The Calculus of Concurrent Systems
Labelled Transition System and Behavioural equivalence

Names (channels)	$a, b, c, \ldots$	
Processes	$P \quad ::= \quad a.P \mid \overline{a}.P \mid P_1 \vert P_2 \mid P_1 + P_2 \mid \mathbf{0} \mid (\boldsymbol{\nu} a)P \mid !a.P$	
Actions	$\mu \quad ::= \quad a \mid \overline{a} \mid \tau$	

$$\text{T-inp}\ \frac{}{a.P \xrightarrow{a} P} \qquad \text{T-out}\ \frac{}{\overline{a}.P \xrightarrow{\overline{a}} P} \qquad \text{T-suml}\ \frac{P \xrightarrow{\mu} P'}{P + Q \xrightarrow{\mu} P'} \qquad \text{T-sumr}\ \frac{Q \xrightarrow{\mu} Q'}{P + Q \xrightarrow{\mu} Q'}$$

$$\text{T-com1}\ \frac{P \xrightarrow{a} P' \qquad Q \xrightarrow{\overline{a}} Q'}{P|Q \xrightarrow{\tau} P'|Q'} \qquad \text{T-com2}\ \frac{P \xrightarrow{\overline{a}} P' \qquad Q \xrightarrow{a} Q'}{P|Q \xrightarrow{\tau} P'|Q'}$$

$$\text{T-parl}\ \frac{P \xrightarrow{\mu} P'}{P|Q \xrightarrow{\mu} P'|Q} \qquad \text{T-parr}\ \frac{Q \xrightarrow{\mu} Q'}{P|Q \xrightarrow{\mu} P|Q'}$$

$$\text{T-res}\ \frac{P \xrightarrow{\mu} P'}{(\boldsymbol{\nu} a)P \xrightarrow{\mu} (\boldsymbol{\nu} a)P'}\ \text{ if } \mu \neq a \text{ and } \mu \neq \overline{a} \qquad \text{T-repl}\ \frac{}{!a.P \xrightarrow{a} !a.P|P}$$

Definition: A relation $\mathcal{R}$ between processes is a *bisimulation* if whenever $P\mathcal{R}Q$, we have the following:

1. if $P \xrightarrow{\mu} P'$, then there exists Q' such that $Q \xrightarrow{\mu} Q'$ and $P'\mathcal{R}Q'$, and
2. if $Q \xrightarrow{\mu} Q'$, then there exists P' such that $P \xrightarrow{\mu} P'$ and $P'\mathcal{R}Q'$.

Definition: *Bisimilarity*, written $\sim$, is a relation between processes defined as follows: we have $P \sim Q$ if there exists a relation $\mathcal{R}$ such that

1. $\mathcal{R}$ is a bisimulation, and
2. $P\mathcal{R}Q$.