Drowsiness detection from polysomnographic data using multivariate self-similarity and eigen-wavelet analysis

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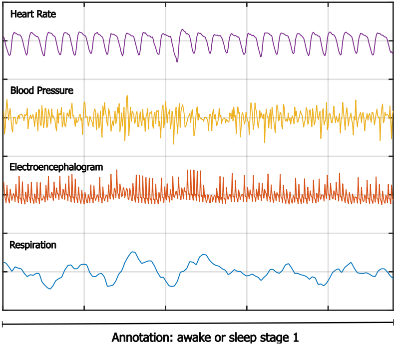
Drowsiness detection

MIT-BIH Polysomnographic database



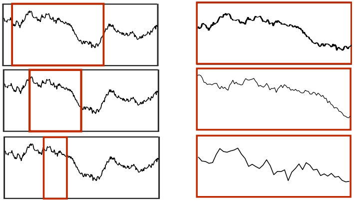
Description:

- from 16 male subjects
- resampling at 4Hz
- 2-minute long window of 480 samples:
  - 753 windows for "awake"
  - 561 windows for "sleep stage 1"

Goal: detection of "awake" vs. "sleep stage1"

Drowsiness and self-similarity

Univariate self-similarity

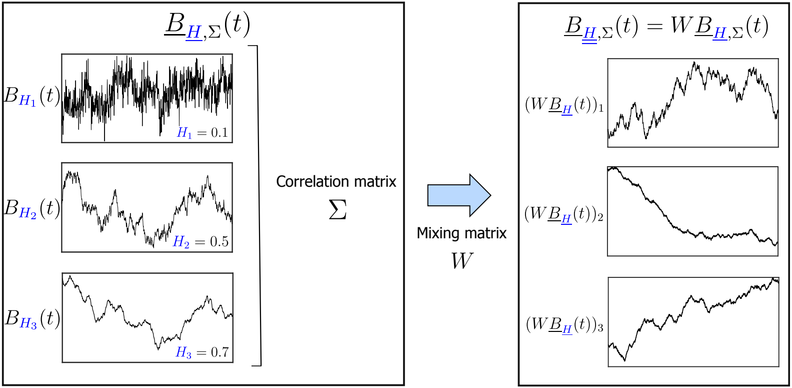


Characterized by $0 < H < 1$

$\Rightarrow$ Multivariate self-similarity analysis: $\underline{H} = (H_1, \ldots, H_M)$

Goal: detection of changes in $\underline{H}$

Multivariate self-similarity model [Didier et al., 2011]

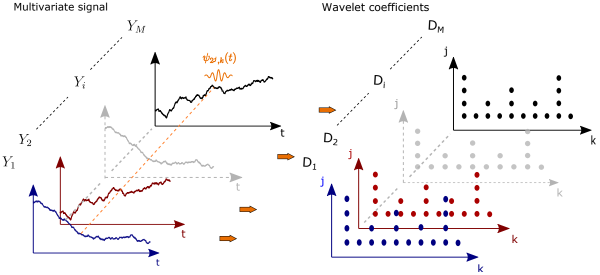


$\underline{B}_{\underline{\underline{H}},\Sigma}(t)$ characterized by the matrix $\underline{\underline{H}} = W\text{diag}(\underline{H})W^{-1}$

Issue: estimation of $\underline{\underline{H}}$

Wavelet spectrum

- Multivariate wavelet transform of $Y = W\underline{B}_{\underline{H},\Sigma}$:
  - ψ_0: mother wavelet
  - $D_m(2^j, k) = \langle 2^{-j/2}\psi_0(2^{-j}t - k) | Y_m(t)\rangle$
  - $D(2^j, k) = (D_1(2^j, k), \ldots, D_M(2^j, k))$



- Wavelet spectrum ($M \times M$ matrix):

$$S_{m_1,m_2}(2^j) = \frac{1}{N_j}\sum_{k=1}^{N_j} D_{m_1}(2^j, k) D_{m_2}(2^j, k)^*, \; N_j = \frac{N}{2^j}, \; N\text{: sample size}$$

Self-similarity parameter estimation

Wavelet spectrum:

$$S(2^j) = \begin{bmatrix} S_{1,1}(2^j) & S_{1,2}(2^j) & \cdots & \cdots & S_{1,M}(2^j) \\ S_{2,1}(2^j) & S_{2,2}(2^j) & \cdots & \cdots & S_{2,M}(2^j) \\ S_{3,1}(2^j) & S_{3,2}(2^j) & \ddots & & \vdots \\ \vdots & \vdots & & \ddots & S_{M-1,M}(2^j) \\ S_{M,1}(2^j) & S_{M,2}(2^j) & \cdots & S_{M,M-1}(2^j) & S_{M,M}(2^j) \end{bmatrix}$$

Eigenvalues of $S(2^j)$:

$$S(2^j) = U(2^j) \begin{bmatrix} \lambda_1(2^j) & 0 & \cdots & \cdots & 0 \\ 0 & \lambda_2(2^j) & \cdots & \cdots & 0 \\ 0 & 0 & \ddots & & \vdots \\ \vdots & \vdots & & \ddots & 0 \\ 0 & 0 & \cdots & 0 & \lambda_M(2^j) \end{bmatrix} U(2^j)^T$$

Eigen-wavelet estimation [Didier and Abry, 2018]

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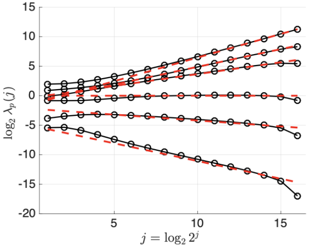
- $Y = W\underline{B}_{\underline{H},\Sigma}$ self-similar
  $\Rightarrow$ asymptotical power law:

$$\lambda_m(2^j) = \xi_m 2^{j(2H_m+1)}$$

- Linear regression on log-eigenvalues:

$$\hat{H}_m^M = \frac{1}{2} \sum_{j=j_1}^{j_2} \omega_j \log_2 \lambda_m(2^j) - \frac{1}{2}$$



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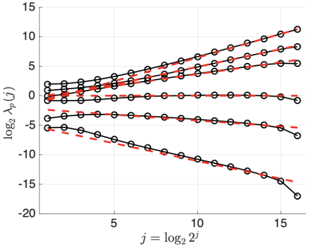


Issue:

Different numbers of wavelet coefficients to compute $S(2^j)$ between scales 2^j

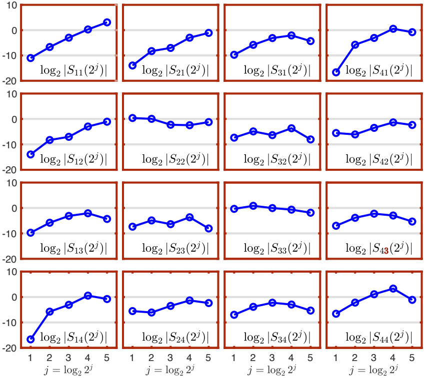
$\Rightarrow$ $\lambda_m(2^j)$ have different bias across scale 2^j

$\Rightarrow$ bias corrected estimation [Lucas et al., EUSIPCO 2021]

4-variate data

Wavelet analysis scales: $2^{j_1} = 2^1$ to $2^{j_2} = 2^4 \Rightarrow$ Analysis frequencies: 1/8 to 2 Hz

Log-wavelet spectrum $\log_2 |S_{m,m'}(2^j)|$



- Classical multivariate self-similarity:

$$\hat{H}_{m,m'} = \frac{1}{2} \sum_{j=j_1}^{j_2} \omega_j \log_2 S_{m,m'}(2^j) - \frac{1}{2}$$

Cross-temporal dynamics $\Rightarrow$ need for a multivariate approach

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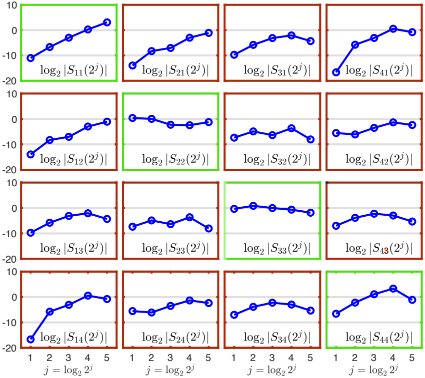

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$$\hat{H}_{m,m'} = \frac{1}{2} \sum_{j=j_1}^{j_2} \omega_j \log_2 S_{m,m'}(2^j) - \frac{1}{2}$$

- Univariate self-similarity:

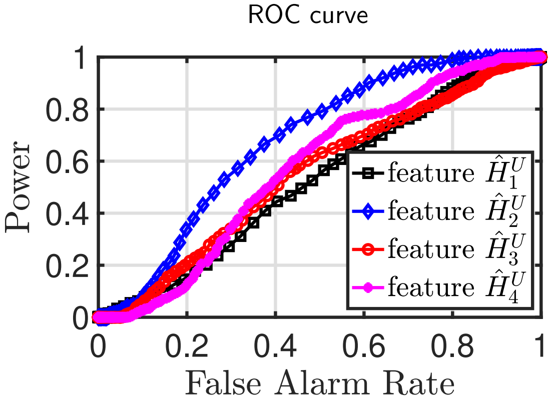
$$\hat{H}^U_m = \frac{1}{2} \sum_{j=j_1}^{j_2} \omega_j \log_2 S_{m,m}(2^j) - \frac{1}{2}$$

$$\Rightarrow \hat{H}^U_m = \hat{H}_{m,m}$$

Cross-temporal dynamics $\Rightarrow$ need for a multivariate approach

Single-feature classification

Comparing $\hat{H}_m^U$ to a threshold:



Low performance ⇒ need for a multi-feature approach

Multi-feature classification

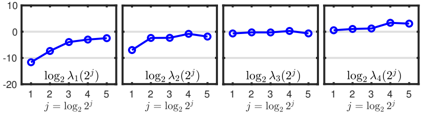
Features:

- *features1* (4): $\{\hat{H}_m^U\}_{m=1,\dots,4}$
- *features2* (8): $\{\{\hat{H}_m^U\}_{m=1,\dots,4}, \{\hat{H}_{m,m'}\}_{m\neq m'}\}$
- *features3* (10): $\{\{\hat{H}_m^U\}_{m=1,\dots,4}, \{\hat{H}_m^M\}_{m=1,\dots,4}\}$

where:

- $\hat{H}_{m,m'} = \frac{1}{2}\sum_{j=j_1}^{j_2} \omega_j \log_2 S_{m,m'}(2^j) - \frac{1}{2}$
- $\hat{H}_m^U = \hat{H}_{m,m}$
- $\hat{H}_m^M = \frac{1}{2}\sum_{j=j_1}^{j_2} \omega_j \log_2 \lambda_m(2^j) - \frac{1}{2}$

Log-eigenvalues $\log_2 \lambda_m(2^j)$



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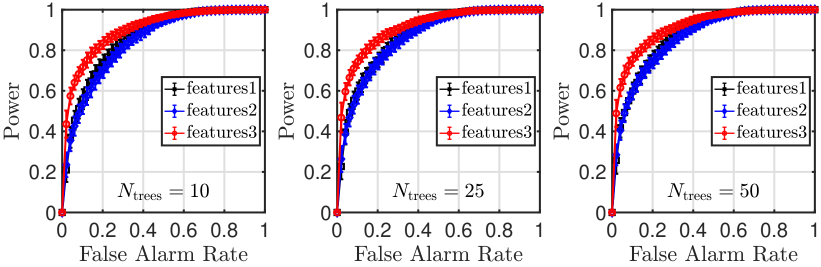
Random Forest Classifiers: $N_{\text{trees}} \in \{10, 25, 50\}$



Conclusion

Self-similarity tools:

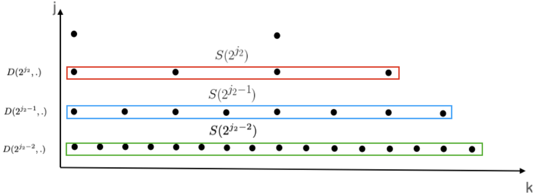
- multivariate self-similarity model
- eigen-wavelet approach
- multivariate self-similarity parameter estimation

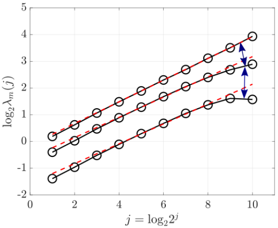
Drowsiness detection:

- cross-temporal dynamics of physiological data
- sleep classification from multivariate parameter estimation

Repulsion effect

Gap between eigenvalues larger than expected at each scale





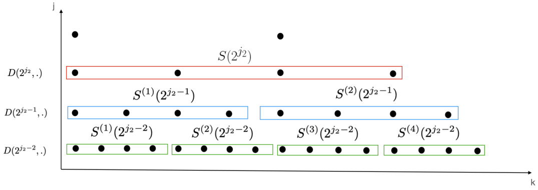
Few coefficients $\Rightarrow$ repulsion effect: important bias when $H_1 = \ldots = H_M$

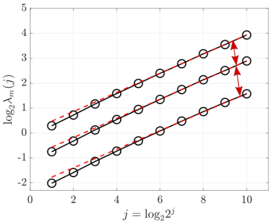
Issue: repulsion effect increases with scale 2^j

Bias corrected estimation

$$S^{(w)}(2^j) \triangleq \frac{1}{n_{j_2}} \sum_{k=1+(w-1)n_{j_2}}^{wn_{j_2}} D(2^j, k)D(2^j, k)^*, \quad w = 1, \ldots, 2^{j-j_2}, \quad n_{j_2} = \frac{N}{2^{j_2}}$$

Wavelet spectra for same numbers of wavelet coefficients





Eigenvalues of $S^{(w)}(2^j)$: $\{\lambda_1^{(w)}(2^j), \ldots, \lambda_M^{(w)}(2^j)\}$

$\rightarrow$ similar repulsion at all scales $j \in \{j_1, \ldots, j_2\}$

Averaged log-eigenvalues: $\vartheta_m(2^j) \triangleq 2^{j_2-j} \sum_{w=1}^{2^{j-j_2}} \log_2(\lambda_m^{(w)}(2^j))$

Linear regression on averaged log-eigenvalues $\vartheta_m(2^j)$