 \\ 6a^2 - 2 & n=0 \\ 0 & \text{otherwise} \end{array} \right) \quad (1.4)$$

>  $\text{co0} := \text{allvalues}(\%) \text{ assuming } n \geq 2;$

$\text{co0} :=$

$$\frac{(-6(a + \sqrt{a^2 - 3})a^3 + 6(a + \sqrt{a^2 - 3})a + 9a^2 - 3)(a + \sqrt{a^2 - 3})^{-n}}{\sqrt{a^2 - 3}(a + \sqrt{a^2 - 3})} \quad (1.5)$$

$$- \frac{(-6(a - \sqrt{a^2 - 3})a^3 + 6(a - \sqrt{a^2 - 3})a + 9a^2 - 3)(a - \sqrt{a^2 - 3})^{-n}}{\sqrt{a^2 - 3}(a - \sqrt{a^2 - 3})}$$

We analyse the signs of the coefficients of these nth powers

**> co:=subs([(a - sqrt(a^2 - 3))^(n)=X[1],(a + sqrt(a^2 - 3))^(n)=X[0]],%);**

$$co := \frac{(-6(a + \sqrt{a^2 - 3})a^3 + 6(a + \sqrt{a^2 - 3})a + 9a^2 - 3)X_0}{\sqrt{a^2 - 3}(a + \sqrt{a^2 - 3})} \quad (1.6)$$

$$- \frac{(-6(a - \sqrt{a^2 - 3})a^3 + 6(a - \sqrt{a^2 - 3})a + 9a^2 - 3)X_1}{\sqrt{a^2 - 3}(a - \sqrt{a^2 - 3})}$$

**> collect(co,[X[0],X[1]],normal@evala@Expand);**

$$- \frac{(3a^2\sqrt{a^2 - 3} + 3a^3 - \sqrt{a^2 - 3} - 5a)X_0}{\sqrt{a^2 - 3}} \quad (1.7)$$

$$- \frac{(3a^2\sqrt{a^2 - 3} - 3a^3 - \sqrt{a^2 - 3} + 5a)X_1}{\sqrt{a^2 - 3}}$$

**> signum(coeff(%,X[1])) assuming a>sqrt(3);**

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(1.8)

The sign in front of the fastest growing term is positive, which shows ultimate positivity.

It is thus sufficient to prove the positivity for n=2.

**> eval(co0,n=2);**

$$- \frac{-6(a + \sqrt{a^2 - 3})a^3 + 6(a + \sqrt{a^2 - 3})a + 9a^2 - 3}{(a + \sqrt{a^2 - 3})^3 \sqrt{a^2 - 3}} \quad (1.9)$$

$$- \frac{-6(a - \sqrt{a^2 - 3})a^3 + 6(a - \sqrt{a^2 - 3})a + 9a^2 - 3}{(a - \sqrt{a^2 - 3})^3 \sqrt{a^2 - 3}}$$

**> evala(Normal(%));**

$$\frac{2a^2}{9} - \frac{2}{3}$$

(1.10)

This is nonnegative for  $a \geq \sqrt{3}$ , concluding the proof.

## Gerhold-Kauers Method

Start with a basic example of quantifier elimination:

**> QuantifierElimination[QuantifierEliminate](exists(x,a\*x^2+b\*x+c=0));**

$$(a = 0 \wedge b = 0 \wedge c = 0) \vee (a = 0 \wedge b \neq 0 \wedge ac^2 = 0) \vee (a < 0 \wedge 4ac - b^2 < 0) \quad (2.1)$$

$$\vee (a \neq 0 \wedge 4ac - b^2 \leq 0) \vee (-a < 0 \wedge 4ac - b^2 < 0)$$

## Turan's inequality

Turan's inequality:

$$\begin{aligned} > \mathbf{P[n](x)^2 - P[n+1](x) * P[n-1](x) >= 0;} \\ & \quad 0 \leq P_n(x)^2 - P_{n+1}(x) P_{n-1}(x) \end{aligned} \quad (2.1.1)$$

where  $P_n$  are the Legendre polynomials

The Legendre polynomials are defined by their recurrence:

$$\begin{aligned} > \mathbf{eqL := (n + 1) * p(n) - (2 * n + 3) * x * p(n + 1) + (n + 2) * p(n + 2) = 0;} \\ & \quad eqL := (n + 1) p(n) - (2 n + 3) x p(n + 1) + (n + 2) p(n + 2) = 0 \end{aligned} \quad (2.1.2)$$

$$\begin{aligned} > \mathbf{Turan := p(n)^2 - p(n+1) * p(n-1);} \\ & \quad Turan := p(n)^2 - p(n + 1) p(n - 1) \end{aligned} \quad (2.1.3)$$

First test: does the inequality hold for arbitrary values of  $p(n)$  and  $p(n-1)$ :

$$\begin{aligned} > \mathbf{m := 0;} \\ & \text{A formula whose unsatisfiability would prove Turan's inequality} \\ > \mathbf{And(-1 < x, x < 1, n >= 1, seq(subs(n=n+i, eqL), i=0..m), seq(subs(n=n+i, Turan) > 0, i=1..m), subs(n=n+m+1, Turan) < 0);} \\ & -1 < x \wedge x < 1 \wedge 1 \leq n \wedge (n + 1) p(n) - (2 n + 3) x p(n + 1) + (n + 2) p(n + 2) = 0 \wedge p(n + 1)^2 - p(n + 2) p(n) < 0 \end{aligned} \quad (2.1.4)$$

Forget the relation between the  $p(n+i)$ :

$$\begin{aligned} > \mathbf{ineqs := subs(seq(p(n+i)=p[i], i=0..m+2), \%);} \\ & ineqs := -1 < x \wedge x < 1 \wedge 1 \leq n \wedge (n + 1) p_0 - (2 n + 3) x p_1 + (n + 2) p_2 = 0 \wedge -p_2 p_0 + p_1^2 < 0 \end{aligned} \quad (2.1.5)$$

$$\begin{aligned} > \mathbf{formula := exists([x, n, seq(p[i], i=0..m+2)], ineqs);} \\ & formula := \exists ([x, n, p_0, p_1, p_2], -1 < x \wedge x < 1 \wedge 1 \leq n \wedge (n + 1) p_0 - (2 n + 3) x p_1 + (n + 2) p_2 = 0 \wedge -p_2 p_0 + p_1^2 < 0) \end{aligned} \quad (2.1.6)$$

$$\begin{aligned} > \mathbf{st := time(): QuantifierElimination[QuantifierEliminate](formula), time() - st;} \\ & \quad true, 4.579 \end{aligned} \quad (2.1.7)$$

Faster code:

$$\begin{aligned} > \mathbf{SMTLIB[Satisfiable]({ineqs});} \\ & \quad true \end{aligned} \quad (2.1.8)$$

This means that there are values of  $p_n(x)$  and  $p_{n+1}(x)$  for which positivity does not hold.

Next, look at the induction  $Turan(n) \implies Turan(n+1)$ :

$$\begin{aligned} > \mathbf{m := 1;} \\ > \mathbf{And(-1 < x, x < 1, n >= 1, seq(subs(n=n+i, eqL), i=0..m), seq(subs(n=n+i, Turan) > 0, i=1..m), subs(n=n+m+1, Turan) < 0);} \\ & -1 < x \wedge x < 1 \wedge 1 \leq n \wedge (n + 1) p(n) - (2 n + 3) x p(n + 1) + (n + 2) p(n + 2) = 0 \wedge (n + 2) p(n + 1) - (2 n + 5) x p(n + 2) + (n + 3) p(n + 3) = 0 \\ & \quad \wedge 0 < p(n + 1)^2 - p(n + 2) p(n) \wedge p(n + 2)^2 - p(n + 3) p(n + 1) < 0 \end{aligned} \quad (2.1.9)$$

$$\begin{aligned} > \mathbf{subs(seq(p(n+i)=p[i], i=0..m+2), \%);} \\ & -1 < x \wedge x < 1 \wedge 1 \leq n \wedge (n + 1) p_0 - (2 n + 3) x p_1 + (n + 2) p_2 = 0 \wedge (n + 2) p_1 - (2 n + 5) x p_2 + (n + 3) p_3 = 0 \wedge 0 < -p_0 p_2 + p_1^2 \wedge -p_3 p_1 + p_2^2 < 0 \end{aligned} \quad (2.1.10)$$

killed after 1 min.

```
> #QuantifierElimination[QuantifierEliminate](exists([x,n,seq
  (p[i],i=0..m+2)],%));
> SMTLIB[Satisfiable]({%});
false (2.1.11)
```

This shows that the induction holds.

The proof of the inequality is then completed by finding a value of  $n \geq 2$  where it holds

```
> expand([seq(orthopoly[P](n,x)^2-orthopoly[P](n+1,x)*
  orthopoly[P](n-1,x),n=1..5)]);
[ -x^2/2 + 1/2, 1/4 - x^4/4, -5/16 x^6 + 5/16 x^4 - 3/16 x^2 + 3/16, 9/64 - 15/32 x^4
  - 35/64 x^8 + 7/8 x^6, -147/128 x^10 + 315/128 x^8 - 119/64 x^6 + 35/64 x^4 - 15/128 x^2
  + 15/128 ] (2.1.12)
```

```
> map(signum,%) assuming x>-1,x<1;
[1, 1, 1, 1, 1] (2.1.13)
```

## An example where it does not terminate

```
> rec:={u(n+2)-3*u(n+1)+2*u(n),u(0)=1,u(1)=3};
rec := {u(n+2) - 3 u(n+1) + 2 u(n), u(0) = 1, u(1) = 3} (2.2.1)
```

```
> gfun:-rectoproc(rec,u(n),list)(20);
[1, 3, 7, 15, 31, 63, 127, 255, 511, 1023, 2047, 4095, 8191, 16383, 32767, 65535,
  131071, 262143, 524287, 1048575, 2097151] (2.2.2)
```

```
> formula:=proc(m) local i;{seq(s[i]>0,i=0..m-1),seq(s[i+2]-3*
  s[i+1]+2*s[i]=0,i=0..m-2),s[m]<0} end:
> formula(4);
{s2 - 3 s1 + 2 s0 = 0, s3 - 3 s2 + 2 s1 = 0, s4 - 3 s3 + 2 s2 = 0, 0 < s0, 0 < s1, 0
  < s2, 0 < s3, s4 < 0} (2.2.3)
```

```
> for i to 30 do i,SMTLIB[Satisfiable](formula(i)) od;
1, true
2, true
3, true
4, true
5, true
6, true
7, true
8, true
9, true
10, true
11, true
12, true
```

13, true  
 14, true  
 15, true  
 16, true  
 17, true  
 18, true  
 19, true  
 20, true  
 21, true  
 22, true  
 23, true  
 24, true  
 25, true  
 26, true  
 27, true  
 28, true  
 29, true  
 30, true (2.2.4)

▼ **Explanation**

> **rsolve**({op(1, rec), u(0)=s[0], u(1)=s[1]}, u(n));  

$$-(s_0 - s_1) 2^n + 2 s_0 - s_1$$
 (2.2.1.1)

> **isolate**(%, s[1]);  

$$s_1 = \frac{-2 s_0 + 2^n s_0}{2^n - 1}$$
 (2.2.1.2)

> **factor**(%);  

$$s_1 = \frac{s_0 (2^n - 2)}{2^n - 1}$$
 (2.2.1.3)

> **normal**(1-1/(2^n-1));  

$$\frac{2^n - 2}{2^n - 1}$$
 (2.2.1.4)

Each step of GK will produce the constraint  $s[1] > (1-1/(2^n-1))s[0]$ , which is stricter than the previous one.

▼ **Kauers-Pillwein**

▼ **Fix the previous example**

Add a constraint  $u(n+1) > (3/2)*u(n)$

> **formula**(3) union {s[2]>3/2\*s[1]};

$$\left\{ s_2 - 3s_1 + 2s_0 = 0, s_3 - 3s_2 + 2s_1 = 0, 0 < s_0, 0 < s_1, 0 < s_2, s_3 < 0, \frac{3s_1}{2} < s_2 \right\} \quad (3.1.1)$$

> **SMTLIB[Satisfiable](%)**; *false* (3.1.2)

Check initial conditions:

> **pp:=gfun:-rectoproc(rec,u(n))**;  
 > **seq(pp(i),i=0..2)**; 1, 3, 7 (3.1.3)

> **pp(2)-3/2\*pp(1)**;  $\frac{5}{2}$  (3.1.4)

Proof of positivity complete.

## ▼ An example with polynomial coefficients

> **rec:={ (n+3)\*f(n+3)-(5\*n+13)\*f(n+2)+(5\*n+12)\*f(n+1)-(n+2)\*f(n), f(0)=1, f(1)=1/4, f(2)=1/10}**;  
 $rec := \left\{ (n+3)f(n+3) - (5n+13)f(n+2) + (5n+12)f(n+1) - (n+2)f(n), f(0) = 1, f(1) = \frac{1}{4}, f(2) = \frac{1}{10} \right\}$  (3.2.1)

> **L:=gfun:-rectoproc(rec,f(n),list)(100)**;  
 > **evalf([seq(L[i+1]/L[i],i=1..nops(L)-1])]**;  
 [0.2500000000, 0.4000000000, 1., 2.125000000, 2.905882353, 3.253036437, 3.399679972, 3.468696886, 3.507789568, 3.534092658, 3.553894293, 3.569755577, 3.582903763, 3.594035408, 3.603600742, 3.611915803, 3.619213659, 3.625671635, 3.631427618, 3.636590657, 3.641248209, 3.645471269, 3.649318089, 3.652836921, 3.656068088, 3.659045545, 3.661798098, 3.664350340, 3.666723402, 3.668935538, 3.671002600, 3.672938423, 3.674755133, 3.676463406, 3.678072680, 3.679591324, 3.681026792, 3.682385739, 3.683674130, 3.684897324, 3.686060150, 3.687166974, 3.688221748, 3.689228060, 3.690189178, 3.691108080, 3.691987490, 3.692829901, 3.693637601, 3.694412692, 3.695157112, 3.695872646, 3.696560944, 3.697223532, 3.697861826, 3.698477137, 3.699070685, 3.699643605, 3.700196955, 3.700731721, 3.701248823, 3.701749123, 3.702233427, 3.702702489, 3.703157020, 3.703597683, 3.704025104, 3.704439871, 3.704842539, 3.705233628, 3.705613631, 3.705983014, 3.706342215, 3.706691650, 3.707031712, 3.707362772, 3.707685184, 3.707999283, 3.708305385, 3.708603793, 3.708894792, 3.709178656, 3.709455644, 3.709726002, 3.709989967, 3.710247761, 3.710499599, 3.710745685, 3.710986214, 3.711221372, 3.711451338,

3.711676280, 3.711896362, 3.712111740, 3.712322563, 3.712528973,  
3.712731109, 3.712929102, 3.713123077, 3.713313156]

These numerical values show that we can try to prove an induction with  $u(n+1) \geq 2u(n)$

> **eq:=isolate(op(1,rec),f(n+3));**

$$eq := f(n+3) = \frac{(5n+13)f(n+2) - (5n+12)f(n+1) + (n+2)f(n)}{n+3} \quad (3.2.3)$$

> **mu:=2;**

$$\mu := 2 \quad (3.2.4)$$

> **X[3]:=subs(seq(f(n+i)=X[i],i=0..2),op(2,eq));**

$$X_3 := \frac{(5n+13)X_2 - (5n+12)X_1 + (n+2)X_0}{n+3} \quad (3.2.5)$$

> **ineq:=X[3]<mu\*X[2];**

$$ineq := \frac{(5n+13)X_2 - (5n+12)X_1 + (n+2)X_0}{n+3} < 2X_2 \quad (3.2.6)$$

> **formula:={n>=0,X[0]>0,seq(X[i+1]>mu\*X[i],i=0..1),ineq};**

$$formula := \left\{ 0 \leq n, 0 < X_0, \frac{(5n+13)X_2 - (5n+12)X_1 + (n+2)X_0}{n+3} < 2X_2, 2X_0 < X_1, 2X_1 < X_2 \right\} \quad (3.2.7)$$

> **SMTLIB[Satisfiable](formula);**

$$false \quad (3.2.8)$$

Proof of the induction complete! (Again, initial conditions have to be checked.)

One can also find mu by quantifier elimination:

> **mu:='mu':**

> **formula:=forall([X[0],X[1],X[2],n],Implies(And(n>=0,X[0]>0,seq(X[i+1]>mu\*X[i],i=0..1),subs(f(n+2)=X[2],f(n+1)=X[1],f(n)=X[0],subs(eq,f(n+3)>mu\*f(n+2))))));**

$$formula := \forall \left( [X_0, X_1, X_2, n], 0 \leq n \wedge 0 < X_0 \wedge \mu X_0 < X_1 \wedge \mu X_1 < X_2 \Rightarrow \mu X_2 < \frac{(5n+13)X_2 - (5n+12)X_1 + (n+2)X_0}{n+3} \right) \quad (3.2.9)$$

> **elim:=QuantifierElimination[QuantifierEliminate](formula);**

$$elim := \left( 3\mu \leq 13 \vee (-9\mu^4 + 78\mu^3 - 205\mu^2 + 162\mu \leq 26 \wedge (9\mu^4 - 78\mu^3 + 205\mu^2 - 162\mu + 26 \neq 0 \vee -9\mu^3 + 78\mu^2 - 205\mu \leq -156)) \right) \wedge (\mu^2 - 4\mu + 1 = 0 \vee \mu^3 - 5\mu^2 + 5\mu \leq 1 \vee -3\mu^4 + 22\mu^3 - 45\mu^2 + 18\mu < 2 \vee \mu^2 - 3\mu \leq -2) \wedge ((3\mu \leq 13 \wedge (3\mu - 13 \neq 0 \vee \mu \leq 5)) \vee (-9\mu^4 + 78\mu^3 - 205\mu^2 + 162\mu \leq 26 \wedge (9\mu^4 - 78\mu^3 + 205\mu^2 - 162\mu + 26 \neq 0 \vee (-6\mu^4 + 56\mu^3 - 157\mu^2 + 130\mu \leq 23 \wedge (6\mu^4 - 56\mu^3 + 157\mu^2 - 130\mu + 23 \neq 0 \vee -\mu^4 + 10\mu^3 - 30\mu^2 + 26\mu \leq 5)))) \wedge (\mu^4 - 10\mu^3 + 30\mu^2 - 26\mu + 5 \neq 0 \vee 6\mu^4 - 56\mu^3 + 157\mu^2 - 130\mu + 23 \neq 0 \vee 9\mu^4$$

$$\begin{aligned}
& -78\mu^3 + 205\mu^2 - 162\mu + 26 \neq 0 \vee (-9\mu^3 + 78\mu^2 - 205\mu \leq -156 \\
& \wedge (9\mu^3 - 78\mu^2 + 205\mu - 156 \neq 0 \vee (-6\mu^3 + 56\mu^2 - 157\mu \leq -125 \\
& \wedge (6\mu^3 - 56\mu^2 + 157\mu - 125 \neq 0 \vee -\mu^3 + 10\mu^2 - 30\mu \leq -25)))))) \\
& ) \wedge (\mu - 5 = 0 \vee \mu \leq 5 \vee (-3\mu^2 + 28\mu \leq 65 \wedge 3\mu^2 - 28\mu + 65 \neq 0) \\
& \vee (\mu^2 - 2\mu \leq -1 \wedge (\mu^2 - 2\mu + 1 \neq 0 \vee -\mu^4 + 10\mu^3 - 30\mu^2 + 26\mu \\
& \leq 5) \wedge (\mu^4 - 10\mu^3 + 30\mu^2 - 26\mu + 5 \neq 0 \vee 6\mu^4 - 56\mu^3 + 157\mu^2 \\
& - 130\mu + 23 \neq 0 \vee 9\mu^4 - 78\mu^3 + 205\mu^2 - 162\mu + 26 \neq 0 \vee (3\mu \leq 5 \\
& \wedge (-5 + 3\mu \neq 0 \vee -\mu^3 + 10\mu^2 - 30\mu \leq -25)))))) \wedge (\mu^2 - 4\mu + 1 \\
& = 0 \vee (\mu^3 - 5\mu^2 + 5\mu \leq 1 \wedge (\mu^3 - 5\mu^2 + 5\mu - 1 \neq 0 \vee \mu \leq 5)) \vee ( \\
& -3\mu^4 + 22\mu^3 - 45\mu^2 + 18\mu \leq 2 \wedge 3\mu^4 - 22\mu^3 + 45\mu^2 - 18\mu + 2 \neq 0) \\
& \vee (-\mu^2 + 2\mu \leq 1 \wedge (\mu^2 - 2\mu + 1 \neq 0 \vee -\mu^4 + 10\mu^3 - 30\mu^2 + 26\mu \\
& \leq 5) \wedge (\mu^4 - 10\mu^3 + 30\mu^2 - 26\mu + 5 \neq 0 \vee 6\mu^4 - 56\mu^3 + 157\mu^2 \\
& - 130\mu + 23 \neq 0 \vee 9\mu^4 - 78\mu^3 + 205\mu^2 - 162\mu + 26 \neq 0 \vee (\mu^2 - 3\mu \\
& \leq -2 \wedge (\mu^2 - 3\mu + 2 \neq 0 \vee (-3\mu^5 + 31\mu^4 - 116\mu^3 + 192\mu^2 - 137\mu \\
& \leq -25 \wedge (3\mu^5 - 31\mu^4 + 116\mu^3 - 192\mu^2 + 137\mu - 25 \neq 0 \vee -\mu^3 \\
& + 10\mu^2 - 30\mu \leq -25)))))) \wedge (\mu^2 - 5\mu + 5 = 0 \vee (3\mu^3 - 20\mu^2 \\
& + 40\mu \leq 25 \wedge (3\mu^3 - 20\mu^2 + 40\mu - 25 \neq 0 \vee \mu \leq 5)) \vee (-3\mu^4 \\
& + 28\mu^3 - 92\mu^2 + 125\mu \leq 60 \wedge 3\mu^4 - 28\mu^3 + 92\mu^2 - 125\mu + 60 \neq 0) \\
& \vee (-3\mu^3 + 14\mu^2 - 21\mu \leq -10 \wedge (3\mu^3 - 14\mu^2 + 21\mu - 10 \neq 0 \vee ( \\
& -3\mu^6 + 34\mu^5 - 142\mu^4 + 266\mu^3 - 210\mu^2 + 30\mu \leq -25 \wedge (3\mu^6 - 34\mu^5 \\
& + 142\mu^4 - 266\mu^3 + 210\mu^2 - 30\mu - 25 \neq 0 \vee -\mu^4 + 10\mu^3 - 30\mu^2 + 26\mu \\
& \leq 5)))) \wedge (\mu^4 - 10\mu^3 + 30\mu^2 - 26\mu + 5 \neq 0 \vee 6\mu^4 - 56\mu^3 + 157\mu^2 \\
& - 130\mu + 23 \neq 0 \vee 9\mu^4 - 78\mu^3 + 205\mu^2 - 162\mu + 26 \neq 0 \vee (-3\mu \\
& \leq -5 \wedge (-5 + 3\mu \neq 0 \vee -\mu^3 + 10\mu^2 - 30\mu \leq -25)))))) \wedge (3\mu^2 \\
& - 13\mu + 12 = 0 \vee 3\mu \leq 13 \vee (-9\mu^5 + 78\mu^4 - 241\mu^3 + 318\mu^2 - 170\mu \\
& \leq -24 \wedge 9\mu^5 - 78\mu^4 + 241\mu^3 - 318\mu^2 + 170\mu - 24 \neq 0) \vee -9\mu^3 \\
& + 78\mu^2 - 205\mu \leq -156) \wedge (\mu^2 - 4\mu + 1 = 0 \vee \mu^3 - 6\mu^2 + 9\mu - 2 = 0 \\
& \vee \mu^3 - 5\mu^2 + 5\mu \leq 1 \vee -3\mu^4 + 22\mu^3 - 45\mu^2 + 18\mu < 2 \vee \mu^2 - 3\mu \\
& \leq -2) \wedge ((\mu^2 - 5\mu + 5 = 0 \wedge 3\mu^2 - 13\mu + 12 = 0) \vee (3\mu \leq 13 \wedge (3\mu \\
& - 13 \neq 0 \vee \mu \leq 5)) \vee (-9\mu^5 + 78\mu^4 - 241\mu^3 + 318\mu^2 - 170\mu \leq -24 \\
& \wedge (9\mu^5 - 78\mu^4 + 241\mu^3 - 318\mu^2 + 170\mu - 24 \neq 0 \vee (-6\mu^5 + 56\mu^4 \\
& - 184\mu^3 + 255\mu^2 - 143\mu \leq -22 \wedge (6\mu^5 - 56\mu^4 + 184\mu^3 - 255\mu^2 \\
& + 143\mu - 22 \neq 0 \vee -\mu^5 + 10\mu^4 - 35\mu^3 + 51\mu^2 - 30\mu \leq -5)))) \wedge (\mu^5
\end{aligned}$$

$$\begin{aligned}
& -10\mu^4 + 35\mu^3 - 51\mu^2 + 30\mu - 5 \neq 0 \vee 6\mu^5 - 56\mu^4 + 184\mu^3 - 255\mu^2 \\
& + 143\mu - 22 \neq 0 \vee 9\mu^5 - 78\mu^4 + 241\mu^3 - 318\mu^2 + 170\mu - 24 \neq 0)) \\
& \vee (-9\mu^3 + 78\mu^2 - 205\mu \leq -156 \wedge (9\mu^3 - 78\mu^2 + 205\mu - 156 \neq 0 \\
& \vee (-6\mu^3 + 56\mu^2 - 157\mu \leq -125 \wedge (6\mu^3 - 56\mu^2 + 157\mu - 125 \neq 0 \vee \\
& -\mu^3 + 10\mu^2 - 30\mu \leq -25)))))) \wedge (\mu - 5 = 0 \vee (\mu^2 - 5\mu + 5 = 0 \wedge 3\mu^2 \\
& - 13\mu + 12 = 0) \vee \mu \leq 5 \vee (-3\mu^2 + 28\mu \leq 65 \wedge 3\mu^2 - 28\mu + 65 \neq 0) \\
& \vee (-3\mu^3 + 11\mu^2 - 13\mu \leq -5 \wedge (3\mu^3 - 11\mu^2 + 13\mu - 5 \neq 0 \vee (6\mu^5 \\
& - 71\mu^4 + 293\mu^3 - 513\mu^2 + 385\mu \leq 100 \wedge (6\mu^5 - 71\mu^4 + 293\mu^3 \\
& - 513\mu^2 + 385\mu - 100 \neq 0 \vee -\mu^5 + 10\mu^4 - 35\mu^3 + 51\mu^2 - 30\mu \\
& \leq -5)))) \wedge (\mu^5 - 10\mu^4 + 35\mu^3 - 51\mu^2 + 30\mu - 5 \neq 0 \vee 6\mu^5 - 56\mu^4 \\
& + 184\mu^3 - 255\mu^2 + 143\mu - 22 \neq 0 \vee 9\mu^5 - 78\mu^4 + 241\mu^3 - 318\mu^2 \\
& + 170\mu - 24 \neq 0) \vee (3\mu \leq 5 \wedge (-5 + 3\mu \neq 0 \vee -\mu^3 + 10\mu^2 - 30\mu \\
& \leq -25))) \wedge (\mu^2 - 4\mu + 1 = 0 \vee (\mu^2 - 5\mu + 5 = 0 \wedge 3\mu^2 - 13\mu + 12 \\
& = 0) \vee (\mu^3 - 5\mu^2 + 5\mu \leq 1 \wedge (\mu^3 - 5\mu^2 + 5\mu - 1 \neq 0 \vee \mu \leq 5))) \vee ( \\
& -3\mu^4 + 22\mu^3 - 45\mu^2 + 18\mu \leq 2 \wedge 3\mu^4 - 22\mu^3 + 45\mu^2 - 18\mu + 2 \neq 0) \\
& \vee (\mu^2 - 3\mu \leq -2 \wedge (\mu^2 - 3\mu + 2 \neq 0 \vee -\mu^5 + 10\mu^4 - 35\mu^3 + 51\mu^2 \\
& - 30\mu \leq -5) \wedge (\mu^5 - 10\mu^4 + 35\mu^3 - 51\mu^2 + 30\mu - 5 \neq 0 \vee 6\mu^5 \\
& - 56\mu^4 + 184\mu^3 - 255\mu^2 + 143\mu - 22 \neq 0 \vee 9\mu^5 - 78\mu^4 + 241\mu^3 \\
& - 318\mu^2 + 170\mu - 24 \neq 0) \vee (\mu^2 - 3\mu \leq -2 \wedge (\mu^2 - 3\mu + 2 \neq 0 \\
& \vee (-3\mu^5 + 31\mu^4 - 116\mu^3 + 192\mu^2 - 137\mu \leq -25 \wedge (3\mu^5 - 31\mu^4 \\
& + 116\mu^3 - 192\mu^2 + 137\mu - 25 \neq 0 \vee -\mu^3 + 10\mu^2 - 30\mu \leq -25)))))) \\
& \wedge (\mu^2 - 5\mu + 5 = 0 \vee (\mu^2 - 5\mu + 5 = 0 \wedge 3\mu^2 - 13\mu + 12 = 0) \vee (3\mu^3 \\
& - 20\mu^2 + 40\mu \leq 25 \wedge (3\mu^3 - 20\mu^2 + 40\mu - 25 \neq 0 \vee \mu \leq 5))) \vee ( \\
& -3\mu^4 + 28\mu^3 - 92\mu^2 + 125\mu \leq 60 \wedge 3\mu^4 - 28\mu^3 + 92\mu^2 - 125\mu + 60 \\
& \neq 0) \vee (-\mu^2 + 3\mu \leq 2 \wedge (\mu^2 - 3\mu + 2 \neq 0 \vee -\mu^5 + 10\mu^4 - 35\mu^3 \\
& + 51\mu^2 - 30\mu \leq -5) \wedge (\mu^5 - 10\mu^4 + 35\mu^3 - 51\mu^2 + 30\mu - 5 \neq 0 \\
& \vee 6\mu^5 - 56\mu^4 + 184\mu^3 - 255\mu^2 + 143\mu - 22 \neq 0 \vee 9\mu^5 - 78\mu^4 \\
& + 241\mu^3 - 318\mu^2 + 170\mu - 24 \neq 0)) \vee (-3\mu \leq -5 \wedge (-5 + 3\mu \neq 0 \\
& \vee -\mu^3 + 10\mu^2 - 30\mu \leq -25))) \wedge (-3\mu \leq -13 \vee (-9\mu^4 + 78\mu^3 \\
& - 205\mu^2 + 162\mu \leq 26 \wedge (9\mu^4 - 78\mu^3 + 205\mu^2 - 162\mu + 26 \neq 0 \vee \\
& -9\mu^3 + 78\mu^2 - 205\mu \leq -156))) \wedge (\mu^2 - 4\mu + 1 = 0 \vee -\mu^3 + 5\mu^2 \\
& - 5\mu \leq -1 \vee -3\mu^4 + 22\mu^3 - 45\mu^2 + 18\mu < 2 \vee \mu^2 - 3\mu \leq -2) \wedge (( \\
& -3\mu \leq -13 \wedge (3\mu - 13 \neq 0 \vee -\mu \leq -5)) \vee (-9\mu^4 + 78\mu^3 - 205\mu^2
\end{aligned}$$

$$\begin{aligned}
& + 162\mu \leq 26 \wedge (9\mu^4 - 78\mu^3 + 205\mu^2 - 162\mu + 26 \neq 0 \vee (-6\mu^4 \\
& + 56\mu^3 - 157\mu^2 + 130\mu \leq 23 \wedge (6\mu^4 - 56\mu^3 + 157\mu^2 - 130\mu + 23 \neq 0 \\
& \vee -\mu^4 + 10\mu^3 - 30\mu^2 + 26\mu \leq 5))) \wedge (\mu^4 - 10\mu^3 + 30\mu^2 - 26\mu + 5 \\
& \neq 0 \vee 6\mu^4 - 56\mu^3 + 157\mu^2 - 130\mu + 23 \neq 0 \vee 9\mu^4 - 78\mu^3 + 205\mu^2 \\
& - 162\mu + 26 \neq 0 \vee (-9\mu^3 + 78\mu^2 - 205\mu \leq -156 \wedge (9\mu^3 - 78\mu^2 \\
& + 205\mu - 156 \neq 0 \vee (-6\mu^3 + 56\mu^2 - 157\mu \leq -125 \wedge (6\mu^3 - 56\mu^2 \\
& + 157\mu - 125 \neq 0 \vee -\mu^3 + 10\mu^2 - 30\mu \leq -25)))))) \wedge (\mu - 5 = 0 \\
& \vee -\mu \leq -5 \vee (-3\mu^2 + 28\mu \leq 65 \wedge 3\mu^2 - 28\mu + 65 \neq 0) \vee (\mu^2 - 2\mu \\
& \leq -1 \wedge (\mu^2 - 2\mu + 1 \neq 0 \vee -\mu^4 + 10\mu^3 - 30\mu^2 + 26\mu \leq 5) \wedge (\mu^4 \\
& - 10\mu^3 + 30\mu^2 - 26\mu + 5 \neq 0 \vee 6\mu^4 - 56\mu^3 + 157\mu^2 - 130\mu + 23 \neq 0 \\
& \vee 9\mu^4 - 78\mu^3 + 205\mu^2 - 162\mu + 26 \neq 0 \vee (3\mu \leq 5 \wedge (-5 + 3\mu \neq 0 \\
& \vee -\mu^3 + 10\mu^2 - 30\mu \leq -25)))) \wedge (\mu^2 - 4\mu + 1 = 0 \vee (-\mu^3 + 5\mu^2 \\
& - 5\mu \leq -1 \wedge (\mu^3 - 5\mu^2 + 5\mu - 1 \neq 0 \vee -\mu \leq -5)) \vee (-3\mu^4 \\
& + 22\mu^3 - 45\mu^2 + 18\mu \leq 2 \wedge 3\mu^4 - 22\mu^3 + 45\mu^2 - 18\mu + 2 \neq 0) \vee ( \\
& -\mu^2 + 2\mu \leq 1 \wedge (\mu^2 - 2\mu + 1 \neq 0 \vee -\mu^4 + 10\mu^3 - 30\mu^2 + 26\mu \leq 5) \\
& \wedge (\mu^4 - 10\mu^3 + 30\mu^2 - 26\mu + 5 \neq 0 \vee 6\mu^4 - 56\mu^3 + 157\mu^2 - 130\mu \\
& + 23 \neq 0 \vee 9\mu^4 - 78\mu^3 + 205\mu^2 - 162\mu + 26 \neq 0 \vee (\mu^2 - 3\mu \leq -2 \\
& \wedge (\mu^2 - 3\mu + 2 \neq 0 \vee (-3\mu^5 + 31\mu^4 - 116\mu^3 + 192\mu^2 - 137\mu \leq -25 \\
& \wedge (3\mu^5 - 31\mu^4 + 116\mu^3 - 192\mu^2 + 137\mu - 25 \neq 0 \vee -\mu^3 + 10\mu^2 \\
& - 30\mu \leq -25)))))) \wedge (\mu^2 - 5\mu + 5 = 0 \vee (-3\mu^3 + 20\mu^2 - 40\mu \\
& \leq -25 \wedge (3\mu^3 - 20\mu^2 + 40\mu - 25 \neq 0 \vee -\mu \leq -5)) \vee (-3\mu^4 \\
& + 28\mu^3 - 92\mu^2 + 125\mu \leq 60 \wedge 3\mu^4 - 28\mu^3 + 92\mu^2 - 125\mu + 60 \neq 0) \\
& \vee (-3\mu^3 + 14\mu^2 - 21\mu \leq -10 \wedge (3\mu^3 - 14\mu^2 + 21\mu - 10 \neq 0 \vee ( \\
& -3\mu^6 + 34\mu^5 - 142\mu^4 + 266\mu^3 - 210\mu^2 + 30\mu \leq -25 \wedge (3\mu^6 - 34\mu^5 \\
& + 142\mu^4 - 266\mu^3 + 210\mu^2 - 30\mu - 25 \neq 0 \vee -\mu^4 + 10\mu^3 - 30\mu^2 + 26\mu \\
& \leq 5))) \wedge (\mu^4 - 10\mu^3 + 30\mu^2 - 26\mu + 5 \neq 0 \vee 6\mu^4 - 56\mu^3 + 157\mu^2 \\
& - 130\mu + 23 \neq 0 \vee 9\mu^4 - 78\mu^3 + 205\mu^2 - 162\mu + 26 \neq 0 \vee (-3\mu \\
& \leq -5 \wedge (-5 + 3\mu \neq 0 \vee -\mu^3 + 10\mu^2 - 30\mu \leq -25)))) \wedge (3\mu^2 \\
& - 13\mu + 12 = 0 \vee -3\mu \leq -13 \vee (-9\mu^5 + 78\mu^4 - 241\mu^3 + 318\mu^2 \\
& - 170\mu \leq -24 \wedge 9\mu^5 - 78\mu^4 + 241\mu^3 - 318\mu^2 + 170\mu - 24 \neq 0) \vee \\
& -9\mu^3 + 78\mu^2 - 205\mu \leq -156) \wedge (\mu^2 - 4\mu + 1 = 0 \vee \mu^3 - 6\mu^2 + 9\mu \\
& - 2 = 0 \vee -\mu^3 + 5\mu^2 - 5\mu \leq -1 \vee -3\mu^4 + 22\mu^3 - 45\mu^2 + 18\mu < 2 \\
& \vee \mu^2 - 3\mu \leq -2) \wedge ((\mu^2 - 5\mu + 5 = 0 \wedge 3\mu^2 - 13\mu + 12 = 0) \vee (-3\mu
\end{aligned}$$

$$\begin{aligned}
&\leq -13 \wedge (3\mu - 13 \neq 0 \vee -\mu \leq -5) \vee (-9\mu^5 + 78\mu^4 - 241\mu^3 \\
&+ 318\mu^2 - 170\mu \leq -24 \wedge (9\mu^5 - 78\mu^4 + 241\mu^3 - 318\mu^2 + 170\mu - 24 \\
&\neq 0 \vee (-6\mu^5 + 56\mu^4 - 184\mu^3 + 255\mu^2 - 143\mu \leq -22 \wedge (6\mu^5 - 56\mu^4 \\
&+ 184\mu^3 - 255\mu^2 + 143\mu - 22 \neq 0 \vee -\mu^5 + 10\mu^4 - 35\mu^3 + 51\mu^2 - 30\mu \\
&\leq -5))) \wedge (\mu^5 - 10\mu^4 + 35\mu^3 - 51\mu^2 + 30\mu - 5 \neq 0 \vee 6\mu^5 - 56\mu^4 \\
&+ 184\mu^3 - 255\mu^2 + 143\mu - 22 \neq 0 \vee 9\mu^5 - 78\mu^4 + 241\mu^3 - 318\mu^2 \\
&+ 170\mu - 24 \neq 0) \vee (-9\mu^3 + 78\mu^2 - 205\mu \leq -156 \wedge (9\mu^3 - 78\mu^2 \\
&+ 205\mu - 156 \neq 0 \vee (-6\mu^3 + 56\mu^2 - 157\mu \leq -125 \wedge (6\mu^3 - 56\mu^2 \\
&+ 157\mu - 125 \neq 0 \vee -\mu^3 + 10\mu^2 - 30\mu \leq -25)))))) \wedge (\mu - 5 = 0 \\
&\vee (\mu^2 - 5\mu + 5 = 0 \wedge 3\mu^2 - 13\mu + 12 = 0) \vee -\mu \leq -5 \vee (-3\mu^2 \\
&+ 28\mu \leq 65 \wedge 3\mu^2 - 28\mu + 65 \neq 0) \vee (-3\mu^3 + 11\mu^2 - 13\mu \leq -5 \\
&\wedge (3\mu^3 - 11\mu^2 + 13\mu - 5 \neq 0 \vee (6\mu^5 - 71\mu^4 + 293\mu^3 - 513\mu^2 + 385\mu \\
&\leq 100 \wedge (6\mu^5 - 71\mu^4 + 293\mu^3 - 513\mu^2 + 385\mu - 100 \neq 0 \vee -\mu^5 \\
&+ 10\mu^4 - 35\mu^3 + 51\mu^2 - 30\mu \leq -5))) \wedge (\mu^5 - 10\mu^4 + 35\mu^3 - 51\mu^2 \\
&+ 30\mu - 5 \neq 0 \vee 6\mu^5 - 56\mu^4 + 184\mu^3 - 255\mu^2 + 143\mu - 22 \neq 0 \vee 9\mu^5 \\
&- 78\mu^4 + 241\mu^3 - 318\mu^2 + 170\mu - 24 \neq 0) \vee (3\mu \leq 5 \wedge (-5 + 3\mu \\
&\neq 0 \vee -\mu^3 + 10\mu^2 - 30\mu \leq -25))) \wedge (\mu^2 - 4\mu + 1 = 0 \vee (\mu^2 - 5\mu \\
&+ 5 = 0 \wedge 3\mu^2 - 13\mu + 12 = 0) \vee (-\mu^3 + 5\mu^2 - 5\mu \leq -1 \wedge (\mu^3 - 5\mu^2 \\
&+ 5\mu - 1 \neq 0 \vee -\mu \leq -5)) \vee (-3\mu^4 + 22\mu^3 - 45\mu^2 + 18\mu \leq 2 \\
&\wedge 3\mu^4 - 22\mu^3 + 45\mu^2 - 18\mu + 2 \neq 0) \vee (\mu^2 - 3\mu \leq -2 \wedge (\mu^2 - 3\mu \\
&+ 2 \neq 0 \vee -\mu^5 + 10\mu^4 - 35\mu^3 + 51\mu^2 - 30\mu \leq -5) \wedge (\mu^5 - 10\mu^4 \\
&+ 35\mu^3 - 51\mu^2 + 30\mu - 5 \neq 0 \vee 6\mu^5 - 56\mu^4 + 184\mu^3 - 255\mu^2 + 143\mu \\
&- 22 \neq 0 \vee 9\mu^5 - 78\mu^4 + 241\mu^3 - 318\mu^2 + 170\mu - 24 \neq 0)) \vee (\mu^2 \\
&- 3\mu \leq -2 \wedge (\mu^2 - 3\mu + 2 \neq 0 \vee (-3\mu^5 + 31\mu^4 - 116\mu^3 + 192\mu^2 \\
&- 137\mu \leq -25 \wedge (3\mu^5 - 31\mu^4 + 116\mu^3 - 192\mu^2 + 137\mu - 25 \neq 0 \vee \\
&-\mu^3 + 10\mu^2 - 30\mu \leq -25)))))) \wedge (\mu^2 - 5\mu + 5 = 0 \vee (\mu^2 - 5\mu + 5 = 0 \\
&\wedge 3\mu^2 - 13\mu + 12 = 0) \vee (-3\mu^3 + 20\mu^2 - 40\mu \leq -25 \wedge (3\mu^3 - 20\mu^2 \\
&+ 40\mu - 25 \neq 0 \vee -\mu \leq -5)) \vee (-3\mu^4 + 28\mu^3 - 92\mu^2 + 125\mu \leq 60 \\
&\wedge 3\mu^4 - 28\mu^3 + 92\mu^2 - 125\mu + 60 \neq 0) \vee (-\mu^2 + 3\mu \leq 2 \wedge (\mu^2 - 3\mu \\
&+ 2 \neq 0 \vee -\mu^5 + 10\mu^4 - 35\mu^3 + 51\mu^2 - 30\mu \leq -5) \wedge (\mu^5 - 10\mu^4 \\
&+ 35\mu^3 - 51\mu^2 + 30\mu - 5 \neq 0 \vee 6\mu^5 - 56\mu^4 + 184\mu^3 - 255\mu^2 + 143\mu \\
&- 22 \neq 0 \vee 9\mu^5 - 78\mu^4 + 241\mu^3 - 318\mu^2 + 170\mu - 24 \neq 0)) \vee (-3\mu \\
&\leq -5 \wedge (-5 + 3\mu \neq 0 \vee -\mu^3 + 10\mu^2 - 30\mu \leq -25))) \wedge (3\mu^2
\end{aligned}$$

$$\begin{aligned}
& -13\mu + 12 = 0 \vee -9\mu^5 + 78\mu^4 - 241\mu^3 + 318\mu^2 - 170\mu \leq -24 \vee 3\mu \\
& < 13) \wedge (\mu \leq 5 \vee 3\mu^2 - 20\mu + 25 = 0 \vee -3\mu^2 + 28\mu < 65 \vee -3\mu^3 \\
& + 11\mu^2 - 13\mu \leq -5) \wedge (-\mu \leq -5 \vee 3\mu^2 - 20\mu + 25 = 0 \vee -3\mu^2 \\
& + 28\mu < 65 \vee -3\mu^3 + 11\mu^2 - 13\mu \leq -5) \wedge ((\mu^2 - 5\mu + 5 = 0 \wedge 3\mu^2 \\
& - 13\mu + 12 = 0) \vee (-9\mu^5 + 78\mu^4 - 241\mu^3 + 318\mu^2 - 170\mu \leq -24 \\
& \wedge (9\mu^5 - 78\mu^4 + 241\mu^3 - 318\mu^2 + 170\mu - 24 \neq 0 \vee (-6\mu^5 + 56\mu^4 \\
& - 184\mu^3 + 255\mu^2 - 143\mu \leq -22 \wedge (6\mu^5 - 56\mu^4 + 184\mu^3 - 255\mu^2 \\
& + 143\mu - 22 \neq 0 \vee -\mu^5 + 10\mu^4 - 35\mu^3 + 51\mu^2 - 30\mu \leq -5)))) \\
& \vee (3\mu \leq 13 \wedge (3\mu - 13 \neq 0 \vee (3\mu \leq 14 \wedge (3\mu - 14 \neq 0 \vee \mu \\
& < 5)))))) \wedge (\mu^2 - 4\mu + 1 = 0 \vee (\mu^2 - 5\mu + 5 = 0 \wedge 3\mu^2 - 13\mu + 12 \\
& = 0) \vee (-3\mu^4 + 22\mu^3 - 45\mu^2 + 18\mu \leq 2 \wedge 3\mu^4 - 22\mu^3 + 45\mu^2 - 18\mu \\
& + 2 \neq 0) \vee (\mu^2 - 3\mu \leq -2 \wedge (\mu^2 - 3\mu + 2 \neq 0 \vee -\mu^5 + 10\mu^4 - 35\mu^3 \\
& + 51\mu^2 - 30\mu \leq -5)) \vee (-2\mu^2 + 3\mu \leq 1 \wedge (2\mu^2 - 3\mu + 1 \neq 0 \vee (- \\
& -\mu^4 + 11\mu^3 - 33\mu^2 + 23\mu \leq 4 \wedge (\mu^4 - 11\mu^3 + 33\mu^2 - 23\mu + 4 \neq 0 \vee \mu \\
& < 5)))))) \wedge (\mu^2 - 5\mu + 5 = 0 \vee (\mu^2 - 5\mu + 5 = 0 \wedge 3\mu^2 - 13\mu + 12 \\
& = 0) \vee (-3\mu^4 + 28\mu^3 - 92\mu^2 + 125\mu \leq 60 \wedge 3\mu^4 - 28\mu^3 + 92\mu^2 \\
& - 125\mu + 60 \neq 0) \vee (-\mu^2 + 3\mu \leq 2 \wedge (\mu^2 - 3\mu + 2 \neq 0 \vee -\mu^5 \\
& + 10\mu^4 - 35\mu^3 + 51\mu^2 - 30\mu \leq -5)) \vee (-6\mu^2 + 19\mu \leq 15 \wedge (6\mu^2 \\
& - 19\mu + 15 \neq 0 \vee (-\mu^4 + 13\mu^3 - 55\mu^2 + 90\mu \leq 50 \wedge (\mu^4 - 13\mu^3 \\
& + 55\mu^2 - 90\mu + 50 \neq 0 \vee \mu < 5)))))) \wedge 3\mu^3 - 13\mu^2 + 12\mu \leq 2 \\
& \wedge (3\mu^3 - 13\mu^2 + 12\mu - 2 \neq 0 \vee 3\mu^2 - 13\mu \leq -12) \wedge (\mu \leq 5 \vee -3\mu^2 \\
& + 28\mu < 65 \vee (\mu^2 - 2\mu \leq -1 \wedge (\mu^2 - 2\mu + 1 \neq 0 \vee 3\mu \leq 5) \wedge (-5 \\
& + 3\mu \neq 0 \vee \mu - 3 \neq 0))) \wedge (-\mu \leq -5 \vee -3\mu^2 + 28\mu < 65 \vee (\mu^2 \\
& - 2\mu \leq -1 \wedge (\mu^2 - 2\mu + 1 \neq 0 \vee 3\mu \leq 5) \wedge (-5 + 3\mu \neq 0 \vee \mu - 3 \\
& \neq 0))) \wedge (\mu^2 - 5\mu + 5 = 0 \vee -3\mu^4 + 28\mu^3 - 92\mu^2 + 125\mu < 60 \vee (- \\
& -2\mu^3 + 9\mu^2 - 13\mu \leq -6 \wedge (2\mu^3 - 9\mu^2 + 13\mu - 6 \neq 0 \vee -6\mu^2 + 19\mu \\
& < 15)))) \wedge (\mu^2 - 4\mu + 1 = 0 \vee -3\mu^4 + 22\mu^3 - 45\mu^2 + 18\mu < 2 \vee (2\mu^2 \\
& - 5\mu \leq -2 \wedge (2\mu^2 - 5\mu + 2 \neq 0 \vee 2\mu^3 - 5\mu^2 + 2\mu \neq 0 \vee -2\mu^2 \\
& + 3\mu < 1))) \wedge (3\mu^3 - 13\mu^2 + 12\mu - 2 \neq 0 \vee (6\mu^3 - 28\mu^2 + 27\mu \leq 5 \\
& \wedge (6\mu^3 - 28\mu^2 + 27\mu - 5 \neq 0 \vee \mu^3 - 5\mu^2 + 5\mu \leq 1))) \wedge (\mu^3 - 5\mu^2 \\
& + 5\mu - 1 \neq 0 \vee 6\mu^3 - 28\mu^2 + 27\mu - 5 \neq 0 \vee 3\mu^3 - 13\mu^2 + 12\mu - 2 \\
& \neq 0 \vee (3\mu^2 - 13\mu \leq -12 \wedge (3\mu^2 - 13\mu + 12 \neq 0 \vee (6\mu^2 - 28\mu \\
& \leq -27 \wedge (6\mu^2 - 28\mu + 27 \neq 0 \vee \mu^2 - 5\mu \leq -5)))))) \wedge (-\mu \leq -5
\end{aligned}$$

$$\begin{aligned}
& \vee (-3\mu^2 + 28\mu \leq 65 \wedge 3\mu^2 - 28\mu + 65 \neq 0) \vee (\mu^2 - 2\mu \leq -1 \wedge (\mu^2 \\
& - 2\mu + 1 \neq 0 \vee (-2\mu^4 + 17\mu^3 - 39\mu^2 + 19\mu \leq -5 \wedge (2\mu^4 - 17\mu^3 \\
& + 39\mu^2 - 19\mu - 5 \neq 0 \vee \mu^3 - 5\mu^2 + 5\mu \leq 1))) \wedge (\mu^3 - 5\mu^2 + 5\mu - 1 \\
& \neq 0 \vee 6\mu^3 - 28\mu^2 + 27\mu - 5 \neq 0 \vee 3\mu^3 - 13\mu^2 + 12\mu - 2 \neq 0 \vee (3\mu \\
& \leq 5 \wedge (-5 + 3\mu \neq 0 \vee (-2\mu^3 + 17\mu^2 - 40\mu \leq -25 \wedge (2\mu^3 - 17\mu^2 \\
& + 40\mu - 25 \neq 0 \vee \mu^2 - 5\mu \leq -5)))))) \wedge (\mu \leq 5 \vee (-3\mu^2 + 28\mu \\
& \leq 65 \wedge 3\mu^2 - 28\mu + 65 \neq 0) \vee (\mu^2 - 2\mu \leq -1 \wedge (\mu^2 - 2\mu + 1 \neq 0 \\
& \vee (-2\mu^4 + 17\mu^3 - 39\mu^2 + 19\mu \leq -5 \wedge (2\mu^4 - 17\mu^3 + 39\mu^2 - 19\mu \\
& - 5 \neq 0 \vee \mu^3 - 5\mu^2 + 5\mu \leq 1))) \wedge (\mu^3 - 5\mu^2 + 5\mu - 1 \neq 0 \vee 6\mu^3 \\
& - 28\mu^2 + 27\mu - 5 \neq 0 \vee 3\mu^3 - 13\mu^2 + 12\mu - 2 \neq 0 \vee (3\mu \leq 5 \wedge (-5 \\
& + 3\mu \neq 0 \vee (-2\mu^3 + 17\mu^2 - 40\mu \leq -25 \wedge (2\mu^3 - 17\mu^2 + 40\mu - 25 \\
& \neq 0 \vee \mu^2 - 5\mu \leq -5)))))) \wedge (\mu^2 - 4\mu + 1 = 0 \vee (-3\mu^4 + 22\mu^3 \\
& - 45\mu^2 + 18\mu \leq 2 \wedge 3\mu^4 - 22\mu^3 + 45\mu^2 - 18\mu + 2 \neq 0) \vee (-2\mu^2 \\
& + 3\mu \leq 1 \wedge (2\mu^2 - 3\mu + 1 \neq 0 \vee \mu^3 - 5\mu^2 + 5\mu \leq 1) \wedge (\mu^3 - 5\mu^2 \\
& + 5\mu - 1 \neq 0 \vee 6\mu^3 - 28\mu^2 + 27\mu - 5 \neq 0 \vee 3\mu^3 - 13\mu^2 + 12\mu - 2 \\
& \neq 0 \vee (2\mu^2 - 5\mu \leq -2 \wedge (2\mu^2 - 5\mu + 2 \neq 0 \vee (-2\mu^5 + 19\mu^4 - 62\mu^3 \\
& + 82\mu^2 - 44\mu \leq -7 \wedge (2\mu^5 - 19\mu^4 + 62\mu^3 - 82\mu^2 + 44\mu - 7 \neq 0 \\
& \vee \mu^2 - 5\mu \leq -5)))))) \wedge (\mu^2 - 5\mu + 5 = 0 \vee (-3\mu^4 + 28\mu^3 - 92\mu^2 \\
& + 125\mu \leq 60 \wedge 3\mu^4 - 28\mu^3 + 92\mu^2 - 125\mu + 60 \neq 0) \vee (-2\mu^3 + 9\mu^2 \\
& - 13\mu \leq -6 \wedge (2\mu^3 - 9\mu^2 + 13\mu - 6 \neq 0 \vee (-2\mu^6 + 23\mu^5 - 99\mu^4 \\
& + 199\mu^3 - 191\mu^2 + 75\mu \leq 5 \wedge (2\mu^6 - 23\mu^5 + 99\mu^4 - 199\mu^3 + 191\mu^2 \\
& - 75\mu + 5 \neq 0 \vee \mu^3 - 5\mu^2 + 5\mu \leq 1))) \wedge (\mu^3 - 5\mu^2 + 5\mu - 1 \neq 0 \\
& \vee 6\mu^3 - 28\mu^2 + 27\mu - 5 \neq 0 \vee 3\mu^3 - 13\mu^2 + 12\mu - 2 \neq 0 \vee (-2\mu \\
& \leq -3 \wedge (2\mu - 3 \neq 0 \vee \mu^2 - 5\mu \leq -5)))) \wedge (3\mu^2 - 13\mu + 12 = 0 \\
& \vee (-9\mu^5 + 78\mu^4 - 241\mu^3 + 318\mu^2 - 170\mu \leq -24 \wedge 9\mu^5 - 78\mu^4 \\
& + 241\mu^3 - 318\mu^2 + 170\mu - 24 \neq 0) \vee 3\mu^2 - 13\mu \leq -12) \wedge (\mu \leq 5 \\
& \vee 3\mu^2 - 20\mu + 25 = 0 \vee -3\mu^2 + 28\mu < 65 \vee (-3\mu^3 + 11\mu^2 - 13\mu \\
& \leq -5 \wedge 3\mu^3 - 11\mu^2 + 13\mu - 5 \neq 0) \vee (3\mu \leq 5 \wedge (-5 + 3\mu \neq 0 \vee \mu \\
& - 3 \neq 0))) \wedge (-\mu \leq -5 \vee 3\mu^2 - 20\mu + 25 = 0 \vee -3\mu^2 + 28\mu < 65 \\
& \vee (-3\mu^3 + 11\mu^2 - 13\mu \leq -5 \wedge 3\mu^3 - 11\mu^2 + 13\mu - 5 \neq 0) \vee (3\mu \\
& \leq 5 \wedge (-5 + 3\mu \neq 0 \vee \mu - 3 \neq 0))) \wedge (\mu^2 - 4\mu + 1 = 0 \vee \mu^3 - 6\mu^2 \\
& + 9\mu - 2 = 0 \vee -3\mu^4 + 22\mu^3 - 45\mu^2 + 18\mu < 2 \vee (2\mu^2 - 5\mu \leq -2 \\
& \wedge (2\mu^2 - 5\mu + 2 \neq 0 \vee 2\mu^3 - 5\mu^2 + 2\mu \neq 0 \vee -2\mu^2 + 3\mu < 1)))
\end{aligned}$$

$$\begin{aligned}
& \wedge ((\mu^2 - 5\mu + 5 = 0 \wedge 3\mu^2 - 13\mu + 12 = 0) \vee (-9\mu^5 + 78\mu^4 - 241\mu^3 \\
& + 318\mu^2 - 170\mu \leq -24 \wedge (9\mu^5 - 78\mu^4 + 241\mu^3 - 318\mu^2 + 170\mu - 24 \\
& \neq 0 \vee (-6\mu^5 + 56\mu^4 - 184\mu^3 + 255\mu^2 - 143\mu \leq -22 \wedge (6\mu^5 - 56\mu^4 \\
& + 184\mu^3 - 255\mu^2 + 143\mu - 22 \neq 0 \vee -\mu^5 + 10\mu^4 - 35\mu^3 + 51\mu^2 - 30\mu \\
& \leq -5))) \wedge (\mu^5 - 10\mu^4 + 35\mu^3 - 51\mu^2 + 30\mu - 5 \neq 0 \vee 6\mu^5 - 56\mu^4 \\
& + 184\mu^3 - 255\mu^2 + 143\mu - 22 \neq 0 \vee 9\mu^5 - 78\mu^4 + 241\mu^3 - 318\mu^2 \\
& + 170\mu - 24 \neq 0) \vee (3\mu^2 - 13\mu \leq -12 \wedge (3\mu^2 - 13\mu + 12 \neq 0 \\
& \vee (6\mu^2 - 28\mu \leq -27 \wedge (6\mu^2 - 28\mu + 27 \neq 0 \vee \mu^2 - 5\mu \leq -5)))))) \\
& \wedge (-\mu \leq -5 \vee (\mu^2 - 5\mu + 5 = 0 \wedge 3\mu^2 - 13\mu + 12 = 0) \vee (-3\mu^2 \\
& + 28\mu \leq 65 \wedge 3\mu^2 - 28\mu + 65 \neq 0) \vee (-3\mu^3 + 11\mu^2 - 13\mu \leq -5 \\
& \wedge (3\mu^3 - 11\mu^2 + 13\mu - 5 \neq 0 \vee (6\mu^5 - 71\mu^4 + 293\mu^3 - 513\mu^2 + 385\mu \\
& \leq 100 \wedge (6\mu^5 - 71\mu^4 + 293\mu^3 - 513\mu^2 + 385\mu - 100 \neq 0 \vee -\mu^5 \\
& + 10\mu^4 - 35\mu^3 + 51\mu^2 - 30\mu \leq -5)))) \wedge (\mu^5 - 10\mu^4 + 35\mu^3 - 51\mu^2 \\
& + 30\mu - 5 \neq 0 \vee 6\mu^5 - 56\mu^4 + 184\mu^3 - 255\mu^2 + 143\mu - 22 \neq 0 \vee 9\mu^5 \\
& - 78\mu^4 + 241\mu^3 - 318\mu^2 + 170\mu - 24 \neq 0) \vee (3\mu \leq 5 \wedge (-5 + 3\mu \\
& \neq 0 \vee (-2\mu^3 + 17\mu^2 - 40\mu \leq -25 \wedge (2\mu^3 - 17\mu^2 + 40\mu - 25 \neq 0 \\
& \vee \mu^2 - 5\mu \leq -5)))))) \wedge (\mu \leq 5 \vee (\mu^2 - 5\mu + 5 = 0 \wedge 3\mu^2 - 13\mu + 12 \\
& = 0) \vee (-3\mu^2 + 28\mu \leq 65 \wedge 3\mu^2 - 28\mu + 65 \neq 0) \vee (-3\mu^3 + 11\mu^2 \\
& - 13\mu \leq -5 \wedge (3\mu^3 - 11\mu^2 + 13\mu - 5 \neq 0 \vee (6\mu^5 - 71\mu^4 + 293\mu^3 \\
& - 513\mu^2 + 385\mu \leq 100 \wedge (6\mu^5 - 71\mu^4 + 293\mu^3 - 513\mu^2 + 385\mu - 100 \\
& \neq 0 \vee -\mu^5 + 10\mu^4 - 35\mu^3 + 51\mu^2 - 30\mu \leq -5)))) \wedge (\mu^5 - 10\mu^4 \\
& + 35\mu^3 - 51\mu^2 + 30\mu - 5 \neq 0 \vee 6\mu^5 - 56\mu^4 + 184\mu^3 - 255\mu^2 + 143\mu \\
& - 22 \neq 0 \vee 9\mu^5 - 78\mu^4 + 241\mu^3 - 318\mu^2 + 170\mu - 24 \neq 0) \vee (3\mu \\
& \leq 5 \wedge (-5 + 3\mu \neq 0 \vee (-2\mu^3 + 17\mu^2 - 40\mu \leq -25 \wedge (2\mu^3 - 17\mu^2 \\
& + 40\mu - 25 \neq 0 \vee \mu^2 - 5\mu \leq -5)))))) \wedge (\mu^2 - 4\mu + 1 = 0 \vee (\mu^2 - 5\mu \\
& + 5 = 0 \wedge 3\mu^2 - 13\mu + 12 = 0) \vee (-3\mu^4 + 22\mu^3 - 45\mu^2 + 18\mu \leq 2 \\
& \wedge 3\mu^4 - 22\mu^3 + 45\mu^2 - 18\mu + 2 \neq 0) \vee (\mu^2 - 3\mu \leq -2 \wedge (\mu^2 - 3\mu \\
& + 2 \neq 0 \vee -\mu^5 + 10\mu^4 - 35\mu^3 + 51\mu^2 - 30\mu \leq -5) \wedge (\mu^5 - 10\mu^4 \\
& + 35\mu^3 - 51\mu^2 + 30\mu - 5 \neq 0 \vee 6\mu^5 - 56\mu^4 + 184\mu^3 - 255\mu^2 + 143\mu \\
& - 22 \neq 0 \vee 9\mu^5 - 78\mu^4 + 241\mu^3 - 318\mu^2 + 170\mu - 24 \neq 0) \vee (2\mu^2 \\
& - 5\mu \leq -2 \wedge (2\mu^2 - 5\mu + 2 \neq 0 \vee (-2\mu^5 + 19\mu^4 - 62\mu^3 + 82\mu^2 \\
& - 44\mu \leq -7 \wedge (2\mu^5 - 19\mu^4 + 62\mu^3 - 82\mu^2 + 44\mu - 7 \neq 0 \vee \mu^2 - 5\mu \\
& \leq -5)))))) \wedge (\mu^2 - 5\mu + 5 = 0 \vee (\mu^2 - 5\mu + 5 = 0 \wedge 3\mu^2 - 13\mu + 12
\end{aligned}$$

$$\begin{aligned}
&= 0) \vee (-3\mu^4 + 28\mu^3 - 92\mu^2 + 125\mu \leq 60 \wedge 3\mu^4 - 28\mu^3 + 92\mu^2 \\
&- 125\mu + 60 \neq 0) \vee (-\mu^2 + 3\mu \leq 2 \wedge (\mu^2 - 3\mu + 2 \neq 0 \vee -\mu^5 \\
&+ 10\mu^4 - 35\mu^3 + 51\mu^2 - 30\mu \leq -5) \wedge (\mu^5 - 10\mu^4 + 35\mu^3 - 51\mu^2 \\
&+ 30\mu - 5 \neq 0 \vee 6\mu^5 - 56\mu^4 + 184\mu^3 - 255\mu^2 + 143\mu - 22 \neq 0 \vee 9\mu^5 \\
&- 78\mu^4 + 241\mu^3 - 318\mu^2 + 170\mu - 24 \neq 0)) \vee (-2\mu \leq -3 \wedge (2\mu \\
&- 3 \neq 0 \vee \mu^2 - 5\mu \leq -5))) \wedge (3\mu^2 - 13\mu + 12 = 0 \vee -3\mu \leq -13 \vee \\
&-9\mu^5 + 78\mu^4 - 241\mu^3 + 318\mu^2 - 170\mu \leq -24 \vee 3\mu < 13) \wedge ((\mu^2 \\
&- 5\mu + 5 = 0 \wedge 3\mu^2 - 13\mu + 12 = 0) \vee (-3\mu \leq -13 \wedge (3\mu - 13 \neq 0 \\
&\vee -\mu \leq -5))) \vee (-9\mu^5 + 78\mu^4 - 241\mu^3 + 318\mu^2 - 170\mu \leq -24 \\
&\wedge (9\mu^5 - 78\mu^4 + 241\mu^3 - 318\mu^2 + 170\mu - 24 \neq 0 \vee (-6\mu^5 + 56\mu^4 \\
&- 184\mu^3 + 255\mu^2 - 143\mu \leq -22 \wedge (6\mu^5 - 56\mu^4 + 184\mu^3 - 255\mu^2 \\
&+ 143\mu - 22 \neq 0 \vee -\mu^5 + 10\mu^4 - 35\mu^3 + 51\mu^2 - 30\mu \leq -5)))) \\
&\vee (3\mu \leq 13 \wedge (3\mu - 13 \neq 0 \vee (3\mu \leq 14 \wedge (3\mu - 14 \neq 0 \vee \mu \\
&< 5)))))) \wedge (\mu^2 - 4\mu + 1 = 0 \vee (\mu^2 - 5\mu + 5 = 0 \wedge 3\mu^2 - 13\mu + 12 \\
&= 0) \vee (-\mu^3 + 5\mu^2 - 5\mu \leq -1 \wedge (\mu^3 - 5\mu^2 + 5\mu - 1 \neq 0 \vee -\mu \\
&\leq -5))) \vee (-3\mu^4 + 22\mu^3 - 45\mu^2 + 18\mu \leq 2 \wedge 3\mu^4 - 22\mu^3 + 45\mu^2 \\
&- 18\mu + 2 \neq 0) \vee (\mu^2 - 3\mu \leq -2 \wedge (\mu^2 - 3\mu + 2 \neq 0 \vee -\mu^5 + 10\mu^4 \\
&- 35\mu^3 + 51\mu^2 - 30\mu \leq -5)) \vee (-2\mu^2 + 3\mu \leq 1 \wedge (2\mu^2 - 3\mu + 1 \\
&\neq 0 \vee (-\mu^4 + 11\mu^3 - 33\mu^2 + 23\mu \leq 4 \wedge (\mu^4 - 11\mu^3 + 33\mu^2 - 23\mu + 4 \\
&\neq 0 \vee \mu < 5)))))) \wedge (\mu^2 - 5\mu + 5 = 0 \vee (\mu^2 - 5\mu + 5 = 0 \wedge 3\mu^2 - 13\mu \\
&+ 12 = 0) \vee (-3\mu^3 + 20\mu^2 - 40\mu \leq -25 \wedge (3\mu^3 - 20\mu^2 + 40\mu - 25 \\
&\neq 0 \vee -\mu \leq -5))) \vee (-3\mu^4 + 28\mu^3 - 92\mu^2 + 125\mu \leq 60 \wedge 3\mu^4 \\
&- 28\mu^3 + 92\mu^2 - 125\mu + 60 \neq 0) \vee (-\mu^2 + 3\mu \leq 2 \wedge (\mu^2 - 3\mu + 2 \\
&\neq 0 \vee -\mu^5 + 10\mu^4 - 35\mu^3 + 51\mu^2 - 30\mu \leq -5)) \vee (-6\mu^2 + 19\mu \\
&\leq 15 \wedge (6\mu^2 - 19\mu + 15 \neq 0 \vee (-\mu^4 + 13\mu^3 - 55\mu^2 + 90\mu \leq 50 \wedge (\mu^4 \\
&- 13\mu^3 + 55\mu^2 - 90\mu + 50 \neq 0 \vee \mu < 5)))))) \wedge (-3\mu \leq -13 \vee (- \\
&-9\mu^4 + 78\mu^3 - 205\mu^2 + 162\mu \leq 26 \wedge (9\mu^4 - 78\mu^3 + 205\mu^2 - 162\mu \\
&+ 26 \neq 0 \vee -9\mu^3 + 78\mu^2 - 205\mu \leq -156) \wedge (9\mu^3 - 78\mu^2 + 205\mu \\
&- 156 \neq 0 \vee 3\mu - 13 \neq 0)) \vee 3\mu < 13) \wedge (\mu^2 - 5\mu + 5 = 0 \vee -3\mu^3 \\
&+ 20\mu^2 - 40\mu \leq -25 \vee -3\mu^4 + 28\mu^3 - 92\mu^2 + 125\mu < 60 \vee (-3\mu^3 \\
&+ 14\mu^2 - 21\mu \leq -10 \wedge 3\mu^3 - 14\mu^2 + 21\mu - 10 \neq 0) \vee -6\mu^2 + 19\mu \\
&< 15) \wedge (\mu^2 - 4\mu + 1 = 0 \vee -\mu^3 + 5\mu^2 - 5\mu \leq -1 \vee -3\mu^4 + 22\mu^3 \\
&- 45\mu^2 + 18\mu < 2 \vee (\mu^2 - 3\mu \leq -2 \wedge (\mu^2 - 3\mu + 2 \neq 0 \vee \mu^3 - 3\mu^2
\end{aligned}$$

$$\begin{aligned}
& + 2\mu \neq 0)) \vee (-2\mu^2 + 3\mu < 1) \wedge ((-3\mu \leq -13 \wedge (3\mu - 13 \neq 0 \vee \\
& -\mu \leq -5)) \vee (-9\mu^4 + 78\mu^3 - 205\mu^2 + 162\mu \leq 26 \wedge (9\mu^4 - 78\mu^3 \\
& + 205\mu^2 - 162\mu + 26 \neq 0 \vee (-6\mu^4 + 56\mu^3 - 157\mu^2 + 130\mu \leq 23 \\
& \wedge (6\mu^4 - 56\mu^3 + 157\mu^2 - 130\mu + 23 \neq 0 \vee -\mu^4 + 10\mu^3 - 30\mu^2 + 26\mu \\
& \leq 5))) \wedge (\mu^4 - 10\mu^3 + 30\mu^2 - 26\mu + 5 \neq 0 \vee 6\mu^4 - 56\mu^3 + 157\mu^2 \\
& - 130\mu + 23 \neq 0 \vee 9\mu^4 - 78\mu^3 + 205\mu^2 - 162\mu + 26 \neq 0 \vee (-9\mu^3 \\
& + 78\mu^2 - 205\mu \leq -156 \wedge (9\mu^3 - 78\mu^2 + 205\mu - 156 \neq 0 \vee (-6\mu^3 \\
& + 56\mu^2 - 157\mu \leq -125 \wedge (6\mu^3 - 56\mu^2 + 157\mu - 125 \neq 0 \vee -\mu^3 \\
& + 10\mu^2 - 30\mu \leq -25)))))) \wedge (\mu^3 - 10\mu^2 + 30\mu - 25 \neq 0 \vee 6\mu^3 \\
& - 56\mu^2 + 157\mu - 125 \neq 0 \vee 9\mu^3 - 78\mu^2 + 205\mu - 156 \neq 0 \vee \mu - 5 \neq 0 \\
& \vee 5\mu - 23 \neq 0 \vee 3\mu - 13 \neq 0)) \vee (3\mu \leq 13 \wedge (3\mu - 13 \neq 0 \vee (3\mu \\
& \leq 14 \wedge (3\mu - 14 \neq 0 \vee \mu < 5)))) \wedge (\mu^2 - 4\mu + 1 = 0 \vee (-\mu^3 + 5\mu^2 \\
& - 5\mu \leq -1 \wedge (\mu^3 - 5\mu^2 + 5\mu - 1 \neq 0 \vee -\mu \leq -5)) \vee (-3\mu^4 \\
& + 22\mu^3 - 45\mu^2 + 18\mu \leq 2 \wedge 3\mu^4 - 22\mu^3 + 45\mu^2 - 18\mu + 2 \neq 0) \vee ( \\
& -\mu^2 + 2\mu \leq 1 \wedge (\mu^2 - 2\mu + 1 \neq 0 \vee -\mu^4 + 10\mu^3 - 30\mu^2 + 26\mu \leq 5) \\
& \wedge (\mu^4 - 10\mu^3 + 30\mu^2 - 26\mu + 5 \neq 0 \vee 6\mu^4 - 56\mu^3 + 157\mu^2 - 130\mu \\
& + 23 \neq 0 \vee 9\mu^4 - 78\mu^3 + 205\mu^2 - 162\mu + 26 \neq 0 \vee (\mu^2 - 3\mu \leq -2 \\
& \wedge (\mu^2 - 3\mu + 2 \neq 0 \vee (-3\mu^5 + 31\mu^4 - 116\mu^3 + 192\mu^2 - 137\mu \leq -25 \\
& \wedge (3\mu^5 - 31\mu^4 + 116\mu^3 - 192\mu^2 + 137\mu - 25 \neq 0 \vee -\mu^3 + 10\mu^2 \\
& - 30\mu \leq -25)))))) \wedge (\mu^3 - 10\mu^2 + 30\mu - 25 \neq 0 \vee 6\mu^3 - 56\mu^2 \\
& + 157\mu - 125 \neq 0 \vee 9\mu^3 - 78\mu^2 + 205\mu - 156 \neq 0 \vee \mu - 5 \neq 0 \vee 5\mu \\
& - 23 \neq 0 \vee 3\mu - 13 \neq 0)) \vee (-2\mu^2 + 3\mu \leq 1 \wedge (2\mu^2 - 3\mu + 1 \neq 0 \\
& \vee (-\mu^4 + 11\mu^3 - 33\mu^2 + 23\mu \leq 4 \wedge (\mu^4 - 11\mu^3 + 33\mu^2 - 23\mu + 4 \neq 0 \\
& \vee \mu < 5)))) \wedge (\mu^2 - 5\mu + 5 = 0 \vee (-3\mu^3 + 20\mu^2 - 40\mu \leq -25 \\
& \wedge (3\mu^3 - 20\mu^2 + 40\mu - 25 \neq 0 \vee -\mu \leq -5)) \vee (-3\mu^4 + 28\mu^3 \\
& - 92\mu^2 + 125\mu \leq 60 \wedge 3\mu^4 - 28\mu^3 + 92\mu^2 - 125\mu + 60 \neq 0) \vee (-3\mu^3 \\
& + 14\mu^2 - 21\mu \leq -10 \wedge (3\mu^3 - 14\mu^2 + 21\mu - 10 \neq 0 \vee (-3\mu^6 + 34\mu^5 \\
& - 142\mu^4 + 266\mu^3 - 210\mu^2 + 30\mu \leq -25 \wedge (3\mu^6 - 34\mu^5 + 142\mu^4 \\
& - 266\mu^3 + 210\mu^2 - 30\mu - 25 \neq 0 \vee -\mu^4 + 10\mu^3 - 30\mu^2 + 26\mu \leq 5)))) \\
& \wedge (\mu^4 - 10\mu^3 + 30\mu^2 - 26\mu + 5 \neq 0 \vee 6\mu^4 - 56\mu^3 + 157\mu^2 - 130\mu \\
& + 23 \neq 0 \vee 9\mu^4 - 78\mu^3 + 205\mu^2 - 162\mu + 26 \neq 0 \vee (-3\mu \leq -5 \wedge ( \\
& -5 + 3\mu \neq 0 \vee -\mu^3 + 10\mu^2 - 30\mu \leq -25))) \wedge (\mu^3 - 10\mu^2 + 30\mu \\
& - 25 \neq 0 \vee 6\mu^3 - 56\mu^2 + 157\mu - 125 \neq 0 \vee 9\mu^3 - 78\mu^2 + 205\mu - 156
\end{aligned}$$

$$\begin{aligned}
& \neq 0 \vee \mu - 5 \neq 0 \vee 5\mu - 23 \neq 0 \vee 3\mu - 13 \neq 0)) \vee (-6\mu^2 + 19\mu \\
& \leq 15 \wedge (6\mu^2 - 19\mu + 15 \neq 0 \vee (-\mu^4 + 13\mu^3 - 55\mu^2 + 90\mu \leq 50 \wedge (\mu^4 \\
& - 13\mu^3 + 55\mu^2 - 90\mu + 50 \neq 0 \vee \mu < 5)))))) \wedge (3\mu^2 - 13\mu + 12 = 0 \\
& \vee -3\mu \leq -13 \vee (-9\mu^5 + 78\mu^4 - 241\mu^3 + 318\mu^2 - 170\mu \leq -24 \\
& \wedge 9\mu^5 - 78\mu^4 + 241\mu^3 - 318\mu^2 + 170\mu - 24 \neq 0) \vee (-9\mu^3 + 78\mu^2 \\
& - 205\mu \leq -156 \wedge (9\mu^3 - 78\mu^2 + 205\mu - 156 \neq 0 \vee 3\mu - 13 \neq 0)) \\
& \vee 3\mu < 13) \wedge (\mu^2 - 4\mu + 1 = 0 \vee \mu^3 - 6\mu^2 + 9\mu - 2 = 0 \vee -\mu^3 + 5\mu^2 \\
& - 5\mu \leq -1 \vee -3\mu^4 + 22\mu^3 - 45\mu^2 + 18\mu < 2 \vee (\mu^2 - 3\mu \leq -2 \\
& \wedge (\mu^2 - 3\mu + 2 \neq 0 \vee \mu^3 - 3\mu^2 + 2\mu \neq 0)) \vee -2\mu^2 + 3\mu < 1) \\
& \wedge ((\mu^2 - 5\mu + 5 = 0 \wedge 3\mu^2 - 13\mu + 12 = 0) \vee (-3\mu \leq -13 \wedge (3\mu \\
& - 13 \neq 0 \vee -\mu \leq -5)) \vee (-9\mu^5 + 78\mu^4 - 241\mu^3 + 318\mu^2 - 170\mu \\
& \leq -24 \wedge (9\mu^5 - 78\mu^4 + 241\mu^3 - 318\mu^2 + 170\mu - 24 \neq 0 \vee (-6\mu^5 \\
& + 56\mu^4 - 184\mu^3 + 255\mu^2 - 143\mu \leq -22 \wedge (6\mu^5 - 56\mu^4 + 184\mu^3 \\
& - 255\mu^2 + 143\mu - 22 \neq 0 \vee -\mu^5 + 10\mu^4 - 35\mu^3 + 51\mu^2 - 30\mu \leq -5) \\
& )) \wedge (\mu^5 - 10\mu^4 + 35\mu^3 - 51\mu^2 + 30\mu - 5 \neq 0 \vee 6\mu^5 - 56\mu^4 + 184\mu^3 \\
& - 255\mu^2 + 143\mu - 22 \neq 0 \vee 9\mu^5 - 78\mu^4 + 241\mu^3 - 318\mu^2 + 170\mu - 24 \\
& \neq 0)) \vee (-9\mu^3 + 78\mu^2 - 205\mu \leq -156 \wedge (9\mu^3 - 78\mu^2 + 205\mu - 156 \\
& \neq 0 \vee (-6\mu^3 + 56\mu^2 - 157\mu \leq -125 \wedge (6\mu^3 - 56\mu^2 + 157\mu - 125 \\
& \neq 0 \vee -\mu^3 + 10\mu^2 - 30\mu \leq -25)))) \wedge (\mu^3 - 10\mu^2 + 30\mu - 25 \neq 0 \\
& \vee 6\mu^3 - 56\mu^2 + 157\mu - 125 \neq 0 \vee 9\mu^3 - 78\mu^2 + 205\mu - 156 \neq 0 \vee \mu \\
& - 5 \neq 0 \vee 5\mu - 23 \neq 0 \vee 3\mu - 13 \neq 0)) \vee (3\mu \leq 13 \wedge (3\mu - 13 \neq 0 \\
& \vee (3\mu \leq 14 \wedge (3\mu - 14 \neq 0 \vee \mu < 5)))))) \wedge (\mu^2 - 4\mu + 1 = 0 \vee (\mu^2 \\
& - 5\mu + 5 = 0 \wedge 3\mu^2 - 13\mu + 12 = 0) \vee (-\mu^3 + 5\mu^2 - 5\mu \leq -1 \wedge (\mu^3 \\
& - 5\mu^2 + 5\mu - 1 \neq 0 \vee -\mu \leq -5)) \vee (-3\mu^4 + 22\mu^3 - 45\mu^2 + 18\mu \\
& \leq 2 \wedge 3\mu^4 - 22\mu^3 + 45\mu^2 - 18\mu + 2 \neq 0) \vee (\mu^2 - 3\mu \leq -2 \wedge (\mu^2 \\
& - 3\mu + 2 \neq 0 \vee -\mu^5 + 10\mu^4 - 35\mu^3 + 51\mu^2 - 30\mu \leq -5) \wedge (\mu^5 \\
& - 10\mu^4 + 35\mu^3 - 51\mu^2 + 30\mu - 5 \neq 0 \vee 6\mu^5 - 56\mu^4 + 184\mu^3 - 255\mu^2 \\
& + 143\mu - 22 \neq 0 \vee 9\mu^5 - 78\mu^4 + 241\mu^3 - 318\mu^2 + 170\mu - 24 \neq 0)) \\
& \vee (\mu^2 - 3\mu \leq -2 \wedge (\mu^2 - 3\mu + 2 \neq 0 \vee (-3\mu^5 + 31\mu^4 - 116\mu^3 \\
& + 192\mu^2 - 137\mu \leq -25 \wedge (3\mu^5 - 31\mu^4 + 116\mu^3 - 192\mu^2 + 137\mu - 25 \\
& \neq 0 \vee -\mu^3 + 10\mu^2 - 30\mu \leq -25)))) \wedge (\mu^3 - 10\mu^2 + 30\mu - 25 \neq 0 \\
& \vee 6\mu^3 - 56\mu^2 + 157\mu - 125 \neq 0 \vee 9\mu^3 - 78\mu^2 + 205\mu - 156 \neq 0 \vee \mu \\
& - 5 \neq 0 \vee 5\mu - 23 \neq 0 \vee 3\mu - 13 \neq 0)) \vee (-2\mu^2 + 3\mu \leq 1 \wedge (2\mu^2
\end{aligned}$$

$$\begin{aligned}
& -3\mu + 1 \neq 0 \vee (-\mu^4 + 11\mu^3 - 33\mu^2 + 23\mu \leq 4 \wedge (\mu^4 - 11\mu^3 + 33\mu^2 \\
& - 23\mu + 4 \neq 0 \vee \mu < 5)))) \wedge (\mu^2 - 5\mu + 5 = 0 \vee (\mu^2 - 5\mu + 5 = 0 \\
& \wedge 3\mu^2 - 13\mu + 12 = 0) \vee (-3\mu^3 + 20\mu^2 - 40\mu \leq -25 \wedge (3\mu^3 - 20\mu^2 \\
& + 40\mu - 25 \neq 0 \vee -\mu \leq -5)) \vee (-3\mu^4 + 28\mu^3 - 92\mu^2 + 125\mu \leq 60 \\
& \wedge 3\mu^4 - 28\mu^3 + 92\mu^2 - 125\mu + 60 \neq 0) \vee (-\mu^2 + 3\mu \leq 2 \wedge (\mu^2 - 3\mu \\
& + 2 \neq 0 \vee -\mu^5 + 10\mu^4 - 35\mu^3 + 51\mu^2 - 30\mu \leq -5) \wedge (\mu^5 - 10\mu^4 \\
& + 35\mu^3 - 51\mu^2 + 30\mu - 5 \neq 0 \vee 6\mu^5 - 56\mu^4 + 184\mu^3 - 255\mu^2 + 143\mu \\
& - 22 \neq 0 \vee 9\mu^5 - 78\mu^4 + 241\mu^3 - 318\mu^2 + 170\mu - 24 \neq 0)) \vee (-3\mu \\
& \leq -5 \wedge (-5 + 3\mu \neq 0 \vee -\mu^3 + 10\mu^2 - 30\mu \leq -25) \wedge (\mu^3 - 10\mu^2 \\
& + 30\mu - 25 \neq 0 \vee 6\mu^3 - 56\mu^2 + 157\mu - 125 \neq 0 \vee 9\mu^3 - 78\mu^2 + 205\mu \\
& - 156 \neq 0 \vee \mu - 5 \neq 0 \vee 5\mu - 23 \neq 0 \vee 3\mu - 13 \neq 0)) \vee (-6\mu^2 \\
& + 19\mu \leq 15 \wedge (6\mu^2 - 19\mu + 15 \neq 0 \vee (-\mu^4 + 13\mu^3 - 55\mu^2 + 90\mu \\
& \leq 50 \wedge (\mu^4 - 13\mu^3 + 55\mu^2 - 90\mu + 50 \neq 0 \vee \mu < 5)))) \wedge (3\mu^2 \\
& - 13\mu + 12 = 0 \vee 3\mu \leq 13 \vee -9\mu^5 + 78\mu^4 - 241\mu^3 + 318\mu^2 - 170\mu \\
& \leq -24 \vee 3\mu < 13) \wedge ((\mu^2 - 5\mu + 5 = 0 \wedge 3\mu^2 - 13\mu + 12 = 0) \vee (3\mu \\
& \leq 13 \wedge (3\mu - 13 \neq 0 \vee \mu \leq 5)) \vee (-9\mu^5 + 78\mu^4 - 241\mu^3 + 318\mu^2 \\
& - 170\mu \leq -24 \wedge (9\mu^5 - 78\mu^4 + 241\mu^3 - 318\mu^2 + 170\mu - 24 \neq 0 \vee (-6\mu^5 \\
& + 56\mu^4 - 184\mu^3 + 255\mu^2 - 143\mu \leq -22 \wedge (6\mu^5 - 56\mu^4 + 184\mu^3 \\
& - 255\mu^2 + 143\mu - 22 \neq 0 \vee -\mu^5 + 10\mu^4 - 35\mu^3 + 51\mu^2 - 30\mu \leq -5) \\
& ))) \vee (3\mu \leq 13 \wedge (3\mu - 13 \neq 0 \vee (3\mu \leq 14 \wedge (3\mu - 14 \neq 0 \vee \mu \\
& < 5)))))) \wedge (\mu^2 - 4\mu + 1 = 0 \vee (\mu^2 - 5\mu + 5 = 0 \wedge 3\mu^2 - 13\mu + 12 \\
& = 0) \vee (\mu^3 - 5\mu^2 + 5\mu \leq 1 \wedge (\mu^3 - 5\mu^2 + 5\mu - 1 \neq 0 \vee \mu \leq 5)) \vee (-3\mu^4 \\
& + 22\mu^3 - 45\mu^2 + 18\mu \leq 2 \wedge 3\mu^4 - 22\mu^3 + 45\mu^2 - 18\mu + 2 \neq 0) \\
& \vee (\mu^2 - 3\mu \leq -2 \wedge (\mu^2 - 3\mu + 2 \neq 0 \vee -\mu^5 + 10\mu^4 - 35\mu^3 + 51\mu^2 \\
& - 30\mu \leq -5)) \vee (-2\mu^2 + 3\mu \leq 1 \wedge (2\mu^2 - 3\mu + 1 \neq 0 \vee (-\mu^4 \\
& + 11\mu^3 - 33\mu^2 + 23\mu \leq 4 \wedge (\mu^4 - 11\mu^3 + 33\mu^2 - 23\mu + 4 \neq 0 \vee \mu \\
& < 5)))))) \wedge (\mu^2 - 5\mu + 5 = 0 \vee (\mu^2 - 5\mu + 5 = 0 \wedge 3\mu^2 - 13\mu + 12 \\
& = 0) \vee (3\mu^3 - 20\mu^2 + 40\mu \leq 25 \wedge (3\mu^3 - 20\mu^2 + 40\mu - 25 \neq 0 \vee \mu \\
& \leq 5)) \vee (-3\mu^4 + 28\mu^3 - 92\mu^2 + 125\mu \leq 60 \wedge 3\mu^4 - 28\mu^3 + 92\mu^2 \\
& - 125\mu + 60 \neq 0) \vee (-\mu^2 + 3\mu \leq 2 \wedge (\mu^2 - 3\mu + 2 \neq 0 \vee -\mu^5 \\
& + 10\mu^4 - 35\mu^3 + 51\mu^2 - 30\mu \leq -5)) \vee (-6\mu^2 + 19\mu \leq 15 \wedge (6\mu^2 \\
& - 19\mu + 15 \neq 0 \vee (-\mu^4 + 13\mu^3 - 55\mu^2 + 90\mu \leq 50 \wedge (\mu^4 - 13\mu^3 \\
& + 55\mu^2 - 90\mu + 50 \neq 0 \vee \mu < 5)))))) \wedge (3\mu \leq 13 \vee (-9\mu^4 + 78\mu^3
\end{aligned}$$

$$\begin{aligned}
& -205\mu^2 + 162\mu \leq 26 \wedge (9\mu^4 - 78\mu^3 + 205\mu^2 - 162\mu + 26 \neq 0 \vee \\
& -9\mu^3 + 78\mu^2 - 205\mu \leq -156) \wedge (9\mu^3 - 78\mu^2 + 205\mu - 156 \neq 0 \vee 3\mu \\
& - 13 \neq 0)) \vee 3\mu < 13) \wedge (\mu^2 - 5\mu + 5 = 0 \vee 3\mu^3 - 20\mu^2 + 40\mu \leq 25 \\
& \vee -3\mu^4 + 28\mu^3 - 92\mu^2 + 125\mu < 60 \vee (-3\mu^3 + 14\mu^2 - 21\mu \leq -10 \\
& \wedge 3\mu^3 - 14\mu^2 + 21\mu - 10 \neq 0) \vee -6\mu^2 + 19\mu < 15) \wedge (\mu^2 - 4\mu + 1 \\
& = 0 \vee \mu^3 - 5\mu^2 + 5\mu \leq 1 \vee -3\mu^4 + 22\mu^3 - 45\mu^2 + 18\mu < 2 \vee (\mu^2 \\
& - 3\mu \leq -2 \wedge (\mu^2 - 3\mu + 2 \neq 0 \vee \mu^3 - 3\mu^2 + 2\mu \neq 0)) \vee -2\mu^2 + 3\mu \\
& < 1) \wedge ((3\mu \leq 13 \wedge (3\mu - 13 \neq 0 \vee \mu \leq 5)) \vee (-9\mu^4 + 78\mu^3 \\
& - 205\mu^2 + 162\mu \leq 26 \wedge (9\mu^4 - 78\mu^3 + 205\mu^2 - 162\mu + 26 \neq 0 \vee ( \\
& -6\mu^4 + 56\mu^3 - 157\mu^2 + 130\mu \leq 23 \wedge (6\mu^4 - 56\mu^3 + 157\mu^2 - 130\mu \\
& + 23 \neq 0 \vee -\mu^4 + 10\mu^3 - 30\mu^2 + 26\mu \leq 5))) \wedge (\mu^4 - 10\mu^3 + 30\mu^2 \\
& - 26\mu + 5 \neq 0 \vee 6\mu^4 - 56\mu^3 + 157\mu^2 - 130\mu + 23 \neq 0 \vee 9\mu^4 - 78\mu^3 \\
& + 205\mu^2 - 162\mu + 26 \neq 0 \vee (-9\mu^3 + 78\mu^2 - 205\mu \leq -156 \wedge (9\mu^3 \\
& - 78\mu^2 + 205\mu - 156 \neq 0 \vee (-6\mu^3 + 56\mu^2 - 157\mu \leq -125 \wedge (6\mu^3 \\
& - 56\mu^2 + 157\mu - 125 \neq 0 \vee -\mu^3 + 10\mu^2 - 30\mu \leq -25)))))) \wedge (\mu^3 \\
& - 10\mu^2 + 30\mu - 25 \neq 0 \vee 6\mu^3 - 56\mu^2 + 157\mu - 125 \neq 0 \vee 9\mu^3 - 78\mu^2 \\
& + 205\mu - 156 \neq 0 \vee \mu - 5 \neq 0 \vee 5\mu - 23 \neq 0 \vee 3\mu - 13 \neq 0)) \vee (3\mu \\
& \leq 13 \wedge (3\mu - 13 \neq 0 \vee (3\mu \leq 14 \wedge (3\mu - 14 \neq 0 \vee \mu < 5)))))) \\
& \wedge (\mu^2 - 4\mu + 1 = 0 \vee (\mu^3 - 5\mu^2 + 5\mu \leq 1 \wedge (\mu^3 - 5\mu^2 + 5\mu - 1 \neq 0 \\
& \vee \mu \leq 5)) \vee (-3\mu^4 + 22\mu^3 - 45\mu^2 + 18\mu \leq 2 \wedge 3\mu^4 - 22\mu^3 + 45\mu^2 \\
& - 18\mu + 2 \neq 0) \vee (-\mu^2 + 2\mu \leq 1 \wedge (\mu^2 - 2\mu + 1 \neq 0 \vee -\mu^4 + 10\mu^3 \\
& - 30\mu^2 + 26\mu \leq 5) \wedge (\mu^4 - 10\mu^3 + 30\mu^2 - 26\mu + 5 \neq 0 \vee 6\mu^4 - 56\mu^3 \\
& + 157\mu^2 - 130\mu + 23 \neq 0 \vee 9\mu^4 - 78\mu^3 + 205\mu^2 - 162\mu + 26 \neq 0 \\
& \vee (\mu^2 - 3\mu \leq -2 \wedge (\mu^2 - 3\mu + 2 \neq 0 \vee (-3\mu^5 + 31\mu^4 - 116\mu^3 \\
& + 192\mu^2 - 137\mu \leq -25 \wedge (3\mu^5 - 31\mu^4 + 116\mu^3 - 192\mu^2 + 137\mu - 25 \\
& \neq 0 \vee -\mu^3 + 10\mu^2 - 30\mu \leq -25)))))) \wedge (\mu^3 - 10\mu^2 + 30\mu - 25 \neq 0 \\
& \vee 6\mu^3 - 56\mu^2 + 157\mu - 125 \neq 0 \vee 9\mu^3 - 78\mu^2 + 205\mu - 156 \neq 0 \vee \mu \\
& - 5 \neq 0 \vee 5\mu - 23 \neq 0 \vee 3\mu - 13 \neq 0)) \vee (-2\mu^2 + 3\mu \leq 1 \wedge (2\mu^2 \\
& - 3\mu + 1 \neq 0 \vee (-\mu^4 + 11\mu^3 - 33\mu^2 + 23\mu \leq 4 \wedge (\mu^4 - 11\mu^3 + 33\mu^2 \\
& - 23\mu + 4 \neq 0 \vee \mu < 5)))))) \wedge (\mu^2 - 5\mu + 5 = 0 \vee (3\mu^3 - 20\mu^2 \\
& + 40\mu \leq 25 \wedge (3\mu^3 - 20\mu^2 + 40\mu - 25 \neq 0 \vee \mu \leq 5)) \vee (-3\mu^4 \\
& + 28\mu^3 - 92\mu^2 + 125\mu \leq 60 \wedge 3\mu^4 - 28\mu^3 + 92\mu^2 - 125\mu + 60 \neq 0) \\
& \vee (-3\mu^3 + 14\mu^2 - 21\mu \leq -10 \wedge (3\mu^3 - 14\mu^2 + 21\mu - 10 \neq 0 \vee (
\end{aligned}$$

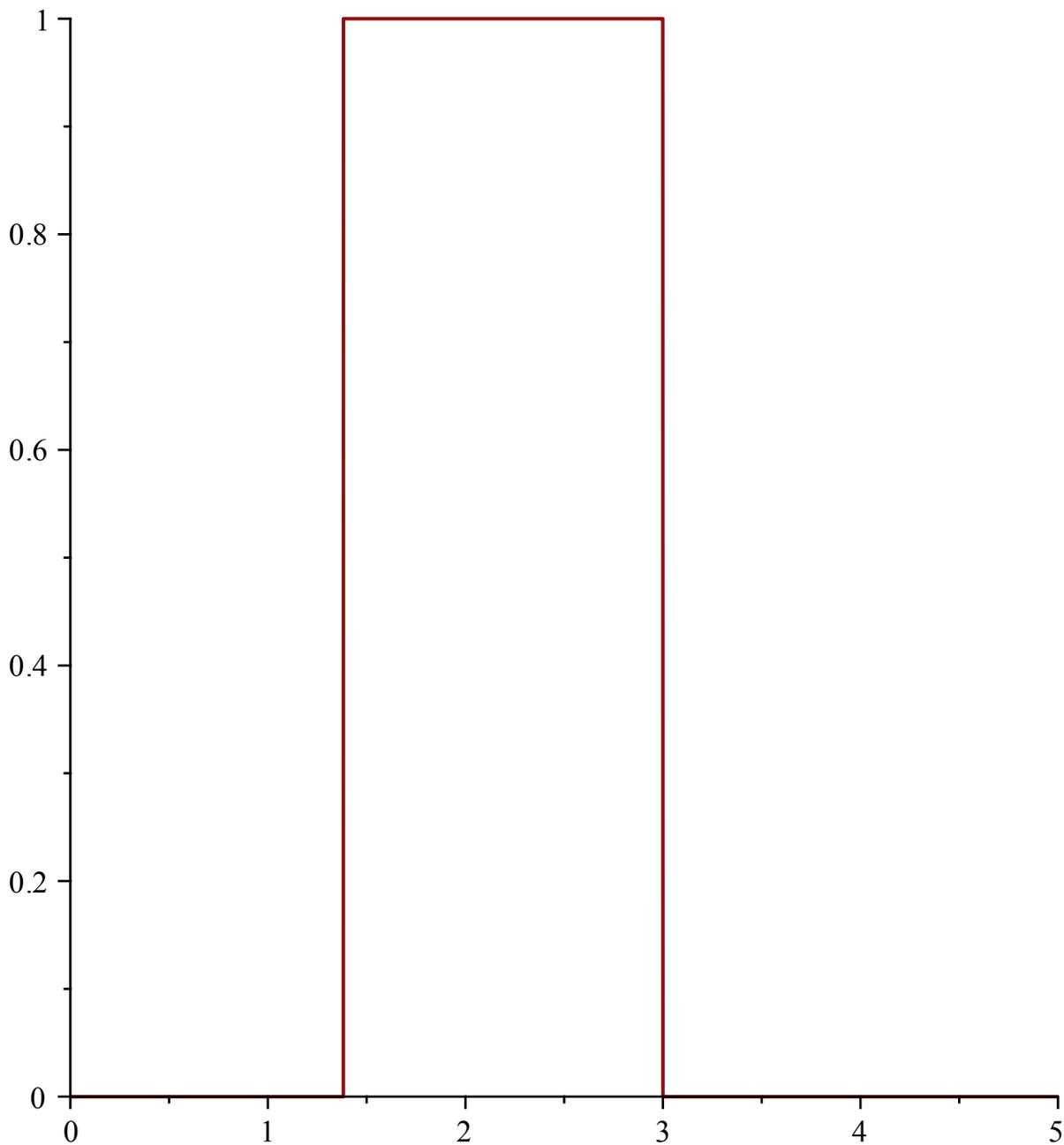
$$\begin{aligned}
& -3\mu^6 + 34\mu^5 - 142\mu^4 + 266\mu^3 - 210\mu^2 + 30\mu \leq -25 \wedge (3\mu^6 - 34\mu^5 \\
& + 142\mu^4 - 266\mu^3 + 210\mu^2 - 30\mu - 25 \neq 0 \vee -\mu^4 + 10\mu^3 - 30\mu^2 + 26\mu \\
& \leq 5)) \wedge (\mu^4 - 10\mu^3 + 30\mu^2 - 26\mu + 5 \neq 0 \vee 6\mu^4 - 56\mu^3 + 157\mu^2 \\
& - 130\mu + 23 \neq 0 \vee 9\mu^4 - 78\mu^3 + 205\mu^2 - 162\mu + 26 \neq 0 \vee (-3\mu \\
& \leq -5 \wedge (-5 + 3\mu \neq 0 \vee -\mu^3 + 10\mu^2 - 30\mu \leq -25))) \wedge (\mu^3 - 10\mu^2 \\
& + 30\mu - 25 \neq 0 \vee 6\mu^3 - 56\mu^2 + 157\mu - 125 \neq 0 \vee 9\mu^3 - 78\mu^2 + 205\mu \\
& - 156 \neq 0 \vee \mu - 5 \neq 0 \vee 5\mu - 23 \neq 0 \vee 3\mu - 13 \neq 0)) \vee (-6\mu^2 \\
& + 19\mu \leq 15 \wedge (6\mu^2 - 19\mu + 15 \neq 0 \vee (-\mu^4 + 13\mu^3 - 55\mu^2 + 90\mu \\
& \leq 50 \wedge (\mu^4 - 13\mu^3 + 55\mu^2 - 90\mu + 50 \neq 0 \vee \mu < 5)))) \wedge (3\mu^2 \\
& - 13\mu + 12 = 0 \vee 3\mu \leq 13 \vee (-9\mu^5 + 78\mu^4 - 241\mu^3 + 318\mu^2 - 170\mu \\
& \leq -24 \wedge 9\mu^5 - 78\mu^4 + 241\mu^3 - 318\mu^2 + 170\mu - 24 \neq 0) \vee (-9\mu^3 \\
& + 78\mu^2 - 205\mu \leq -156 \wedge (9\mu^3 - 78\mu^2 + 205\mu - 156 \neq 0 \vee 3\mu - 13 \\
& \neq 0)) \vee 3\mu < 13) \wedge (\mu^2 - 4\mu + 1 = 0 \vee \mu^3 - 6\mu^2 + 9\mu - 2 = 0 \vee \mu^3 \\
& - 5\mu^2 + 5\mu \leq 1 \vee -3\mu^4 + 22\mu^3 - 45\mu^2 + 18\mu < 2 \vee (\mu^2 - 3\mu \leq -2 \\
& \wedge (\mu^2 - 3\mu + 2 \neq 0 \vee \mu^3 - 3\mu^2 + 2\mu \neq 0)) \vee -2\mu^2 + 3\mu < 1) \\
& \wedge ((\mu^2 - 5\mu + 5 = 0 \wedge 3\mu^2 - 13\mu + 12 = 0) \vee (3\mu \leq 13 \wedge (3\mu - 13 \\
& \neq 0 \vee \mu \leq 5)) \vee (-9\mu^5 + 78\mu^4 - 241\mu^3 + 318\mu^2 - 170\mu \leq -24 \\
& \wedge (9\mu^5 - 78\mu^4 + 241\mu^3 - 318\mu^2 + 170\mu - 24 \neq 0 \vee (-6\mu^5 + 56\mu^4 \\
& - 184\mu^3 + 255\mu^2 - 143\mu \leq -22 \wedge (6\mu^5 - 56\mu^4 + 184\mu^3 - 255\mu^2 \\
& + 143\mu - 22 \neq 0 \vee -\mu^5 + 10\mu^4 - 35\mu^3 + 51\mu^2 - 30\mu \leq -5))) \wedge (\mu^5 \\
& - 10\mu^4 + 35\mu^3 - 51\mu^2 + 30\mu - 5 \neq 0 \vee 6\mu^5 - 56\mu^4 + 184\mu^3 - 255\mu^2 \\
& + 143\mu - 22 \neq 0 \vee 9\mu^5 - 78\mu^4 + 241\mu^3 - 318\mu^2 + 170\mu - 24 \neq 0)) \\
& \vee (-9\mu^3 + 78\mu^2 - 205\mu \leq -156 \wedge (9\mu^3 - 78\mu^2 + 205\mu - 156 \neq 0 \\
& \vee (-6\mu^3 + 56\mu^2 - 157\mu \leq -125 \wedge (6\mu^3 - 56\mu^2 + 157\mu - 125 \neq 0 \vee \\
& -\mu^3 + 10\mu^2 - 30\mu \leq -25))) \wedge (\mu^3 - 10\mu^2 + 30\mu - 25 \neq 0 \vee 6\mu^3 \\
& - 56\mu^2 + 157\mu - 125 \neq 0 \vee 9\mu^3 - 78\mu^2 + 205\mu - 156 \neq 0 \vee \mu - 5 \neq 0 \\
& \vee 5\mu - 23 \neq 0 \vee 3\mu - 13 \neq 0)) \vee (3\mu \leq 13 \wedge (3\mu - 13 \neq 0 \vee (3\mu \\
& \leq 14 \wedge (3\mu - 14 \neq 0 \vee \mu < 5)))) \wedge (\mu^2 - 4\mu + 1 = 0 \vee (\mu^2 - 5\mu \\
& + 5 = 0 \wedge 3\mu^2 - 13\mu + 12 = 0) \vee (\mu^3 - 5\mu^2 + 5\mu \leq 1 \wedge (\mu^3 - 5\mu^2 \\
& + 5\mu - 1 \neq 0 \vee \mu \leq 5)) \vee (-3\mu^4 + 22\mu^3 - 45\mu^2 + 18\mu \leq 2 \wedge 3\mu^4 \\
& - 22\mu^3 + 45\mu^2 - 18\mu + 2 \neq 0) \vee (\mu^2 - 3\mu \leq -2 \wedge (\mu^2 - 3\mu + 2 \neq 0 \\
& \vee -\mu^5 + 10\mu^4 - 35\mu^3 + 51\mu^2 - 30\mu \leq -5) \wedge (\mu^5 - 10\mu^4 + 35\mu^3 \\
& - 51\mu^2 + 30\mu - 5 \neq 0 \vee 6\mu^5 - 56\mu^4 + 184\mu^3 - 255\mu^2 + 143\mu - 22
\end{aligned}$$

$$\begin{aligned}
& \neq 0 \vee 9\mu^5 - 78\mu^4 + 241\mu^3 - 318\mu^2 + 170\mu - 24 \neq 0)) \vee (\mu^2 - 3\mu \\
& \leq -2 \wedge (\mu^2 - 3\mu + 2 \neq 0 \vee (-3\mu^5 + 31\mu^4 - 116\mu^3 + 192\mu^2 - 137\mu \\
& \leq -25 \wedge (3\mu^5 - 31\mu^4 + 116\mu^3 - 192\mu^2 + 137\mu - 25 \neq 0 \vee -\mu^3 \\
& + 10\mu^2 - 30\mu \leq -25))) \wedge (\mu^3 - 10\mu^2 + 30\mu - 25 \neq 0 \vee 6\mu^3 - 56\mu^2 \\
& + 157\mu - 125 \neq 0 \vee 9\mu^3 - 78\mu^2 + 205\mu - 156 \neq 0 \vee \mu - 5 \neq 0 \vee 5\mu \\
& - 23 \neq 0 \vee 3\mu - 13 \neq 0)) \vee (-2\mu^2 + 3\mu \leq 1 \wedge (2\mu^2 - 3\mu + 1 \neq 0 \\
& \vee (-\mu^4 + 11\mu^3 - 33\mu^2 + 23\mu \leq 4 \wedge (\mu^4 - 11\mu^3 + 33\mu^2 - 23\mu + 4 \neq 0 \\
& \vee \mu < 5)))))) \wedge (\mu^2 - 5\mu + 5 = 0 \vee (\mu^2 - 5\mu + 5 = 0 \wedge 3\mu^2 - 13\mu \\
& + 12 = 0) \vee (3\mu^3 - 20\mu^2 + 40\mu \leq 25 \wedge (3\mu^3 - 20\mu^2 + 40\mu - 25 \neq 0 \\
& \vee \mu \leq 5)) \vee (-3\mu^4 + 28\mu^3 - 92\mu^2 + 125\mu \leq 60 \wedge 3\mu^4 - 28\mu^3 + 92\mu^2 \\
& - 125\mu + 60 \neq 0) \vee (-\mu^2 + 3\mu \leq 2 \wedge (\mu^2 - 3\mu + 2 \neq 0 \vee -\mu^5 \\
& + 10\mu^4 - 35\mu^3 + 51\mu^2 - 30\mu \leq -5) \wedge (\mu^5 - 10\mu^4 + 35\mu^3 - 51\mu^2 \\
& + 30\mu - 5 \neq 0 \vee 6\mu^5 - 56\mu^4 + 184\mu^3 - 255\mu^2 + 143\mu - 22 \neq 0 \vee 9\mu^5 \\
& - 78\mu^4 + 241\mu^3 - 318\mu^2 + 170\mu - 24 \neq 0)) \vee (-3\mu \leq -5 \wedge (-5 \\
& + 3\mu \neq 0 \vee -\mu^3 + 10\mu^2 - 30\mu \leq -25) \wedge (\mu^3 - 10\mu^2 + 30\mu - 25 \neq 0 \\
& \vee 6\mu^3 - 56\mu^2 + 157\mu - 125 \neq 0 \vee 9\mu^3 - 78\mu^2 + 205\mu - 156 \neq 0 \vee \mu \\
& - 5 \neq 0 \vee 5\mu - 23 \neq 0 \vee 3\mu - 13 \neq 0)) \vee (-6\mu^2 + 19\mu \leq 15 \wedge (6\mu^2 \\
& - 19\mu + 15 \neq 0 \vee (-\mu^4 + 13\mu^3 - 55\mu^2 + 90\mu \leq 50 \wedge (\mu^4 - 13\mu^3 \\
& + 55\mu^2 - 90\mu + 50 \neq 0 \vee \mu < 5))))))
\end{aligned}$$

**> elim2:=eval(elim,[And=`and`,Or=`or`]):**

**> pp:=proc(mm) if eval(elim2,mu=mm) then 1 else 0 fi end:**

**> plot(pp,0..5);**



This shows that any value of  $\mu$  in the interval between roughly 1.4 and 3 would work (the actual values are algebraic numbers that can be extracted from the formula above).

## Recurrence in action

The aim is to show that the sequence defined by

$$\begin{aligned}
 &> \text{rec} := (16*n+1)*u(n+3) - (32*n-2)*u(n+2) + (20*n-4)*u(n+1) - (5*n-3)*u(n) = 0; \\
 \text{rec} &:= (16n + 1)u(n + 3) - (32n - 2)u(n + 2) + (20n - 4)u(n + 1) - (5n - 3)u(n) = 0 \quad (4.1)
 \end{aligned}$$

and

$$\begin{aligned}
 &> \text{ini} := \{u(0)=5, u(1)=5, u(2)=1\}; \\
 \text{ini} &:= \{u(0) = 5, u(1) = 5, u(2) = 1\} \quad (4.2)
 \end{aligned}$$

is  $>0$  for all  $n$ .

**> with(LinearAlgebra):**

## Check the first values

> **N:=20:**

> **L:=gfun:-rectoproc({rec,op(ini)},u(n),list)(N);**

$$L := \left[ 5, 5, 1, 3, \frac{84}{17}, \frac{3491}{561}, \frac{193118}{27489}, \frac{389756}{51051}, \frac{79660582}{9648639}, \frac{765516302}{85083453}, \right. \\ \left. \frac{1047161107676}{105758732079}, \frac{116430530626}{10600525593}, \frac{734444332400872}{59945972228415}, \right. \\ \left. \frac{1456409614649458996}{106164316816522965}, \frac{41346669999908131424}{2684440582360652115}, \right. \\ \left. \frac{62846624664665278410632}{3626679226769241007365}, \frac{14803158060137689023488776}{757975958394771370539285}, \right. \\ \left. \frac{250610819473108627336486352}{11369639375921570558089275}, \frac{68251690040442831245398165856}{2740083089597098504499515275}, \right. \\ \left. \frac{19843[\dots 22 \text{ digits} \dots]07128}{704201354026454315656375425675}, \frac{18588[\dots 23 \text{ digits} \dots]31504}{58256[\dots 21 \text{ digits} \dots]39675} \right] \quad (4.1.1)$$

> **evalf(%);**

[5., 5., 1., 3., 4.941176471, 6.222816399, 7.025282840, 7.634639870, 8.256147007, 8.997240650, 9.901415109, 10.98346772, 12.25177114, 13.71844758, 15.40234128, 17.32897252, 19.52985170, 22.04210804, 24.90862058, 28.17855618, 31.90818239] (4.1.2)

looks good.

## Animation

```
norm2:=proc(L) local j; sqrt(add(j^2,j=L)) end;
direction:=proc(L) local j,n:=norm2(L); [seq(j/n,j=L)] end;
makecone:=proc(param,t,length:=.65,npt:=integer:=50,V:=list(numeric):=[1,1,1])
local base,sc,i;
base:=evalc(param);
sc:=add(V[i]*base[i],i=1..3);
base:=[seq(length/sc*base[i],i=1..3)];
plots[polygonplot3d]([[0$3],seq(eval(base,t=2*Pi*i/npt),i=0..npt)],style=surface)
end;
```

The intersection of a sphere with  $R^3_{>0}$ :

> **sph:=plot3d(.6,0..Pi/2,0..Pi/2,coords=spherical,color=aquamarine,style=patchnograd):**

The directions of the vectors constructed from successive triples of elements of the sequences

> **WW:=[sph,seq(plottools:-line([0\$3],direction(L[i..i+2]),color=red,thickness=3),i=1..N-2)]:**

> **WW2:=[seq(plots[display]([seq(WW[j],j=1..i)]),i=1..nops(WW))]:**

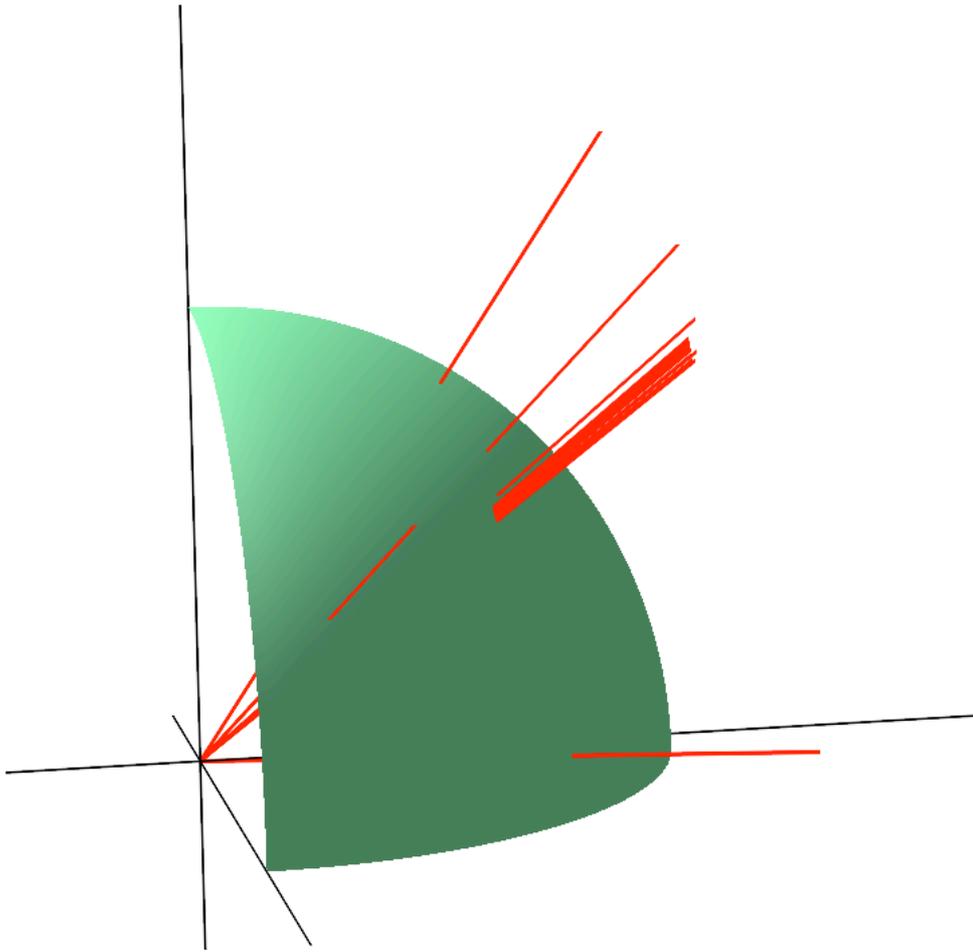
> **opts:=axes=normal,view=[(-0.25..1)\$3],tickmarks=[0\$3],orientation=[-14,76,-6]:**

Animation:

> **#plots[display](WW2,insequence=true,opts);**

Final picture:

> **plots[display](WW2[-1],opts);**



**End of the proof of positivity tomorrow**