Dynamic Dictionary of Mathematical Functions

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Project started in September 2007

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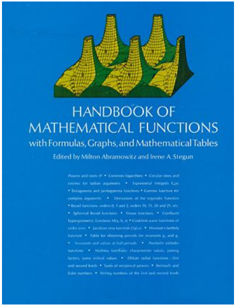
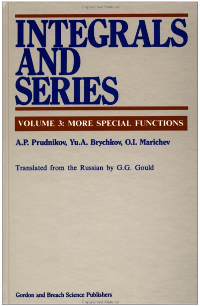
Alexandre Benoit & Marc Mezzarobba (PhD students)

I Introduction

Context

1. First, there were mathematical handbooks.
   Among the most cited documents in the scientific literature.





Thousands of useful mathematical formulas,
computed, compiled and edited by hand.

Context

2. Then, came computer algebra.
   Computation with exact mathematical objects.
   Several million users.

30 years of algorithmic progress in effective mathematics.

Context

3. Last, came the Web.

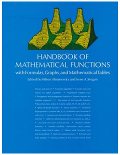
New kinds of interaction with documents.

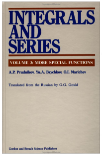
Dynamic Dictionary of Mathematical Functions

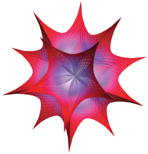
Aim of the project

DDMF = Mathematical Handbooks + Computer Algebra + Web

1. Develop and use computer algebra algorithms to generate the formulas;
2. Provide web-like interaction with the document and the computation.



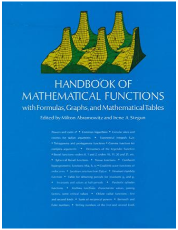




II Computer Algebra Algorithms

Equations as a Data-Structure

- Classical:
  polynomials represent their roots better than radicals.
  Algorithms: Euclidean division and algorithm, Gröbner bases.
- Recent [SaZi94,ChSa98,Chyzak00,. . .]:
  same for linear differential equations.
  Algorithms: non-commutative analogues.



60% of Abramowitz & Stegun.

First step [MeSa03]: ESF http://algo.inria.fr/esf

Recent Progress

- Automatic computation of bounds, fast evaluation at high precision (M. Mezzarobba);
- Chebyshev expansions (A. Benoit);
- Faster algorithms for various operations (A. Bostan);
- More formulas accessible to computer algebra (F. Chyzak, M. Kauers, BS).

New (since last Sunday!)

$$\int_0^\infty x^{k-1}\zeta(n,\alpha+\beta x)\,dx = \beta^{-k}B(k,n-k)\zeta(n-k,\alpha),$$

$$\int_0^\infty x^{\alpha-1}\,\mathrm{Li}_n(-xy)\,dx = \frac{\pi(-\alpha)^n y^{-\alpha}}{\sin(\alpha\pi)},$$

$$\int_0^\infty x^{k-1}\exp(xy)\Gamma(n,xy)\,dx = \frac{\pi y^{-k}}{\sin((n+k)\pi)}\frac{\Gamma(k)}{\Gamma(1-n)},$$

$$\int_0^\infty \frac{x}{(x^2+y^2)\sin(xz)}\,dx = \frac{\pi}{2\sinh(yz)},$$

not accessible to computer algebra before.

- How can we compute them?
- Why do they exist?

New (since last Sunday!)

$$\int_0^\infty x^{k-1}\zeta(n,\alpha+\beta x)\,dx = \beta^{-k}B(k,n-k)\zeta(n-k,\alpha),$$
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Algorithm:

1. Creative telescoping;
2. Confinement in finite-dim vector spaces.

III DynaMoW: Dynamic Mathematics on the Web. Version 0.7: Oct 2008

Principles

1. Dynamic generation of documents by computer algebra
   $\longrightarrow$ interactivity and incremental computations
2. Zoom on the symbolic computation as part of the output
   $\longrightarrow$ Automatic writing of proofs / certification of results
3. Extraction of computer-algebra code

DDMF = ESF + DynaMoW

Language

Syntax	Meaning
[s: extended HTML :s] { *instruction sequence* }	Sections (can be nested)
[a: extended HTML :a] { *function call* }	Anchors
[t: extended HTML :t]	Inline text
[d: extended HTML :d]	Displayed formulas
<* extended symbolic code *>	Symbolic computations

+ common imperative features (if, for,...)

Example

```
unit singularExpansions ( string s, symbolic def ; ) {
  [s: Local expansions at singularities :s] {
    symbolic eqn := <* op(remove(type, $(def), equation)): *> ;
    integer order := <* nops(indets($(eqn), function)) - 1: *> ;
    <* coeff($(eqn), diff(y(x), x $ $(order))): *> ;
    symbolic sing := <* sort(convert( {solve(%)} minus {0}, list)): *> ;
    integer nb_sing := <* nops($(sing)): *>  ;
    [t: The differential equation above has $(nb_sing) non-zero
        finite singular point(s). :t]
    [t: <ul> :t]
    for integer i in [1..nb_sing] do
      asymptExpansion(s, def, <* $(sing)[$(i)]: *>) ;
    done ;
    asymptExpansion(s, def, <* infinity: *>) ;
    [t: </ul> :t]
  }
}
```

Demo DDMF

http://ddmf.msr-inria.inria.fr/

IV Conclusion

Soon to Come

- Complete port of ESF to DynaMoW & Automatic proofs (S. Gerhold);
- More bounds & numerics (M. Mezzarobba);
- More on orthogonal polynomials (A. Benoit);
- Integral transforms (B. Salvy, L. Pech);
- Better and better DynaMoW (F. Chyzak);
- New algorithms in Computer Algebra (ALL).