Amorphous order

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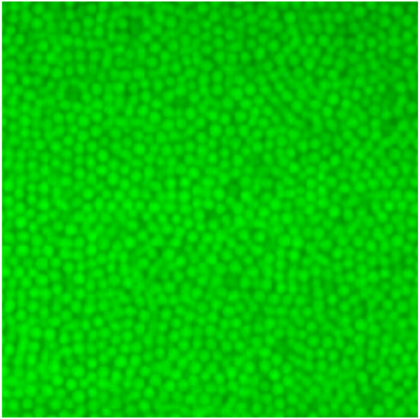
Lyon

with:

Dov Levine



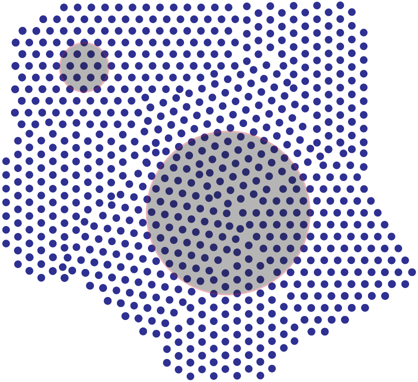
Liquid or Glass ?

How far can one go just looking

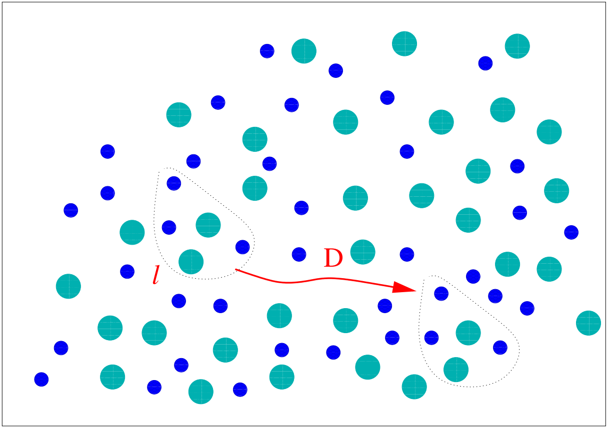
at configurations?

Patch - recurrence length $D(l)$ crossover l_o

detects crystallite length.



Generalize this to general systems



Complexity:

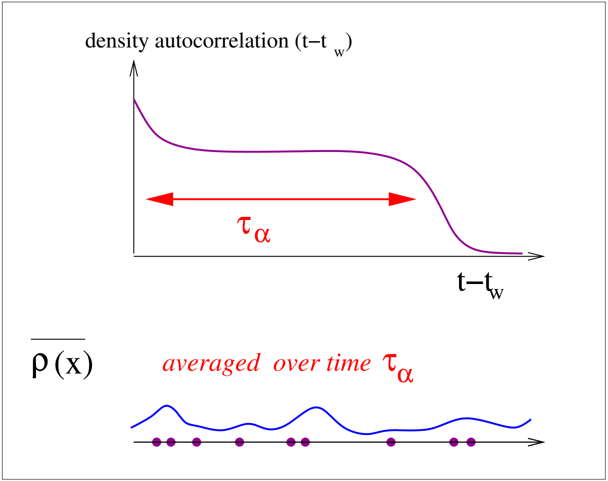
$\ln \{$ **average repetition rate** $\}$

will depend on patch size ℓ, and on presicion ϵ

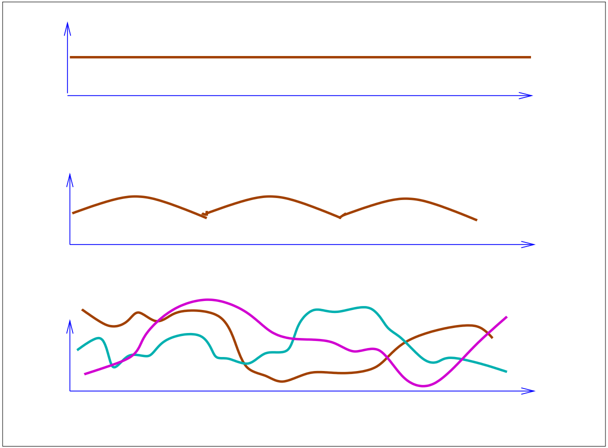
Before going on, we need to specify

how are we going to identify patches

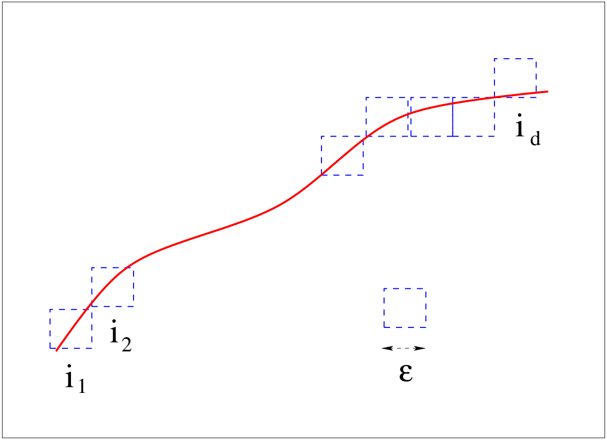
Particle systems: supercooled liquid



Average density profiles: constant, periodic, 'chaotic'



We need to count profiles ↔ identify patches



inspiration from dynamic systems

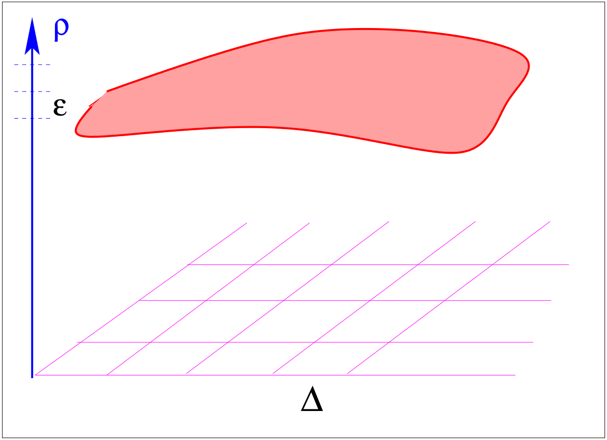
The limit is well-defined:

$$K_1 \sim - \lim_{\tau \to 0} \lim_{\epsilon \to \infty} \lim_{d \to \infty} \frac{1}{\tau d} \sum_{i_1, \ldots, i_d} P_\epsilon(i_1, \ldots, i_d) \ln P_\epsilon(i_1, \ldots, i_d)$$

Renyi: a measure of 'rare' patches (very frequent or very unfrequent):

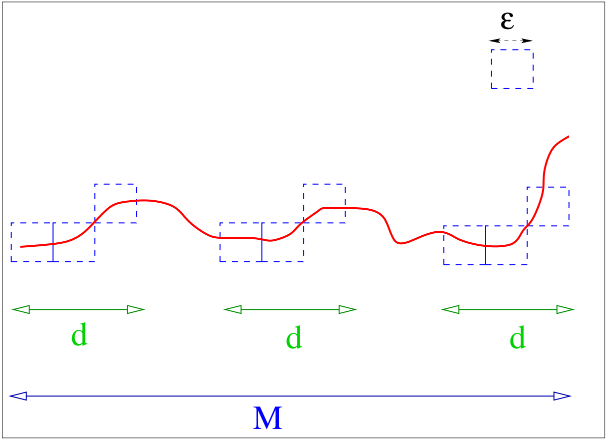
$$K_q \sim - \lim_{\tau \to 0} \lim_{\epsilon \to \infty} \lim_{d \to \infty} \frac{1}{\tau d (q-1)} \ln \left(\sum_{i_1, \ldots, i_d} P_\epsilon(i_1, \ldots, i_d)^q \right)$$

$\ldots \to \mathcal{P}[P_\epsilon]$ **by Legendre transform.**



$$t \to \vec{r} \qquad x \to \rho$$

Grassberger-Procaccia:



count the number of repetitions n_i of a patch of size d within a large box M and average over patches

$$P_\epsilon(i_1, ..., i_d)^q \sim \frac{1}{M} \sum_i [n_i^d(\epsilon)]^{q-1} \sim \epsilon^\phi \; e^{\tau(q-1)d \, K_q}$$

So that:

$$K_d \sim \lim_{\tau \to 0} \lim_{\epsilon \to \infty} \lim_{d \to \infty} \frac{1}{\tau(q-1)} \frac{\delta}{\delta d} \ln \left[\sum_i [n_i^d(\epsilon)]^{q-1} \right]$$

for K_1 we use $\left[\sum_i \ln[n_i^d(\epsilon)]\right]$

practical because we work at finite precision

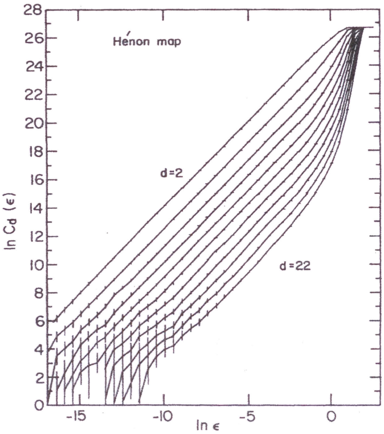
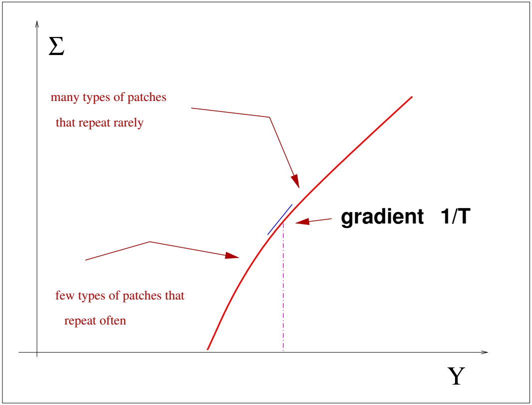


FIG. 3. Same as Fig. 1, but for the Hénon map. The values of d are $d=2$ (top curve), 4,6,8, . . . ,22 (bottom curve).

Y: **log [number of repetitions]**

Σ : **log [patches having e^Y repetitions**

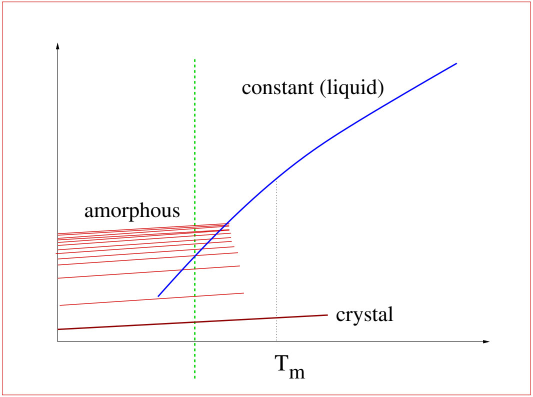


$e^{\Sigma(Y)+Y}$ **max [$\Sigma + Y$] dominates**

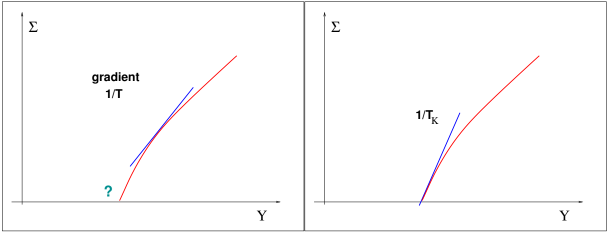
$$e^{qVK_{q+1}} = \langle [n_a]^q \rangle \sim \int dY e^{V(qY+\Sigma)}$$

Renyi entopy $\leftrightarrow x$ **parameter**

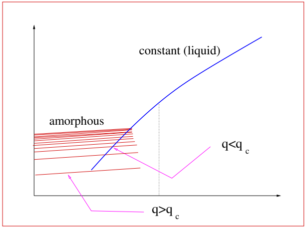
Kauzmann/ Random First Order scenario



$$Y = e^{-(f-f_o)/T} \qquad ; \qquad \Sigma(f)\text{: } \textbf{complexity}$$

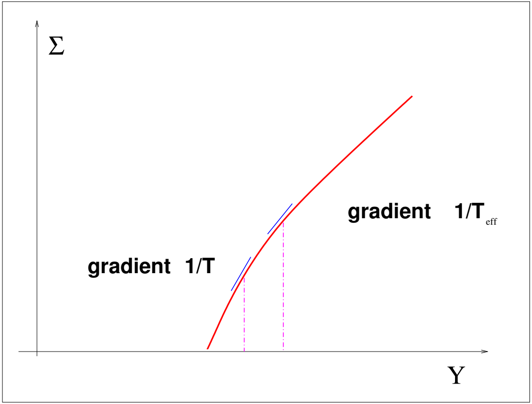


Transition within Kauzmann/RFOT



Rare versus frequent patches

Renyi entropy R_q vanishes for $q > x$ where x is the Parisi parameter



Effective / Fictive temperatures

We may frame the discussion in terms of

well-defined, measurable quantities

and perhaps prove general things.

Open question: does complexity $+$ Adam-Gibbs suffice?