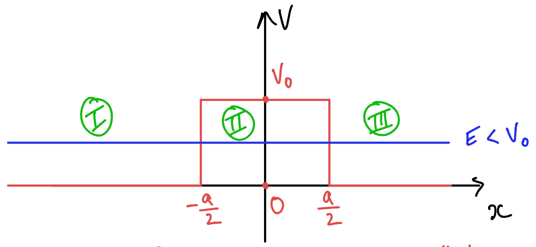
I.

$$i\hbar \frac{\partial \psi}{\partial t} = -\frac{\hbar^2}{2m}\frac{\partial^2 \psi}{\partial x^2} + V\psi \xrightarrow[V(x,\cancel{t})]{} E\psi = -\frac{\hbar^2}{2m}\psi(x)'' + V\psi$$

Barrière tunnel



I, III $\quad \psi'' + k^2\psi = 0$

II $\quad \psi'' - q^2\psi = 0$

$$k \equiv \sqrt{\frac{2mE}{\hbar^2}}$$

$$q \equiv \sqrt{\frac{2m(V_0-E)}{\hbar^2}}$$

Région

I $\quad \psi(x) = \underbrace{A_I e^{ikx}}_{\text{incidente}} + \underbrace{B_I e^{-ikx}}_{\text{réfléchie}}$

II $\quad \psi(x) = A_{II}\, ch(qx) + B_{II}\, sh(qx)$ mieux pour la symétrie

III $\quad \psi(x) = \underbrace{A_{III} e^{ikx}}_{\text{transmise}} + \cancel{A_{III} e^{-ikx}}$ pas d'onde $\leftarrow \infty$

Démarche : 4 équations $\begin{cases}\psi(\pm a/2)\\ \psi'(\pm a/2)\end{cases}$ $\quad q\times\psi(\pm\frac{a}{2}) \pm \psi'(\pm\frac{a}{2})$ m̂ signe $\quad$ élimine $A_{II} \pm B_{II}$ $\longrightarrow$ on trouve R et T

Conditions limites :

en $\psi(-\frac{a}{2})$ $\quad A_I e^{-\frac{ika}{2}} + B_I e^{+\frac{ika}{2}} = A_{II}\, ch\left(\frac{qa}{2}\right) - B_{II}\, sh\left(\frac{qa}{2}\right) \quad (1)$

$\psi'(-\frac{a}{2})$ $\quad A_I ik\, e^{-\frac{ika}{2}} + B_I(-ik)\, e^{\frac{ika}{2}} = -A_{II}\, q\, sh\left(\frac{qa}{2}\right) + qB_{II}\, ch\left(\frac{qa}{2}\right) \quad (2)$

$\psi(\frac{a}{2})$ $\quad A_{III} e^{\frac{ika}{2}} = A_{II}\, ch\left(\frac{qa}{2}\right) + B_{II}\, sh\left(\frac{qa}{2}\right) \quad (3)$

$\psi'(\frac{a}{2})$ $\quad A_{III}\, ik\, e^{\frac{ika}{2}} = A_{II}\, q\, sh\left(\frac{qa}{2}\right) + qB_{II}\, ch\left(\frac{qa}{2}\right) \quad (4)$

↳ 4 équations pour 5 inconnues → on exprime tout en fonction de A_I.

Pas de quantification de l'énergie car pas de confinement

Coefficients de réflexion / transmission

$$R = \frac{|B_I|^2}{|A_I|^2} = \frac{|\vec{j}_r|^2}{|\vec{j}_i|^2} \qquad T = \frac{|A_{III}|^2}{|A_I|^2} = \frac{|\vec{j}_t|^2}{|\vec{j}_i|^2}$$

- $q\times(1) + (2)$ $\quad A_I(q+ik)\, e^{-\frac{ika}{2}} + B_I(q-ik)\, e^{\frac{ika}{2}} = q(A_{II}+B_{II})\underbrace{\left(ch\left(\frac{qa}{2}\right) - sh\left(\frac{qa}{2}\right)\right)}_{e^{-\frac{qa}{2}}}$
- $q\times(3) + (4)$ $\quad A_{III}(q+ik)\, e^{\frac{ika}{2}} = q(A_{II}+B_{II})\underbrace{\left(ch\left(\frac{qa}{2}\right) + sh\left(\frac{qa}{2}\right)\right)}_{e^{\frac{qa}{2}}}$

↳ $A_I(q+ik)\, e^{-\frac{ika}{2}} + B_I(q-ik)\, e^{\frac{ika}{2}} = e^{-qa} \cdot A_{III}(q+ik)\, e^{\frac{ika}{2}}$

- $q\times(1) - (2)$ $\quad A_I(q-ik)\, e^{-\frac{ika}{2}} + B_I(q+ik)\, e^{\frac{ika}{2}} = q(A_{II}-B_{II})\underbrace{\left(ch\left(\frac{qa}{2}\right) + sh\left(\frac{qa}{2}\right)\right)}_{e^{+\frac{qa}{2}}}$
- $q\times(3) - (4)$ $\quad A_{III}(q-ik)\, e^{\frac{ika}{2}} = q(A_{II}-B_{II})\underbrace{\left(ch\left(\frac{qa}{2}\right) - sh\left(\frac{qa}{2}\right)\right)}_{e^{-\frac{qa}{2}}}$

↳ $A_I(q-ik)\, e^{-\frac{ika}{2}} + B_I(q+ik)\, e^{\frac{ika}{2}} = A_{III}\, e^{qa}(q-ik)\, e^{\frac{ika}{2}}$

$$A_I(q+ik)\,e^{-\frac{ika}{2}} + B_I(q-ik)\,e^{\frac{ika}{2}} = e^{-2qa}\,\frac{q+ik}{q-ik}\left(A_I(q-ik)\,e^{-\frac{ika}{2}} + B_I(q+ik)\,e^{\frac{ika}{2}}\right)$$

$$B_I\left((q-ik)\,e^{\frac{ika}{2}} - e^{-2qa}\,\frac{(q+ik)^2}{q-ik}\cdot e^{\frac{ika}{2}}\right) = A_I\left(-(q+ik)\,e^{-\frac{ika}{2}} + e^{-2qa}(q+ik)\,e^{-\frac{ika}{2}}\right)$$

$|q-ik| = |q+ik|$

$|a| = |\bar{a}|$

$$|B_I|^2\left(|q-ik|^2 + e^{-4qa}|q+ik|^2 - 2e^{-2qa}\,\underbrace{\mathrm{Re}\,\frac{(q+ik)^3}{(q-ik)}}\right) = |A_I|^2(q^2+k^2)\left(e^{-2qa}-1\right)^2$$

$$\mathrm{Re}\,\frac{(q+ik)^4}{q^2+k^2} = \frac{1}{q^2+k^2}\left(q^4 - 6q^2k^2 + k^4\right)$$

$$R = \frac{\left(e^{-2qa}-1\right)^2}{1 + e^{-4qa} - 2e^{-2qa}\underbrace{\frac{(q^4-6q^2k^2+k^4)}{(q^2+k^2)^2}}_{1-\frac{8q^2k^2}{(q^2+k^2)^2}}} = \frac{1}{1 - \frac{16e^{-2qa}q^2k^2}{(e^{-2qa}-1)^2(q^2+k^2)^2}}$$

$\frac{4m^2E(V_0-E)}{\hbar^4}$

$\left(\frac{2mV_0}{\hbar^2}\right)^2$

$$\frac{1}{(e^{-qa}-e^{qa})^2} = \frac{1}{4\,\mathrm{sh}^2(qa)}$$

$$\boxed{R = \frac{1}{1 + \frac{4(V_0-E)E}{V_0^2}\cdot\frac{1}{\mathrm{sh}(qa)}} \qquad T = \frac{1}{1 + \frac{V_0^2}{4E(V_0-E)}\cdot\mathrm{sh}^2(qa)}}$$

$$\frac{1}{1+k\,\underbrace{\mathrm{sh}^2(qa)}_{\sim\frac{e^{2qa}}{4}}} \underset{qa\gg1}{\simeq} \frac{1}{\cancel{1+}\,k\,\frac{e^{2qa}}{4}} \sim \frac{4}{k}\cdot e^{-2qa}$$

· limite des barrières épaisses : $\boxed{T \underset{qa\gg1}{\simeq} \frac{16\,E(V_0-E)}{V_0^2}\exp\left(-\frac{2a}{\delta}\right)}$

$\rightarrow$ $\delta = \frac{1}{q} = \frac{\hbar}{\sqrt{2m(V_0-E)}}$

3. <u>ODG</u>

e^- : $m = 9.1\cdot10^{-31}$ kg, $V_0 = 5$ eV, $E = 1$ eV $\rightarrow$ $\delta \sim 1$ Å — transmission appréciable si $a \ll \delta$

skieur : $m = 80$ kg, $V_0 = mgh_{colline} = 8.10^3$ J (10 m), $E = mgh_0 = 8.10^2$ J (élan 1 m) $\rightarrow$ $\delta \sim 10^{-37}$ m $\ll$ échelle macroscopique

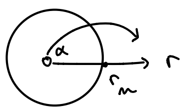
II 1. ex : $^{238}_{92}U \rightarrow {}^{234}_{90}Th + \underbrace{^{4}_{2}He}_{\alpha}$

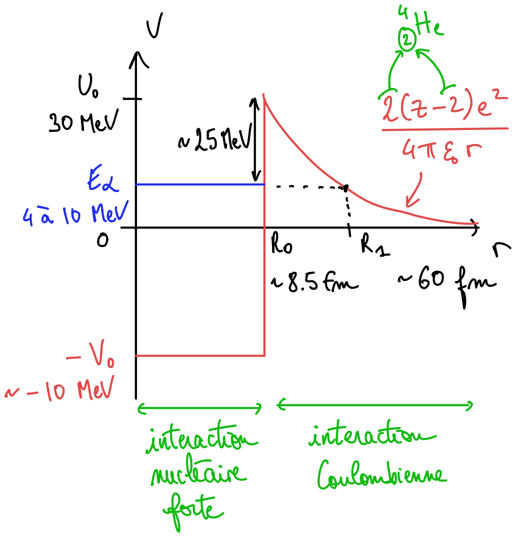
$$(A, Z) \rightarrow (A-4, Z-2) + \alpha$$

odg : temps de demi-vie $\left.\begin{matrix} 10^{15} \text{ ans } {}^{142}_{58}Ce \\ 10^{-7} \text{ s } {}^{212}_{84}Po \end{matrix}\right\}$ 30 odg

modèle de Gamow : effet tunnel

2.





$E_\alpha > 0$ effet tunnel possible

$E_\alpha < 0$ noyau stable à la désintégration α

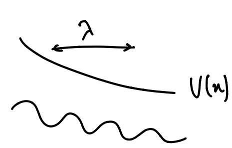
Approximation WKB :

on injecte dans l'éq. de Schrödinger stationnaire $-\frac{\hbar^2}{2m}\psi'' + (V-E)\psi = 0$

$\psi'' - q^2(x)\psi = 0$ $q^2(x) = \frac{(V(x)-E)2m}{\hbar^2}$

A(x) si $E > V_0$, mais ici on reste général $\phi \leftrightarrow \phi - \ln A(x)$

ansatz $\psi(x) = A\, e^{-\phi(x)}$ car $E < V_0$



$$\psi'' = A\left(-\phi'' e^{-\phi(x)} + \phi'^2 e^{-\phi(x)}\right)$$

↳ injecte $-\phi'' + \phi'^2 - q^2 = 0$

Qui domine : ϕ'' ou ϕ'^2 ?

hyp : si $|\phi''| \ll \phi'^2$, alors $\phi' = q$ et donc

$$|\phi''| \ll \phi'^2 \iff |q'| \ll q^2 \quad \text{or} \quad q' = V'(x)\frac{m}{\hbar^2}\cdot\frac{1}{q} = \frac{m}{2\pi\hbar^2}\lambda(x)\left|\frac{dV}{dx}\right| \ll q^2$$

$$\boxed{|\phi''| \ll \phi'^2 \iff \left|\frac{dV}{dx}\right| \ll \frac{\hbar^2 q^2}{m}\cdot q}$$ WKB $\to$ odg ?

alors $\phi' = q \to \phi(x) = C + \int_{x_0}^{x} q(s)\,ds$

fonction de transfert $T = \left|\frac{\psi(x_2)}{\psi(x_1)}\right|^2 = e^{-2\int_{x_1}^{x_2} q(x)\,dx} = e^{-2\int_{x_1}^{x_2}\frac{\sqrt{2m(V-E)}}{\hbar}dx}$

$E_\alpha R_1 = U_0 R_0$

déf R_1

$V(r) = \frac{E_\alpha \cdot R_1}{r}$

$E_\alpha = \frac{1}{2}m v_\alpha^2$

$$\tau = \frac{\Delta t}{T} = \frac{2R_0}{v_\alpha}\cdot\exp\left(2\int_{R_0}^{R_1}\sqrt{\frac{2m(V-E_\alpha)}{\hbar^2}}\,dr\right)$$

$$\ln\tau = \underbrace{\ln\left(2R_0\cdot\sqrt{\frac{m}{2E_\alpha}}\right)}_{\text{admis} \sim \text{cste, varie lentement}} + \frac{2\sqrt{2mE_\alpha}}{\hbar}\int_{R_0}^{R_1}\left(\frac{R_1}{r}-1\right)^{\frac{1}{2}}dr$$

$$R_1\cdot\int_{\frac{R_0}{R_1}}^{1}\left(\frac{1}{u}-1\right)^{\frac{1}{2}}du \sim \frac{1}{E_\alpha}$$

Geiger-Nuttal $\ln\tau = A + \frac{B}{\sqrt{E_\alpha}}$

II