Mini-workshop on glasses
ENS at Lyon , April 12, 2011

Chiral Order in Spin Glasses

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Review article

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H.K. J.Phys.Conf.Ser. 233, 012012 (2010)

Canonical SG

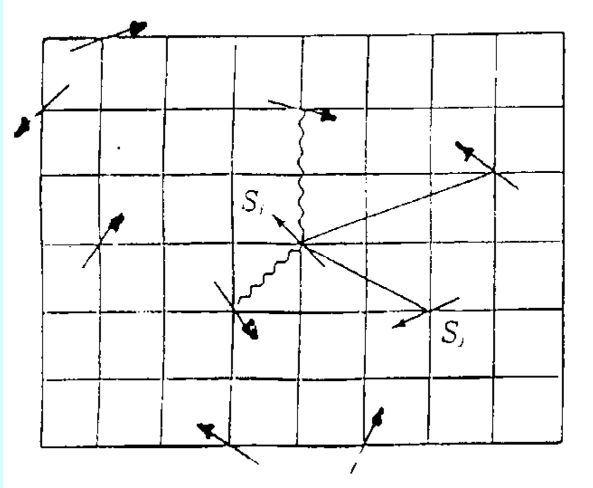
CuMn, AuFe, etc.

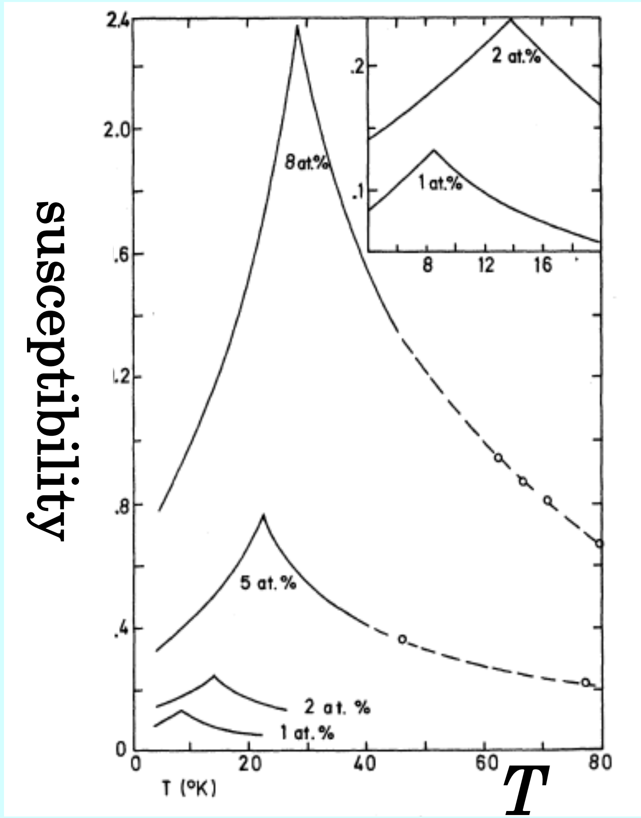
Heisenberg system with weak random magnetic anisotropy

Frustration & randomenss

Experimentally, thermodynamic SG transition and a SG ordered state has been established.

The true nature of the SG transition and the SG order state ? → still at issue





[Canella and Mydosh, 1972]

Model of spin glasses

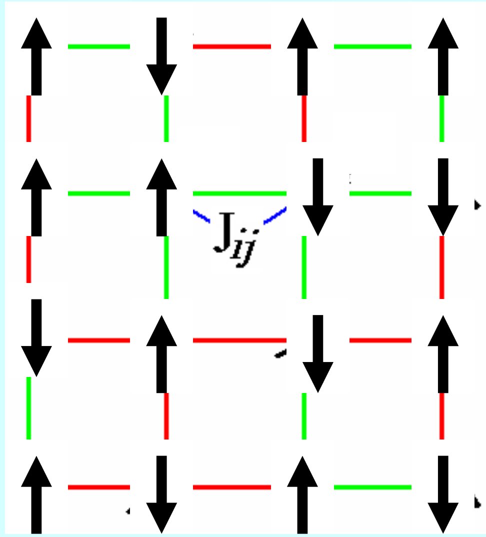
3D Edwards-Anderson model

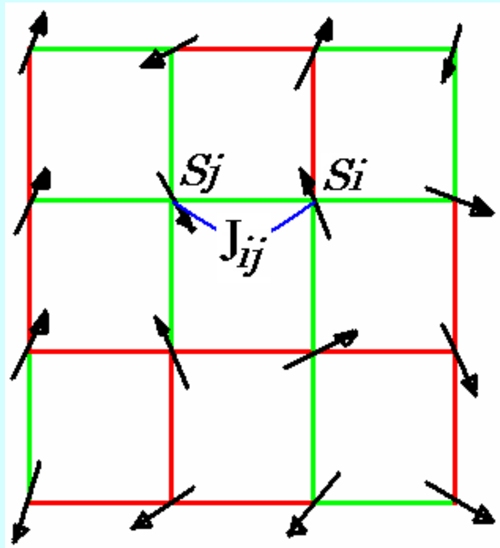
$$H = - J_{ij} \Sigma_{ij} S_i \cdot S_j$$

$\begin{cases} Si = S_{iz} : \text{Ising spin} \quad (\text{FeMnTiO}_3) \\ S_i = (S_{ix}, S_{iy}, S_{iz}) : \text{Heisenberg spin} \quad (\text{canonical SG}) \end{cases}$

J_{ij} : nearest-neighbor random coupling with zero mean and variance J^2

Gaussian or binary ($\pm J$)





Numerical results on the 3D EA SG models

* 3D Ising SG

The existence of a finite-temperature SG transition established in zero field. The nature of the ordered state still under debate.

* 3D isotropic Heisenberg SG

Earlier studies suggested no finite-T transition
[Olive,Young, Sherrington, '86, F. Matsubara et al '91]

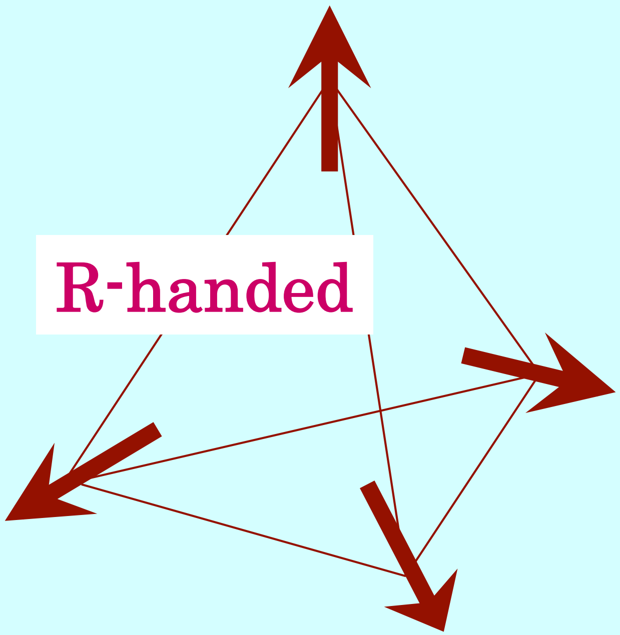
The possibility of a finite-T transition in the *chiral* sector was suggested [H.K., '92] --- chiral glass state

spin-chirality decoupling $T_{SG} < T_{CG}$

Chirality scenario of experimental SG transition [H.K. '92]

Heisenberg SG possesses a new degree of freedom which is absent in Ising SG

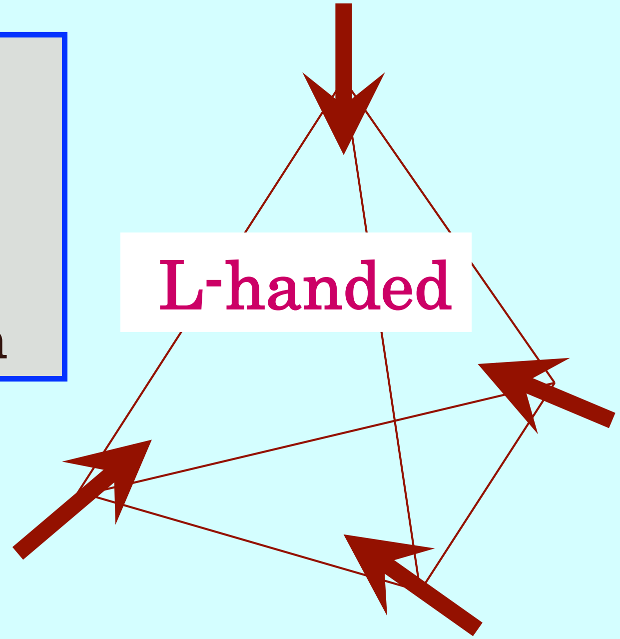
→ **scalar chirality**



chirality −

$Z_2 \times SO(3)$

↑ chirality ↑ spin rotation



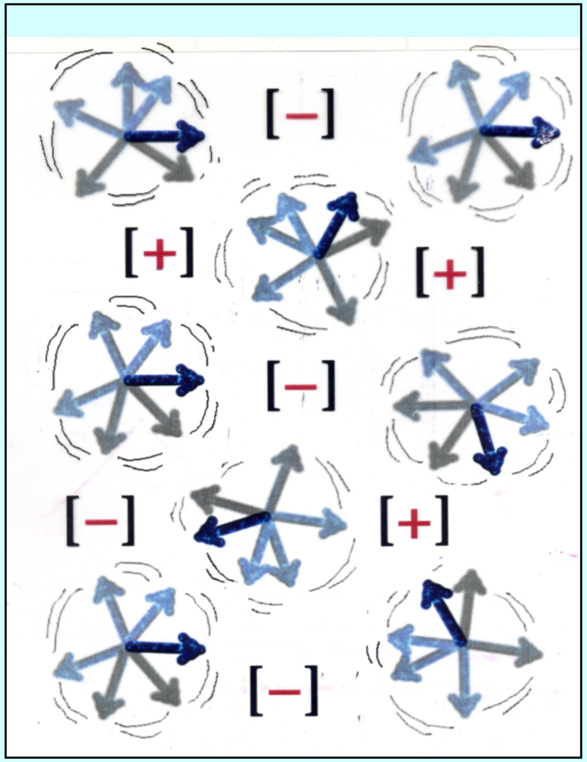
chirality +

$\chi = S_i \cdot S_j \times S_k$ → **Unconventional anomalous Hall effect**

Chirality scenario of SG transiton [H.K. '92~]

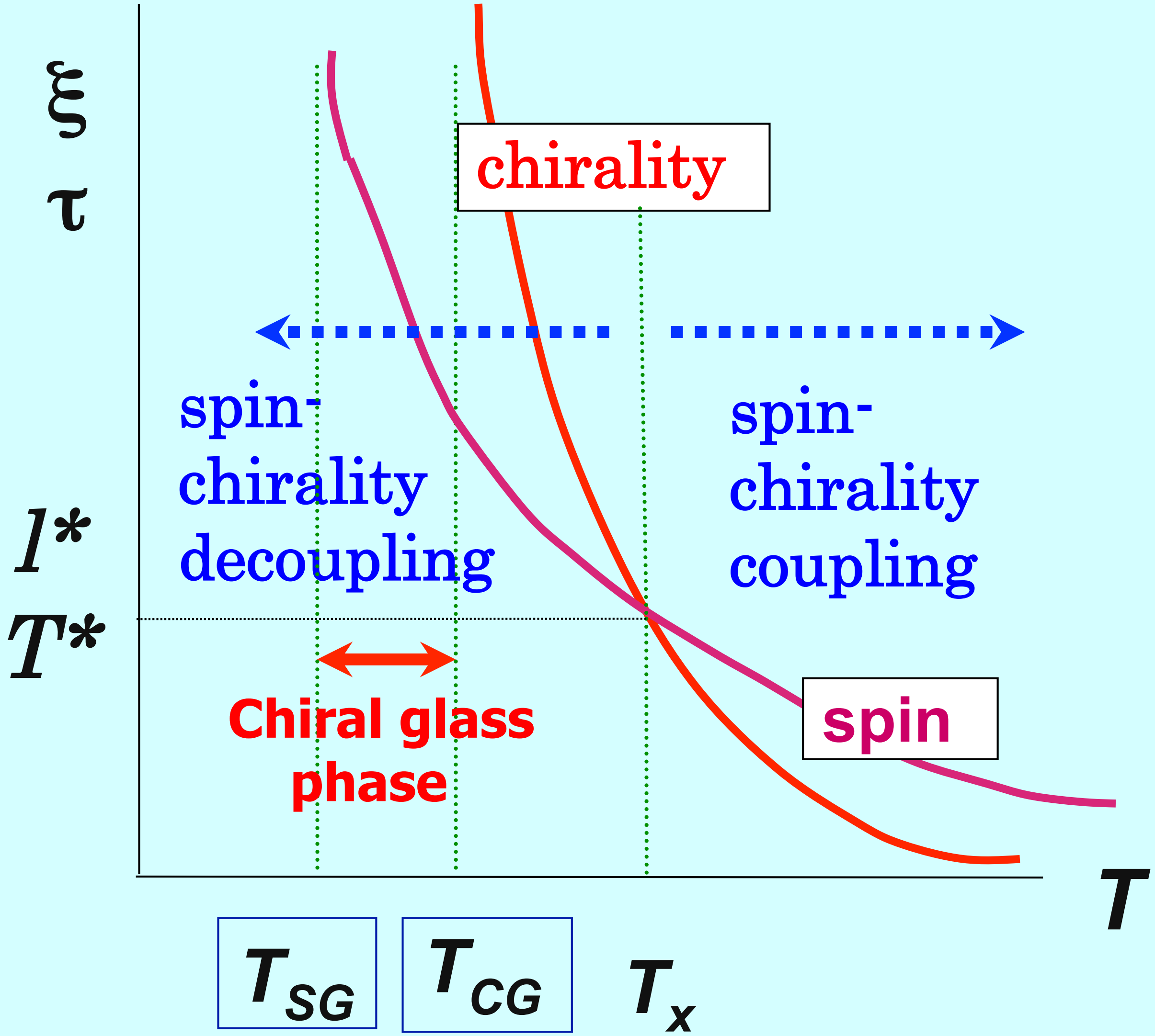
* Isotropic Heisenberg SG in 3D exhibits a spin-chirality decoupling, with the chiral-glass ordered phase not accompanying the standard SG order.

* Chirality is a hidden order parameter of real SG transitions. Experimental SG transition is a "disguized" chiral-glass transition: The spin is recoupled to the chirality, *i.e.*, mixed into the chirality via the random magnetic anisotropy.



[chiral-glass state]

Spin-chirality decoupling in the 3D isotropic Heisenberg SG



How the spin and chiral correlations grow ?

$T_{CG} > T_{SG}$

Recent controversy on the 3D Heisenberg SG

Due to the progress in the computer ability and simulation teqnique, significant numerical study now becomes possible for the 3D Heisenberg SG.

Consensus in recent numerical studies:

The 3D Heisenberg SG exhibits a finite-T transition.

However, its nature has still been largely controversial.

Spin & chirality are decoupled or not ?

* Yes, decoupling occurs ($T_{SG} < T_{CG}$)

H.K.'98, K.Hukushima & H.K. '00 '05

* No decoupling ($T_{SG} = T_{CG} > 0$)

F.Matsubara, T.Shirakura et al,
B.W.Lee & A.P.Young '03; '07
I.Campos et al '06
L.Fernandez et al '09

Chirality scenario has been contested !

Our MC simulation on the 3D Gaussian Heisenberg SG

[D.X.Viet and H.K, PRL102, 027202 ('09); PRB80, 064418 ('10)]

* Heat bath and over-relaxation method, combined with the temperature exchange technique
* System as large as L=32 equilibrated well below T_g
* Sample # large (∼ 10^3 samples averaged)

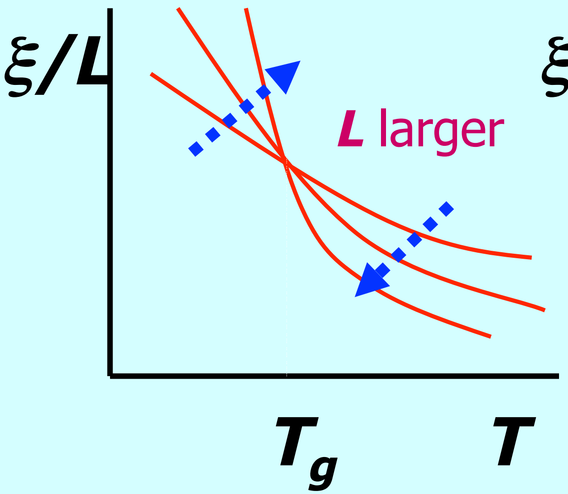
Various independent quantities including the correlation length ratio ξ/L, Binder ratio g, and the glass order parameter $q^{(2)}$ are calculated,

Spin and chirality correlation length ratio ξ/L

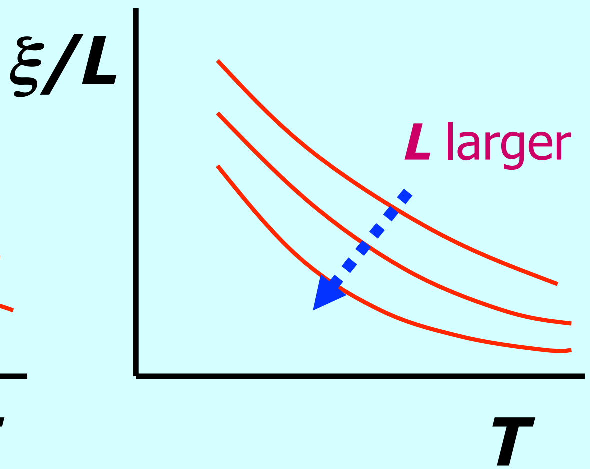
--- a quantity most intensively studied

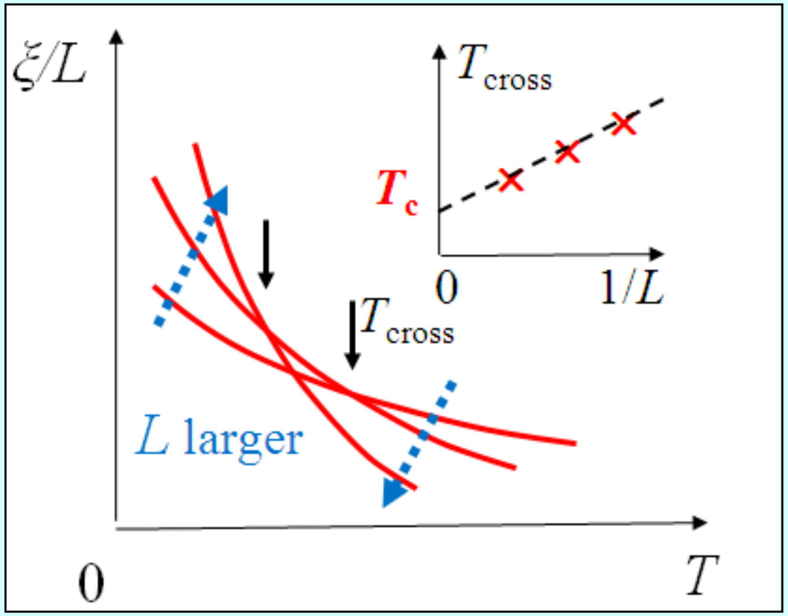
The transition temperature can be estimated from the size and temperature dependence of the dimensionless ratio ξ/L

T>0 transition



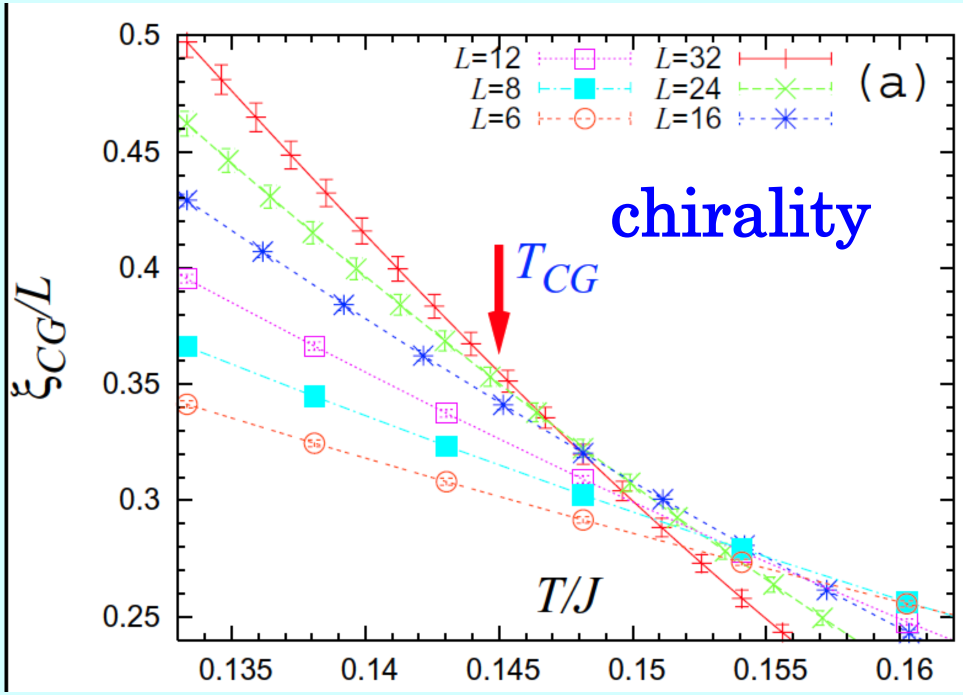
T=0 transition

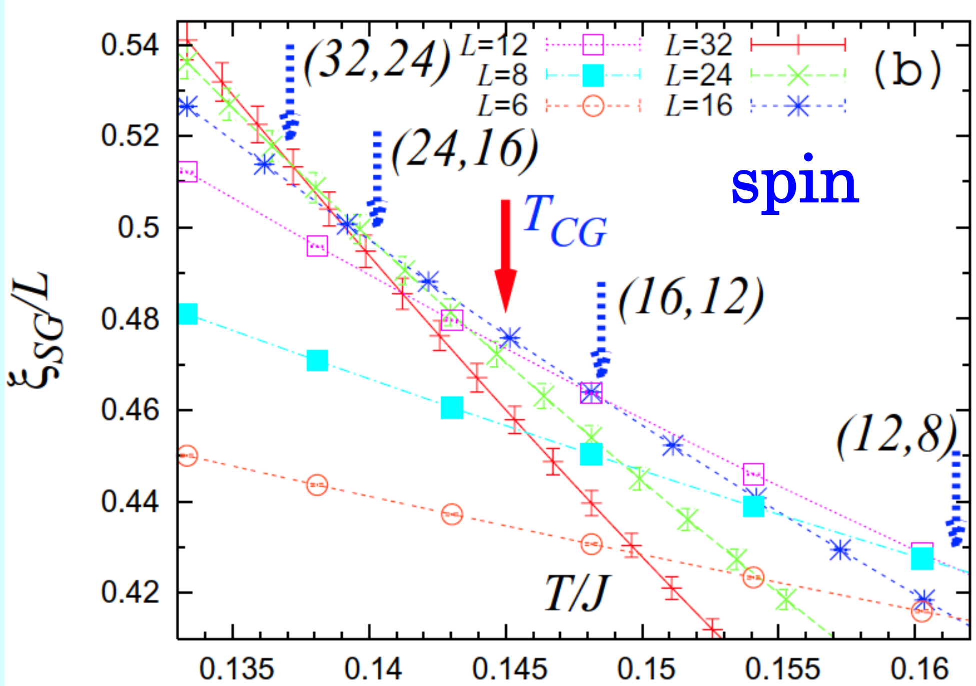


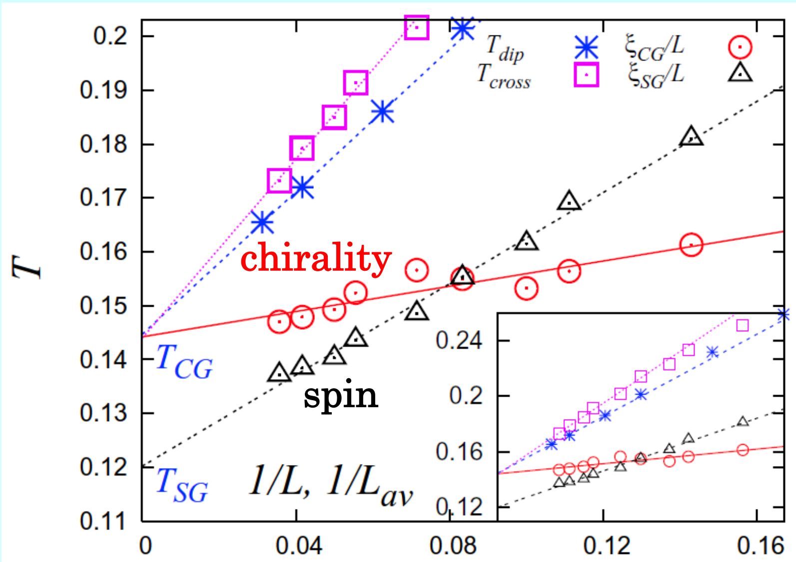


In reality ...

Correlation length ratios ξ/L [D.X. Viet and H.K., '10]







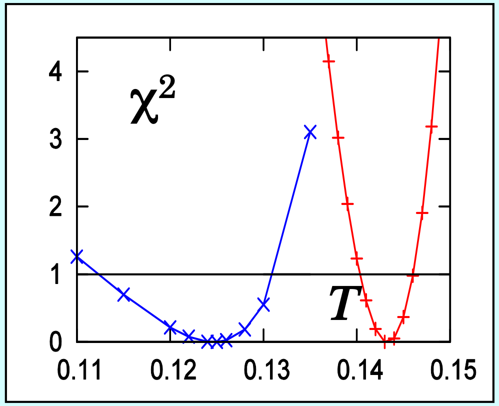
T_{cross} extrapolated to $L=\infty$

$T_{CG}=0.145\pm0.004$

$T_{SG}=0.120\pm0.006$

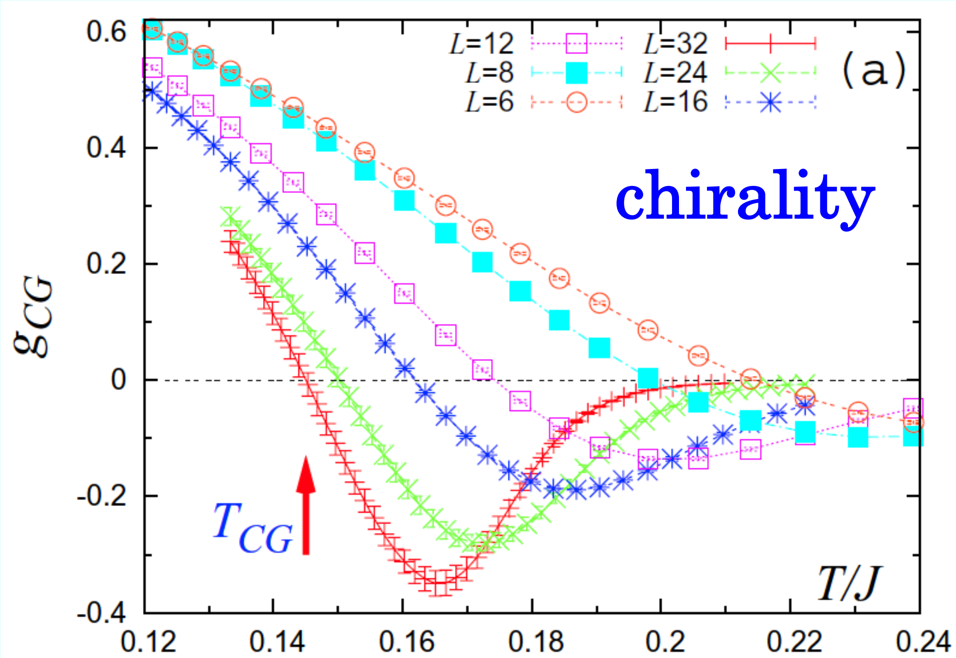
$T_{CG} > T_{SG}$ by ~15%

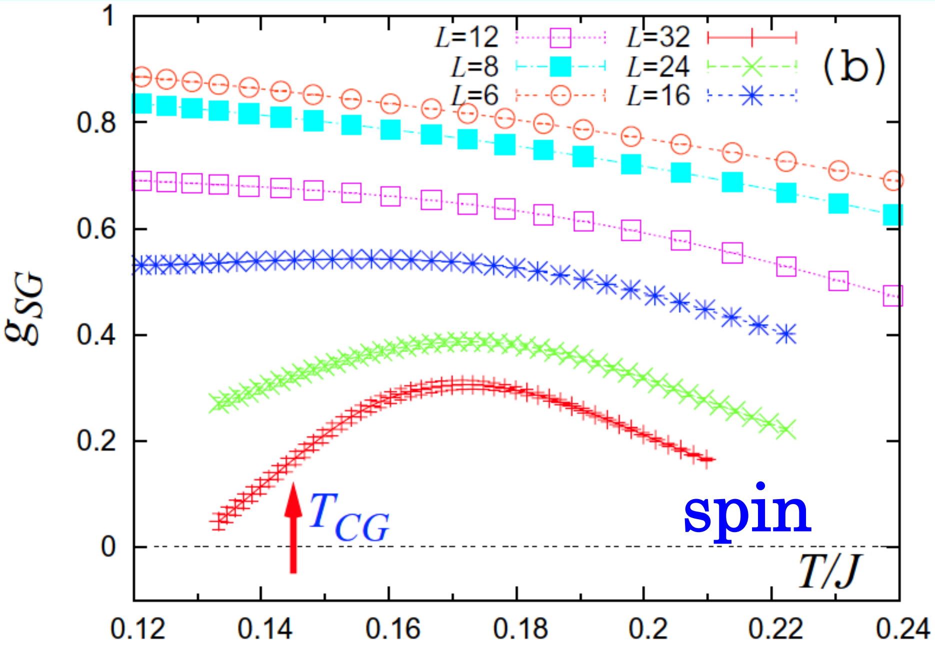
χ^2 -analysis



Binder ratio

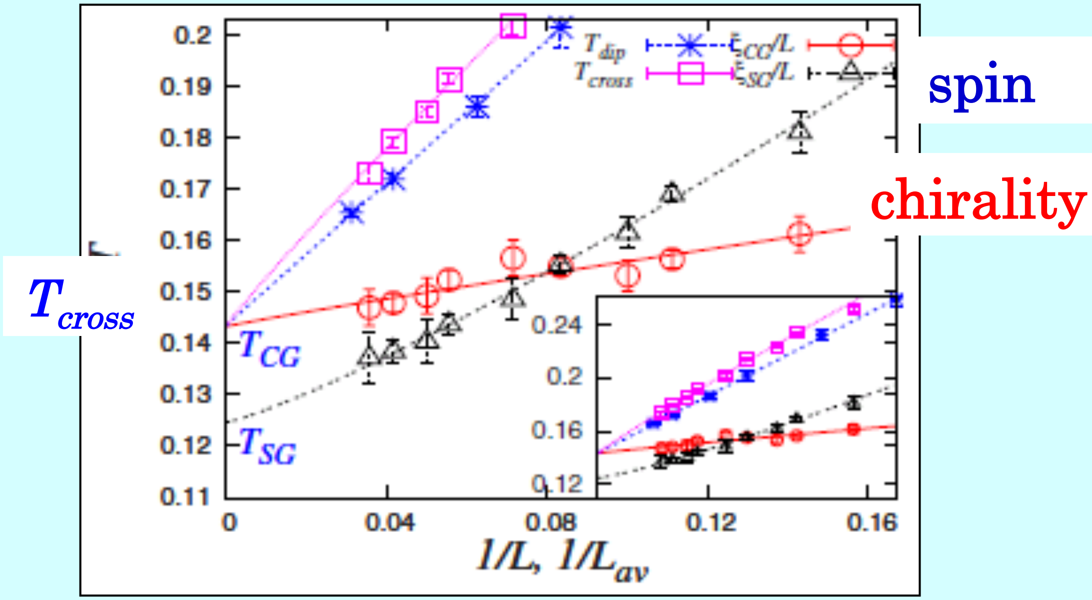
[D.X. Viet and H.K., '10]





growing negative dip

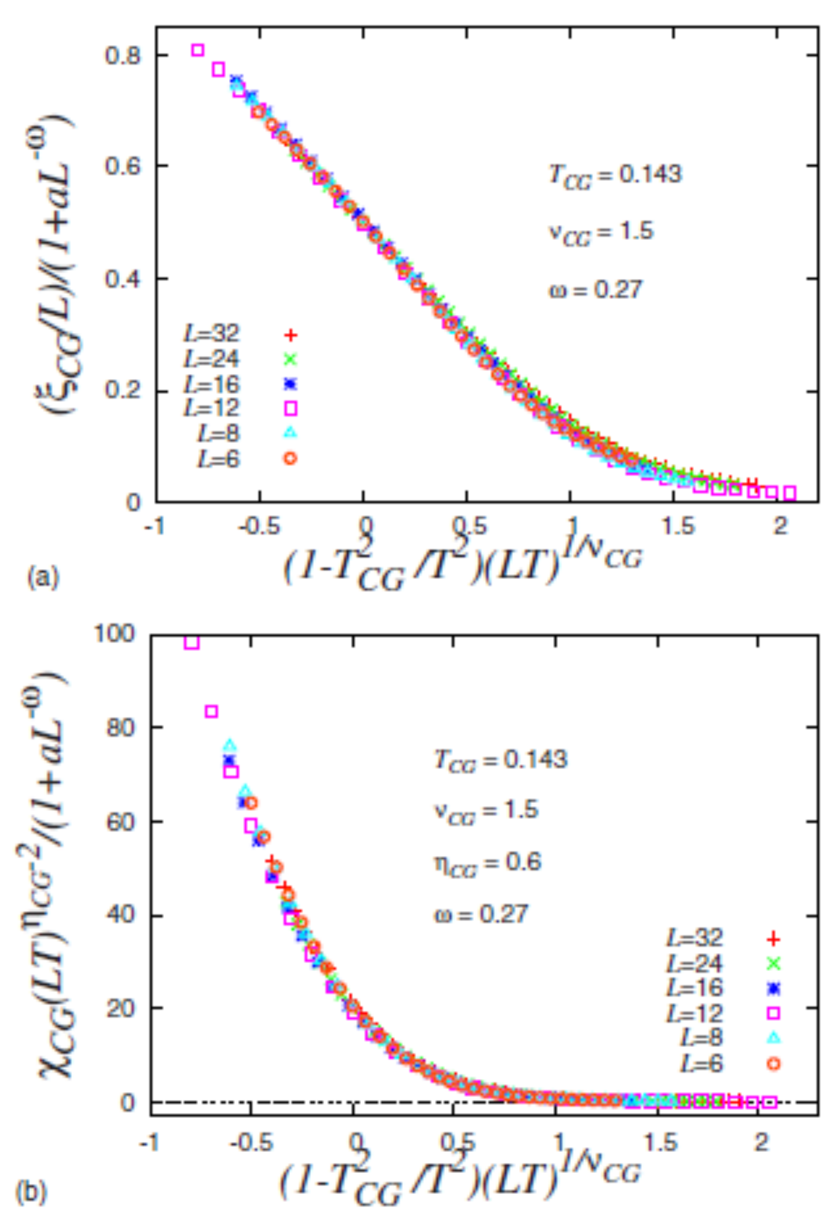
→ consistent with 1-step RSB



Critical properties of the chiral-glass transition of the 3D Gaussian Heisenberg SG

Chiral glass correlation length

Chiral glass susceptibility



Finite-size scaling plots with the leading correction-to-scaling

chiral-glass exponents

$\nu_{CG} = 1.4(2)$
$\eta_{CG} = 0.6(2)$

differ from the 3D Ising values !

$\nu \sim 2.5$

$\eta \sim -0.40$

Chirality hypothsis

[H.K. 1992]

Isotropic (ideal) system

→ "spin-chirality decoupling"

Real system is weakly anisotropic !

Spin is "recoupled " to the chirality due to the weak random magnetic anisotropy *D*.

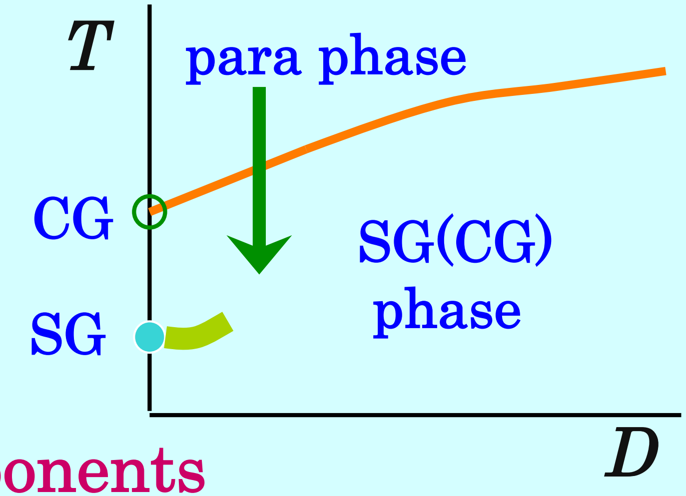
Z_2 [chiral] × ~~**SO(3)** [spin-rotation]~~

The chiral-glass transition now appears as the SG transition.

↓

"spin-chirality recoupling"

experimental SG exponents = CG exponents



Critical properties of canonical SG

	β	γ	ν	η
CuMn & AgMn [de Courtenary *et al.*]	1.0±0.1	2.2±0.1	≃ 1.4	≃ 0.4
AgMn [Bouchiat]	1.0±0.1	2.2±0.2	≃ 1.4	≃ 0.4
AgMn [Levy *et al.*]	0.9±0.2	2.1±0.1	≃ 1.3	≃ 0.4
CuAlMn [Simpsons]	≃ 1.0	≃ 1.9	≃ 1.3	≃ 0.5
PdMn [Coles and Williams]	0.9±0.15	2.0±0.2	≃ 1.3	≃ 0.4
AuFe [Taniguchi & Miyako]	1.0±0.2	2.0±0.2	≃ 1.3	≃ 0.5
$CdCr_{2\times0.85}In_{2\times0.85}S_4$ [Vincent *et al.*]	0.75±0.10	2.3±0.4	≃ 1.3	≃ 0.2
±*J* Campbell *et al.*	≃ 0.82	≃ 6.5	2.72±0.08	−0.40±0.04
±*J* Hasenbusch *et al.*	≃ 0.77	≃ 5.8	2.45±0.15	−0.375±0.010
$Fe_{0.5}Mn_{0.5}TiO_3$ [Gunnarsson *et al.*]	≃ 0.54	4.0±0.3	≃ 1.7	≃ −0.35

Canonical SG Exp.

3D Ising

Chiral (Heisenberg)

β~1 γ~2 ν~1.4 η~0.6

Direct test of the chirality scenario

→ needs to measure the chirality directly

Use an anomalous Hall effect

as a probe of chiral order !

Measurements of linear and nonlinear chiral susceptibilities, X_χ & $X_{\chi nl}$, becomes possible via measurements of Hall coefficient R_s.

[G. Tatara & H.K. '02, H.K. '03]

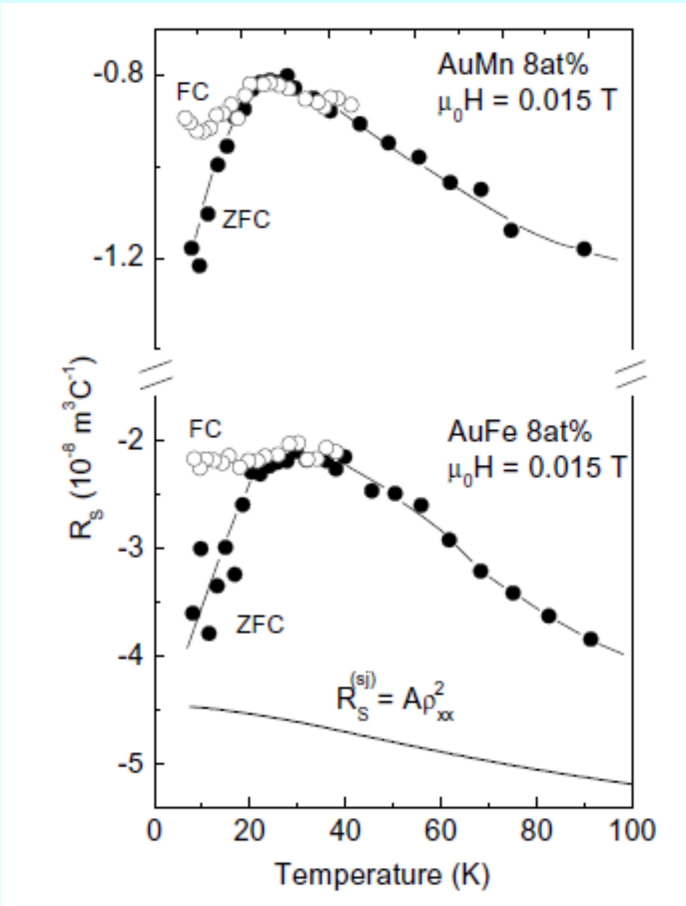
$$R_s = \rho_{xy} / M$$

Chiral contribution

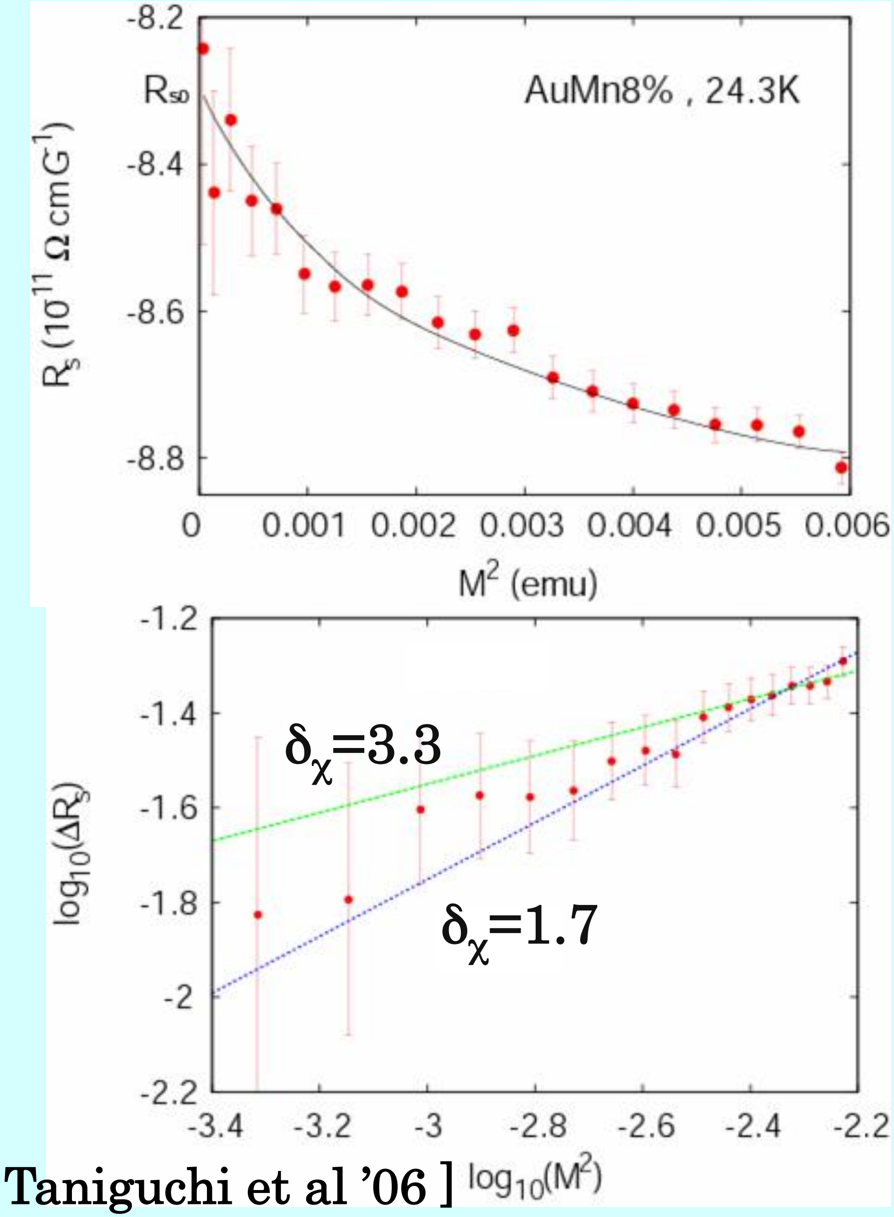
$$= -A\rho - B\rho^2 - CD\,[X_\chi + X_{\chi nl}\,(DM)^2 + \dots]$$

Anomalous Hall coefficient R_s of SG

Kageyama et al ('03) FeAl (RSG); T.Taniguchi et al ('04) AuFe
P.Pureur et al ('04) AuMn;
T. Taniguchi et al ('06) AuMn



[P.Pureur et al ('04)]



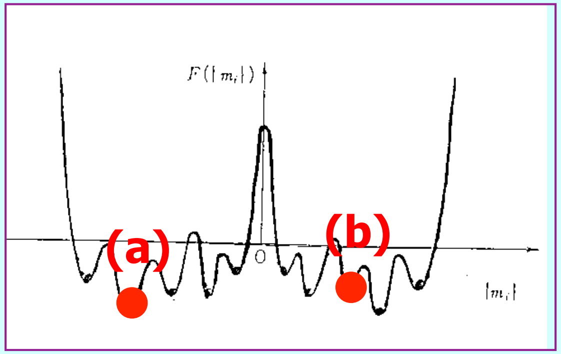
[T. Taniguchi et al '06]

What is the nature of
the chiral-glass ordered state ?

Is there an RSB ?
If so, what type ?

SG ordered state might possess a complex phase-space structure

→ RSB (replica-symmetry breaking)



Possible multi-valley structure in SG

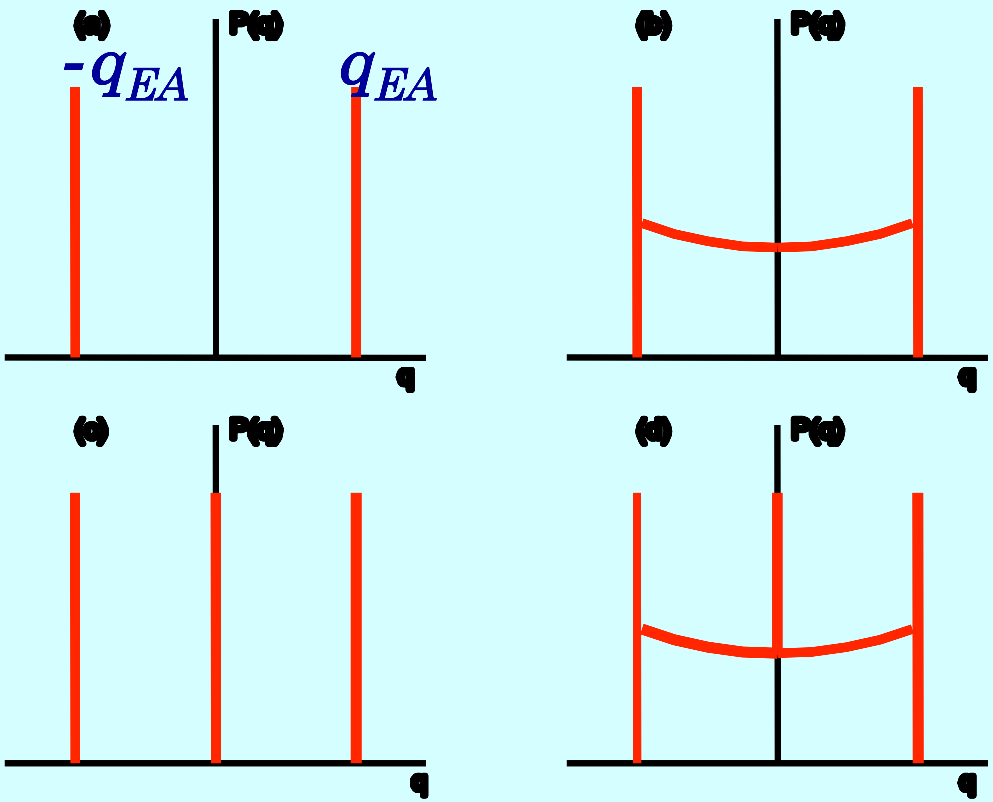
q: overlap

$q=(1/N)\,\Sigma_i\, S_i^{(a)} S_i^{(b)}$

Droplet picture (No RSB)
Fisher & Huse

Hierarchical RSB
Parisi

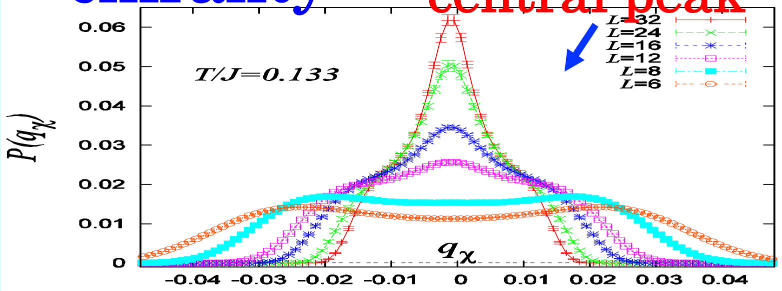
One-step RSB



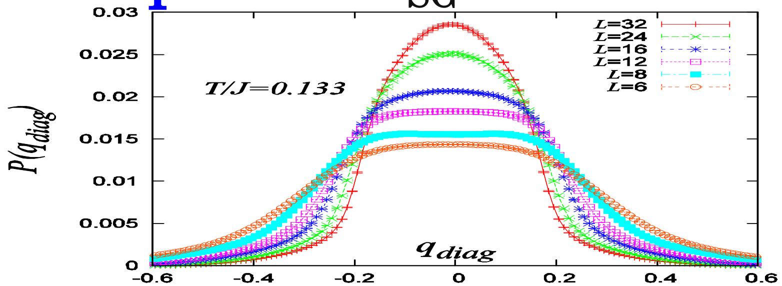
Overlap distribution function: $P(q)$

Overlap distribution of the isotropic model

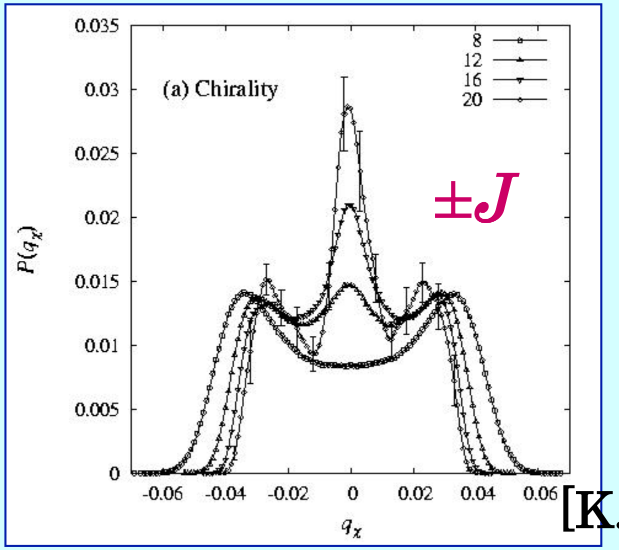
chirality central peak



spin $[T_{SG} < T = 0.133 < T_{CG}]$



$-q_{EA}$ peak q_{EA} peak



$\pm J$

Chiral $P(q_\chi)$:

central peak

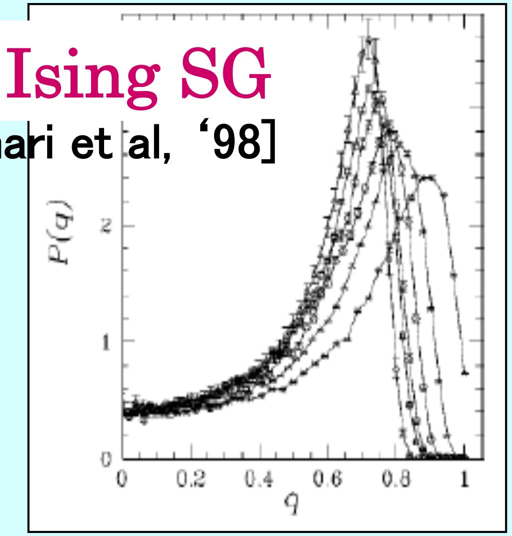
& $\pm q_{EA}$ peak

→ 1-step-like RSB ?

[K.Hukushima & HK '05]

3D Ising SG

[E. Marinari et al, '98]



How to measure $P(q)$ experimentally ?

In equilibrium, FDT holds

SG in off-equilibrium

$R(t_1,t_2)$; response function
$C(t_1,t_2)$; correlation function

***T*; heat bath temperature**

$$R(t_1,t_2)=(1/k_B T)\,[\mathrm{d}C(t_1,t_2)\,/\,\mathrm{d}t_1]$$

In off-equilibrium, FDT does not hold, but there is an off-equilibrium counterpart

$$R(t_1,t_2) = (X(t_1,t_2)\,/\,k_B T)\;[\mathrm{d}C(t_1,t_2)\,/\,\mathrm{d}t_1]$$

$1/k_B T_{eff}$

In the limit of $t_1, t_2 \rightarrow \infty$,

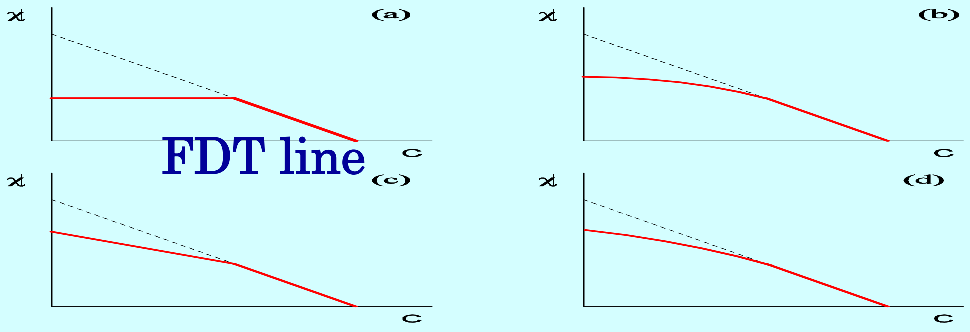
$$X(t_1,t_2) \quad \rightarrow \quad X(C(t_1,t_2))$$

[L.F. Kugliandolo and J. Kurchan '93]

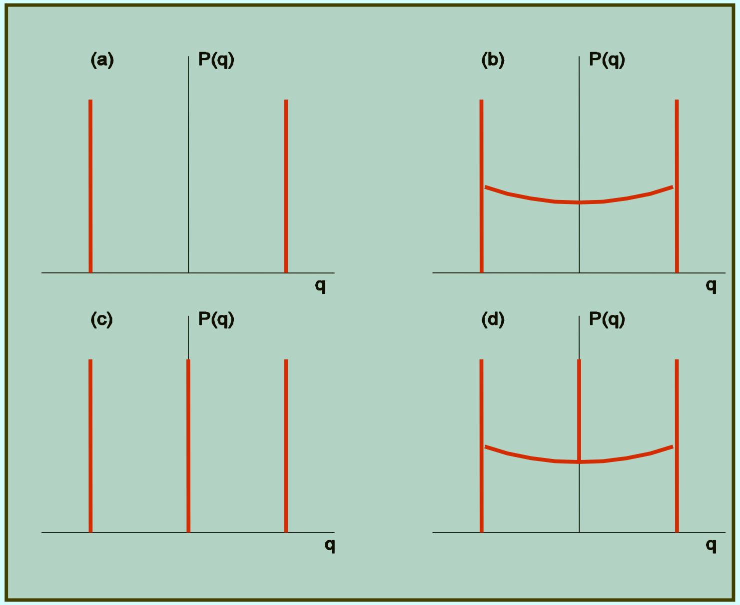
$$P(q) = \mathrm{d}\,X(q)\,/\,\mathrm{d}q$$

$P(q)$; overlap distribution function

Susceptibility $\chi(t_1,t_2)$ *vs.* Correlation $C(t_1,t_2)$



(a) No RSB
(b) full-step RSB
(c) 1-step RSB
(d) combination of (b)+(c)

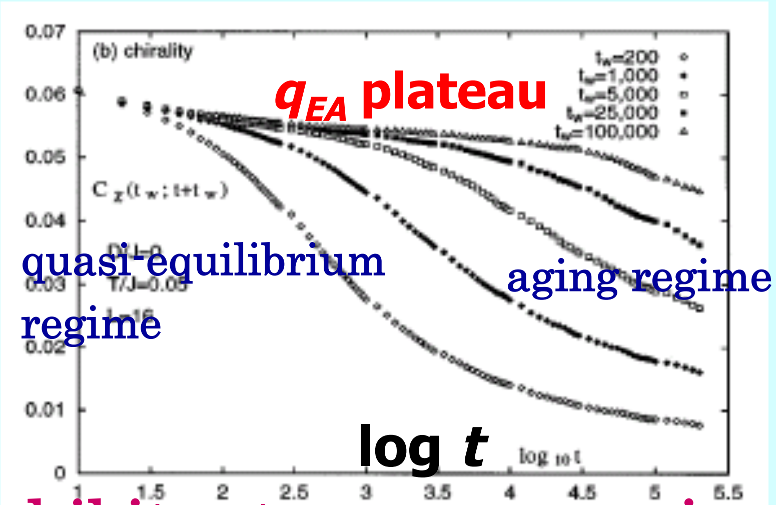


Off-equilibrium MC simulation of the isotropic 3D Gaussian Heisenberg SG

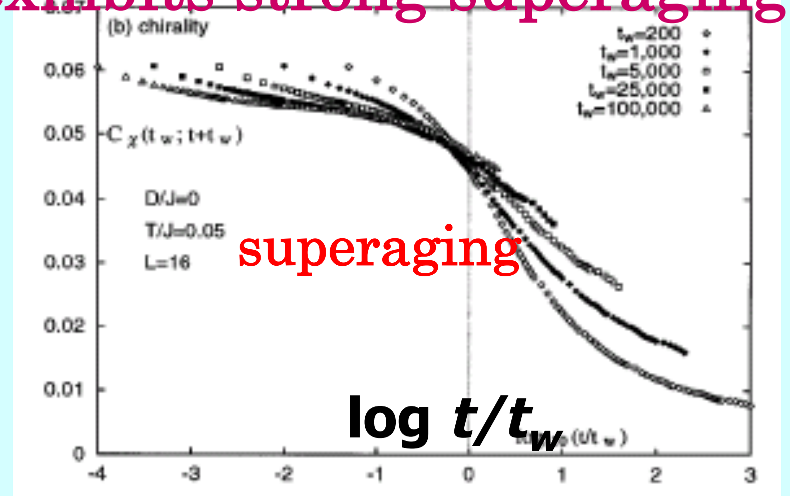
[HK, '98]

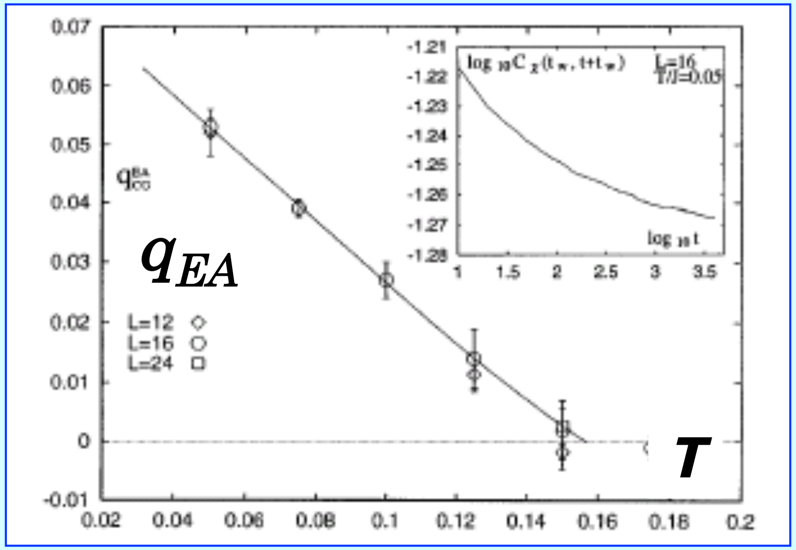
Chiral autocorrelations $C_\chi(t,t_w)$ at $T=0.05$

tw: waiting time



exhibits strong superaging !





Chiral EA parameter q_{EA}

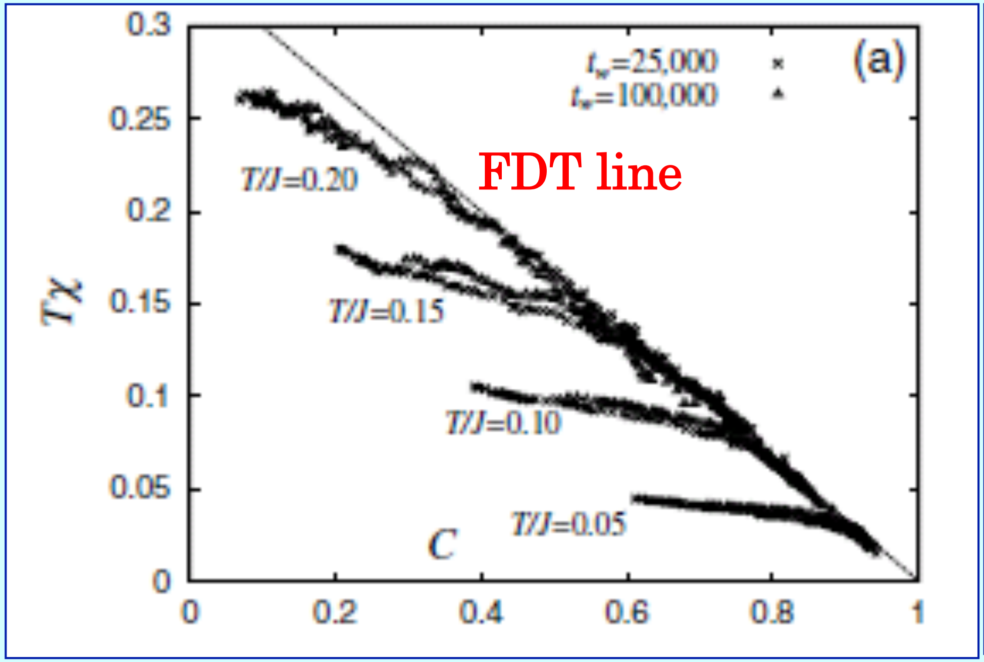
$T_{CG} \sim 0.15$
$\beta \sim 1$

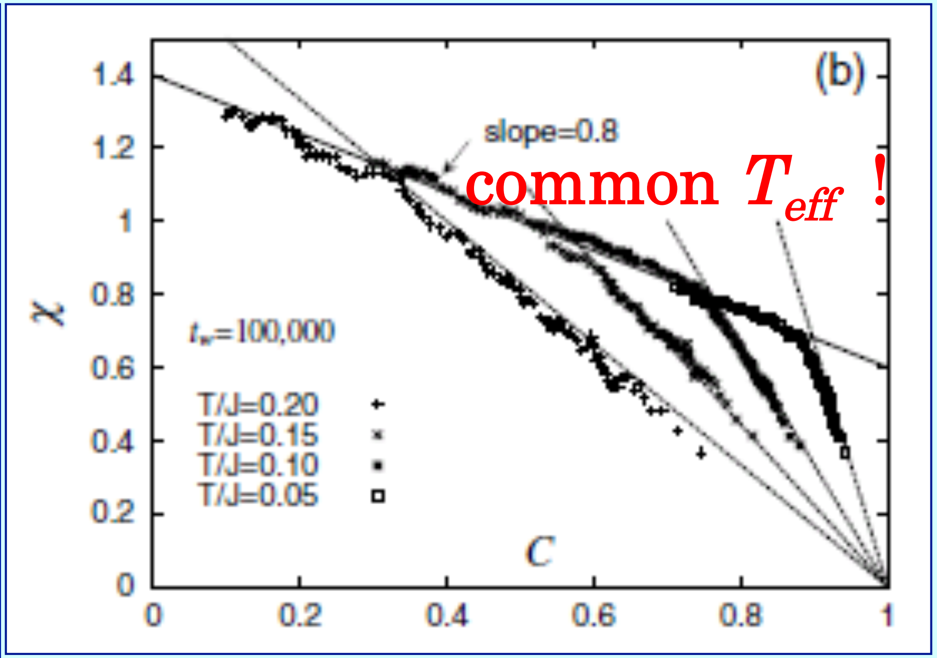
Off-equilibrium simulation of the weakly anisotropic 3D $\pm J$ Heisenberg SG

[HK, '03]

χ-*C* plot for the spin

$Tg \sim 0.21$ $D=0.01$





1-step-like behavior

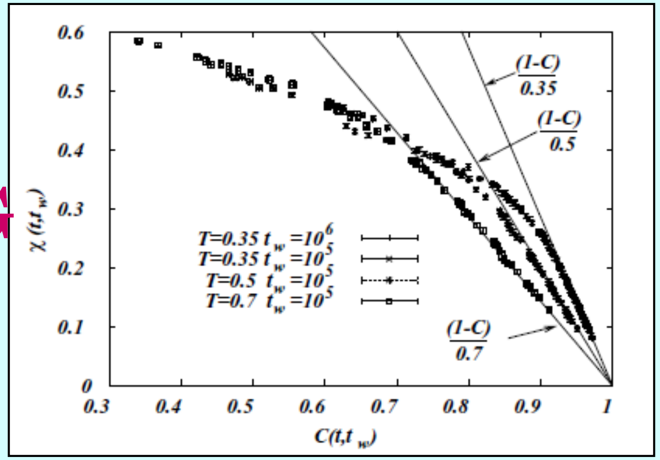
$T_{eff} \sim 2T_g$

irrespective of T

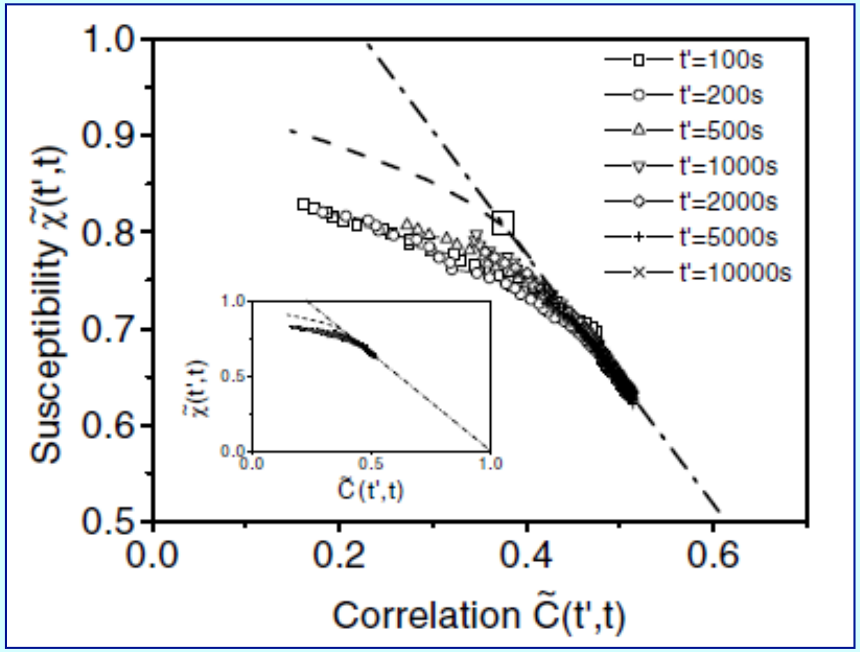
c.f.

3D Ising SG

[E. Marinari et al, '00]



Experiment on Heisenberg-like SG



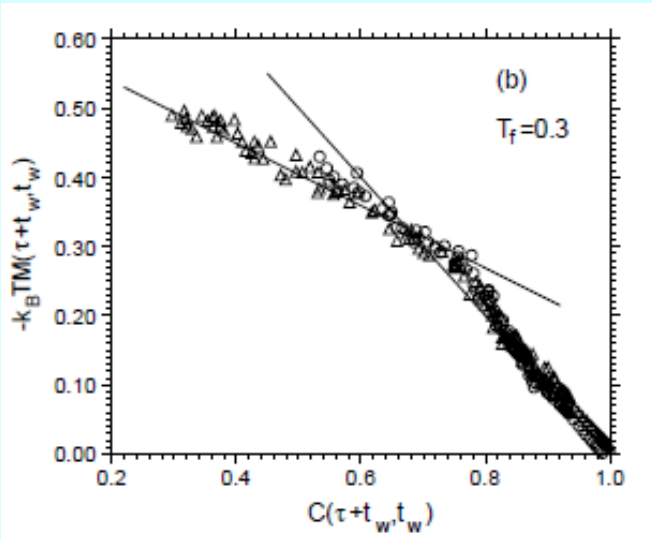
$CdCr_{1.7}In_{0.3}S_4$

[D.Herisson and M.Ocio, '02]

1-step-like ?

$T_{eff} \sim 1.9\, T_g$

(measured at T=$0.8T_g$)



***c.f.* Lennard-Jones fluid simulation**

[J.-L. Barrat and W. Kob, '99]

$T_{eff} \sim 1.5 T_{mc}$ (irrespective of T)

T_{mc} : mode-coupling temperature

SG (chiral-glass) ordered state of canonical SG might exhibit a one-step-like RSB

Analogy to molecular glasses

Dynamical equations describing structural glass are similar to MF SG models exhibiting a 1-step RSB

[T.E. Kirkpatrick, D. Thirumalai, P.G. Wolynes '87]

1. Discontinuous 1-step RSB (discontinuous q_{EA} at $T=Tg$)

   $p>2$-spin MF SG, $p>4$ state MF Potts SG

   dynamical T_D and static Tg ($T_D > Tg$)

   → **structural glass**

2. Continuous 1-step RSB (continuous q_{EA} at $T=Tg$)

   $2<p<4$ state MF Potts SG

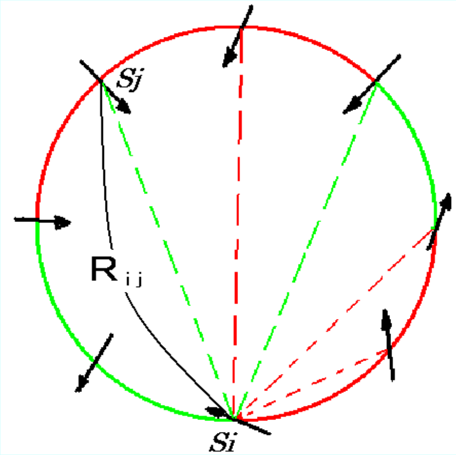
   No dynamical T_D

   → **Heisenberg-like SG**

Reference model

1D Heisenberg SG model with a long-range power-law interaction

$$H = -\sum_{i,j} J_{ij} \vec{S}_i \cdot \vec{S}_j \qquad J_{ij} = c(\sigma)\frac{\epsilon_{ij}}{r_{ij}^{\sigma}}$$



ε_{ij} : Gaussian with zero mean and standard deviation unity

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σ-*d* correspondence expected

Varying *σ* of 1D LR model ~ varying *d* of SR modes

$\sigma \to 0 \quad \Leftrightarrow \quad d = \infty$

σ large $\quad \Leftrightarrow \quad d = 1$

***σ* =2/3 ⇔ *d*=6 (ucd)**

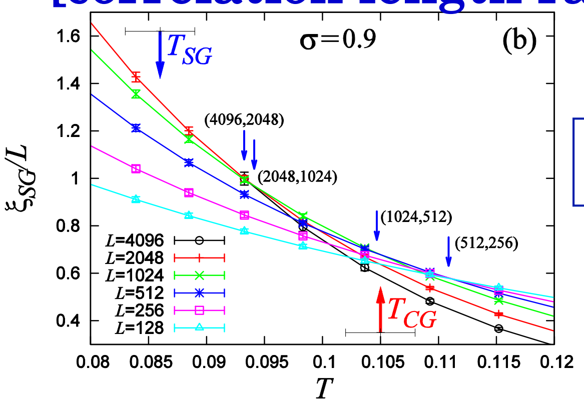
***σ* ~0.9 ⇔ *d*=3**

σ =1.0 ⇔ between *d*=2 & *d*=3

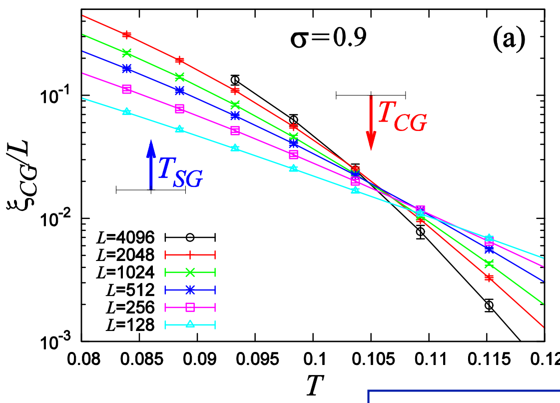
σ=0.9

[D.X.Viet & H.K. PRL 105, 097206 (2010); JPSJ 79, 104708 (2010)]

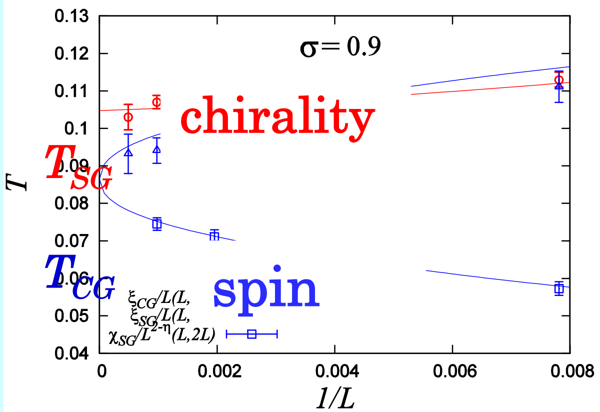
[correlation length ratio]



spin

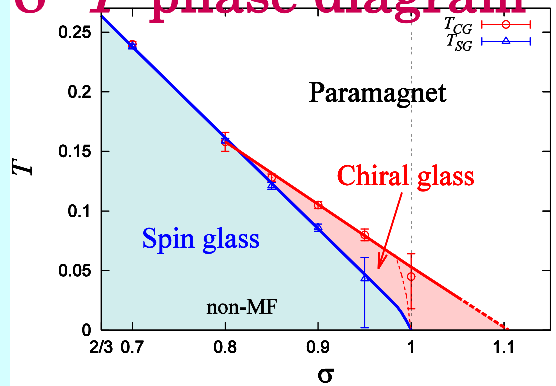


chirality



T_{cross}

σ - T phase diagram



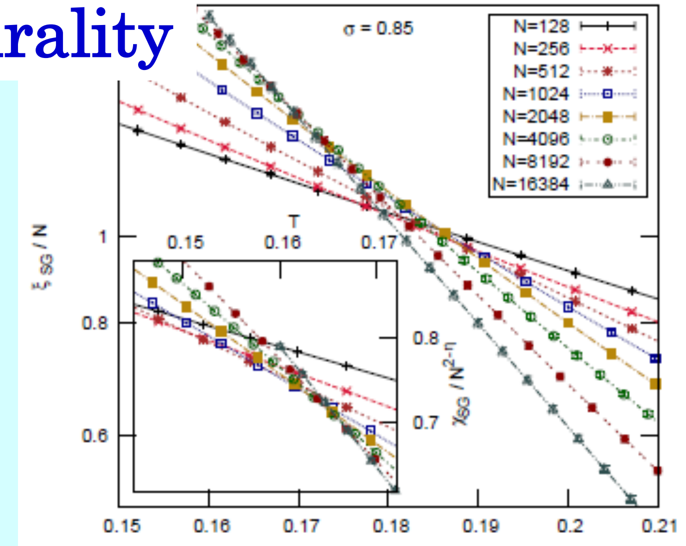
Another 1D LR model

- randomly diluted model

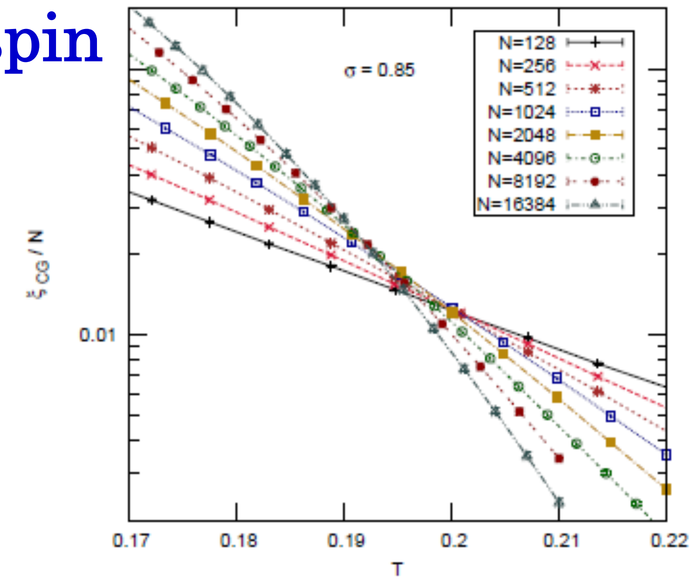
[A. Sharma & A.P. Young, '11]

σ = 0.85

chirality



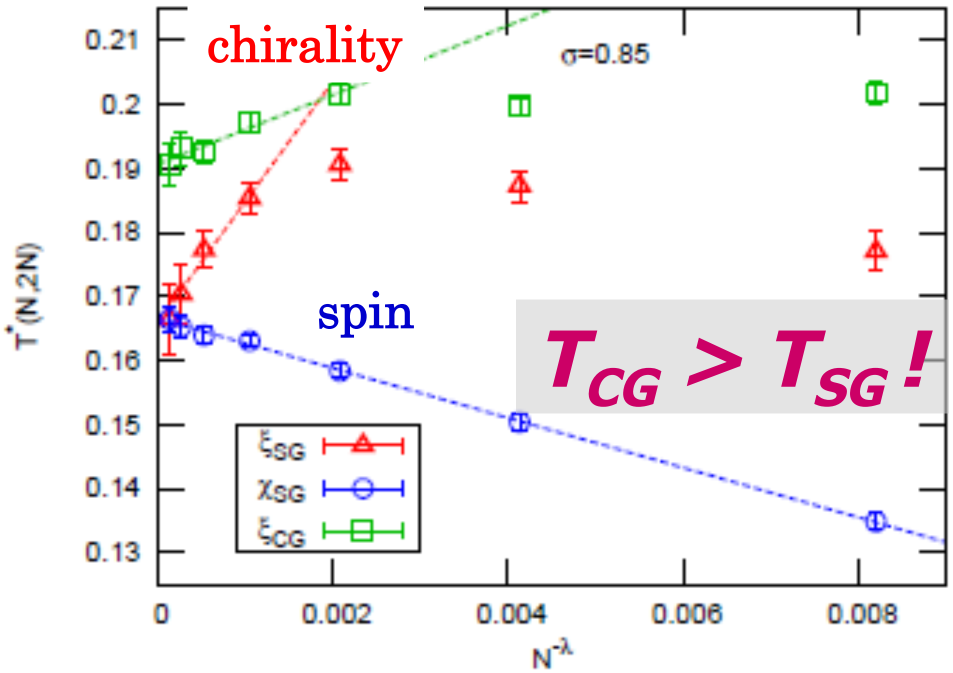
spin



Choose nonzero J_{ij}

with probability $z \times p_{ij}$ (z=6)

$$p_{ij} = \frac{r_{ij}^{-2\sigma}}{\sum_{j\,(j\neq i)} r_{ij}^{-2\sigma}},$$



Summary

*An intriguing “spin-chirality decoupling” phenomenon occurs in Heisenberg-like SGs.

* SG (chiral-glass) ordered state of canonical SG might exhibit a 1-step-like RSB.

* Chirality scenario might solve the long-standing puzzles of experimental SG ordering.

Chirality might be a missing link in spin glasses ?!