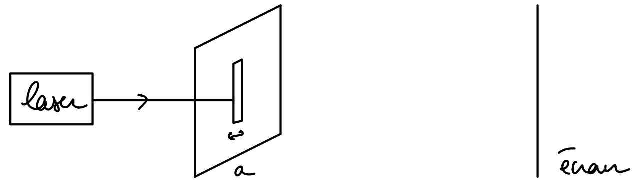
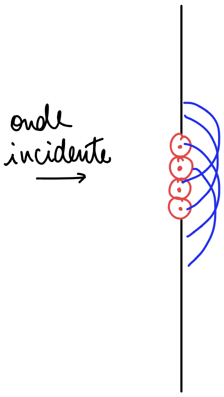
Intro

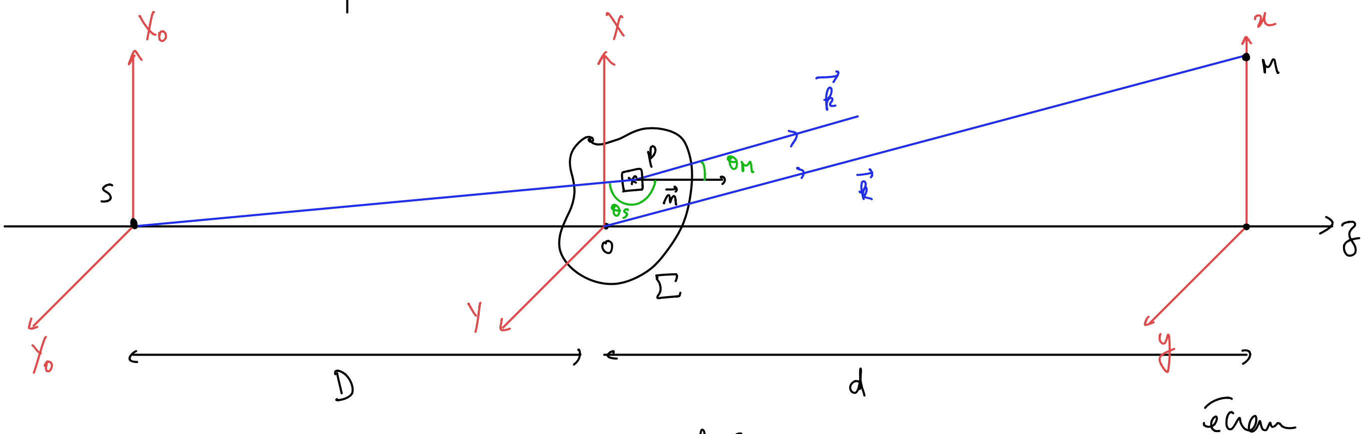


diffraction : $\boxed{\lambda \gtrsim a}$

I) 1.



$$t(X,Y) = \frac{A(X,Y,z=0^+)}{A(X,Y,z=0^-)}$$



$$s(M) = \int_\Sigma d^2P \,.\, K(\theta_S,\theta_M)\,.\, \underbrace{s_0 \frac{e^{ikSP}}{SP}}_{s_0(P)} \,.\, \underbrace{t(P)}_{\text{objet}} \,.\, \underbrace{\frac{e^{ikMP}}{MP}}_{\text{H-F}}$$

2. Cadre : source lointaine, écran lointain $\}$ $D, d \gg x, y, X, Y$

(i) rayons peu inclinés $\rightarrow$ $K(\theta_S,\theta_M) = K$ constante

(ii) $MP = \sqrt{(x-X)^2+(y-Y)^2+d^2} \simeq d\left(1+\frac{1}{2}\frac{(x-X)^2}{d^2}+\frac{1}{2}\frac{(y-Y)^2}{d^2}+O\left(\frac{(x-X)^4}{d^4},\frac{(y-Y)^4}{d^4}\right)\right)$

$\rightarrow$ décroissance $\frac{1}{MP} \simeq \frac{1}{d}$

$\rightarrow$ dans l'exponentielle, il faut comparer les variations de MP à la longueur d'onde

(iii) idem avec SP

$x \leftrightarrow X_0 = 0$
$y \leftrightarrow Y_0 = 0$
$d \leftrightarrow D$

$$s(x,y) = \underbrace{\frac{K s_0}{d.D} e^{ikd} e^{ikD}}_{C} \cdot \underbrace{e^{ik\left(\frac{x^2+y^2}{2d}\right)}}_{\substack{\text{ignoré car} \\ \text{on observe } |s|^2}} \iint_\Sigma dX dY \cdot t(X,Y) \cdot e^{-ik\left(\frac{xX+yY}{d}\right) + \underbrace{ik\frac{(X^2+Y^2)}{2}\left(\frac{1}{d}+\frac{1}{D}\right)}_{(\mathrm{Q})}}$$

Diffraction de Fraunhofer : le terme quadratique (Q) de phase peut être négligé.

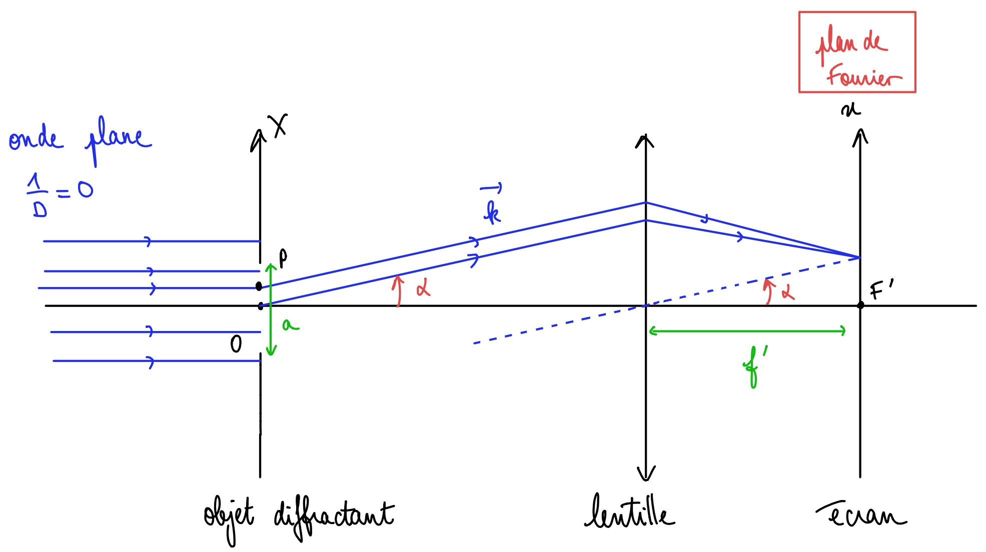
Dans ce cas,
$$\boxed{\begin{aligned} s(x,y) &= C. \iint_\Sigma dX dY\ t(X,Y) \cdot e^{-ik(\theta_x . X + \theta_y Y)} \\ &= C.\ TF[t](q_x, q_y) \end{aligned}}$$

$\theta_x = \frac{x}{d}$, $\theta_y = \frac{y}{d}$

angles sous lesquels P voit l'objet diffractant

$q_x = k\theta_x \quad q_y = k\theta_y \qquad$ fréquences spatiales

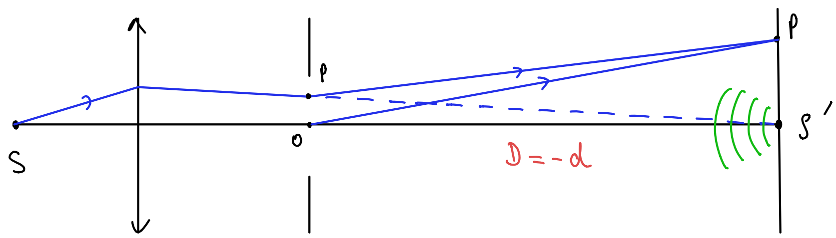
3. (i) onde plane $\left(\frac{1}{D}=0\right)$ | à l'infini $\left(\frac{1}{d}=0\right)$
| au foyer d'une lentille.



(ii) onde plane $\left(\frac{1}{D}=0\right)$ en régime de Fraunhofer $\boxed{N_F = \frac{a^2}{\lambda d} \ll 1}$

(iii) au voisinage de l'image géométrique

$$\frac{1}{D} + \frac{1}{d} = 0$$



translation - modulation $\quad t(x+x_0) \xrightarrow{TF} e^{ifx_0}\, TF[t](f)$

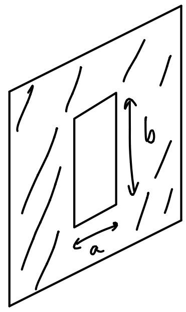
similitude $\quad t(ax) \xrightarrow{TF} \frac{1}{|a|} TF[t]\left(\frac{f}{a}\right)$

linéarité $\Rightarrow$ $\quad 1-t \xrightarrow{TF} \delta(f) - TF[t](f)$

produit - convolution $\quad f*g \xrightarrow{TF} TF[f] \times TF[g]$

itération $\quad f(x) \xrightarrow{TF} \xrightarrow{TF} TF[TF[f]](x) = f(-x)$

II. 1.



Éclairage par une onde plane

$$A(x,y) = C \int_{-a/2}^{a/2} dX \int_{-b/2}^{b/2} dY \; e^{-iq_x X} e^{-iq_y Y}$$

$$= C \left[\frac{e^{-iq_x X}}{-iq_x} \right]_{-a/2}^{a/2} \left[\frac{e^{-iq_y Y}}{-iq_y} \right]_{-b/2}^{b/2}$$

$$= C \left(\frac{e^{-iq_x\frac{a}{2}} - e^{+iq_x\frac{a}{2}}}{-iq_x} \right) \left(\frac{e^{-iq_y\frac{b}{2}} - e^{iq_y\frac{b}{2}}}{-iq_y} \right)$$

$$= C \operatorname{sinc}\left(\frac{q_x a}{2}\right) \operatorname{sinc}\left(\frac{q_y b}{2}\right)$$

$$I(x,y) = I_0 \operatorname{sinc}^2\left(\frac{kxa}{2d}\right) \operatorname{sinc}^2\left(\frac{kyb}{2d}\right)$$

$I=0$ lorsque $\pi = \frac{kxa}{2d}$ ie $\boxed{\theta_x = \frac{2\pi}{ka} = \frac{\lambda}{a}}$

on retrouve la formule du lycée

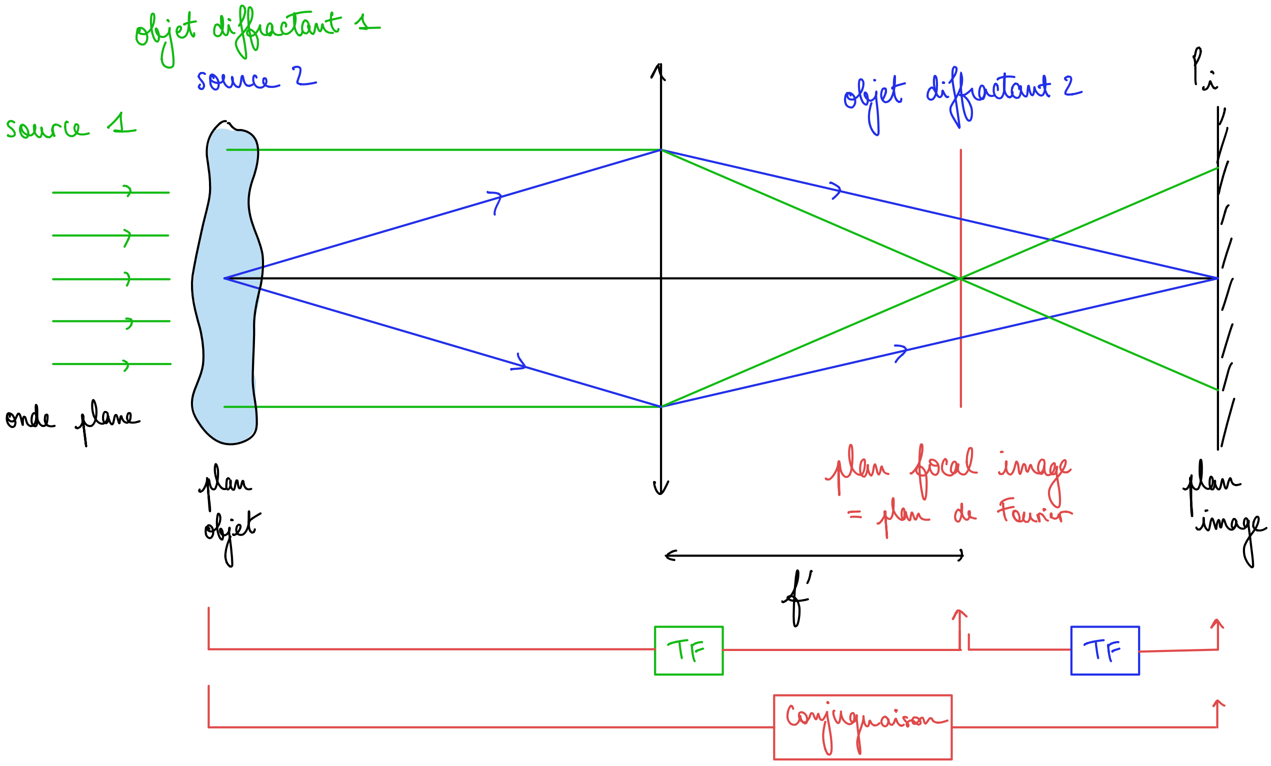
2. $s(x,y) = C \; TF(t)[q_x, q_y]$

modulé par la fig diffraction 1 fente

$$t_{bifente} = t_{fente} * \left(\delta_{-\frac{a}{2}} + \delta_{\frac{a}{2}}\right) \rightarrow \hat{t}_{bifente}(q_x) \propto \hat{t}_{fente}(q_x) \cdot \cos\left(\underbrace{\frac{a}{2} q_x}_{2\pi \cdot \frac{a}{2} \cdot \frac{x}{\lambda d}}\right)$$

interfrange, $i = \frac{\lambda d}{a}$
car $|\hat{t}|^2$ a la période double de t

IV) 1.



sans filtrage

$TF(TF(f(x)) = f(-x)$

image inversée