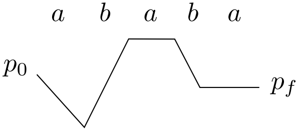
Explorable Automata

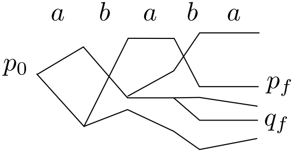
Emile Hazard, **Denis Kuperberg**

CSL 2023, Warsaw, February 16th 2023

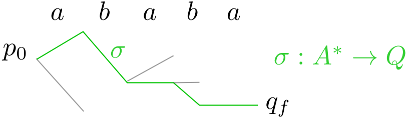
History-Deterministic Automata



Deterministic

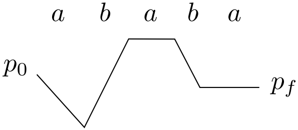


Non-deterministic

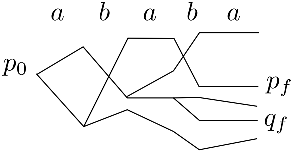


History-Deterministic
(a.k.a. Good-for-Games)

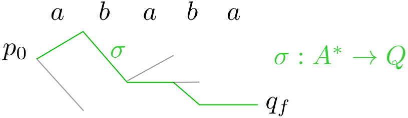
History-Deterministic Automata



Deterministic



Non-deterministic



History-Deterministic
(a.k.a. Good-for-Games)

Motivations

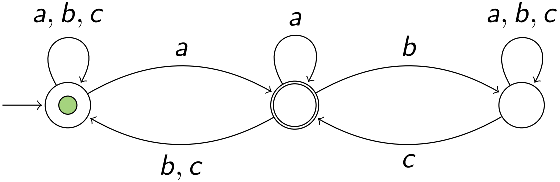
- Solve Church Synthesis more efficiently
- Intermediate model between Det. and Nondet.
- Exponential Succinctness wrt Det. [K., Skrzycpzak '15]

Definition of HD via a game

$\mathcal{A}$ ND automaton on finite or infinite words.

Letter game of $\mathcal{A}$:
Adam plays letters:

Eve: resolves non-deterministic choices for transitions



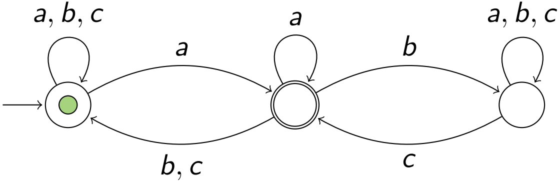
Definition of HD via a game

$\mathcal{A}$ ND automaton on finite or infinite words.

Letter game of $\mathcal{A}$:

Adam plays letters: a

Eve: resolves non-deterministic choices for transitions

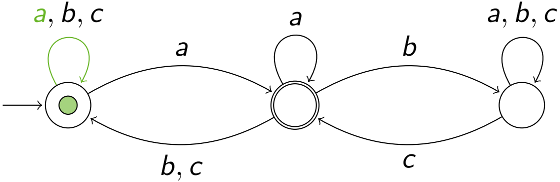


Definition of HD via a game

$\mathcal{A}$ ND automaton on finite or infinite words.

<u>**Letter game of $\mathcal{A}$**</u>:
Adam plays letters: *a*

Eve: resolves non-deterministic choices for transitions

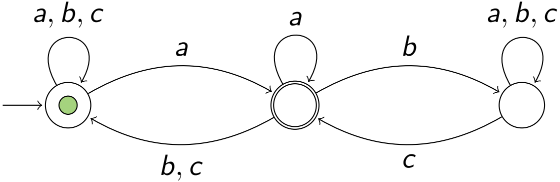


Definition of HD via a game

$\mathcal{A}$ ND automaton on finite or infinite words.

Letter game of $\mathcal{A}$:
Adam plays letters: a a

Eve: resolves non-deterministic choices for transitions

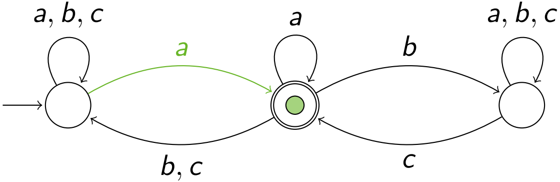


Definition of HD via a game

$\mathcal{A}$ ND automaton on finite or infinite words.

Letter game of $\mathcal{A}$:
Adam plays letters: a a

Eve: resolves non-deterministic choices for transitions

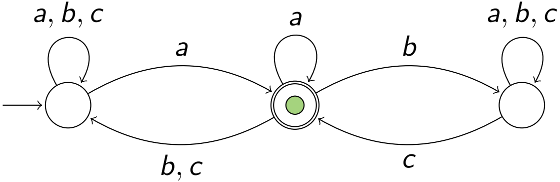


Definition of HD via a game

$\mathcal{A}$ ND automaton on finite or infinite words.

Letter game of $\mathcal{A}$:
Adam plays letters: $a \;\; a \;\; b$

Eve: resolves non-deterministic choices for transitions

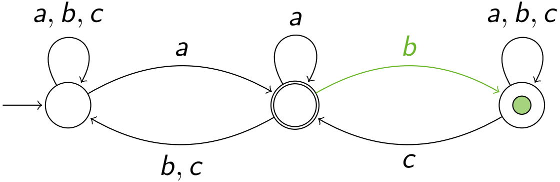


Definition of HD via a game

$\mathcal{A}$ ND automaton on finite or infinite words.

Letter game of $\mathcal{A}$:
Adam plays letters: $a \;\; a \;\; b$

Eve: resolves non-deterministic choices for transitions

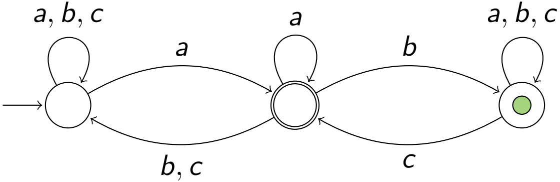


Definition of HD via a game

$\mathcal{A}$ ND automaton on finite or infinite words.

Letter game of $\mathcal{A}$:
Adam plays letters: $a \;\; a \;\; b \;\; c$

Eve: resolves non-deterministic choices for transitions

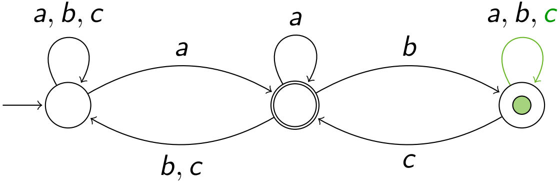


Definition of HD via a game

$\mathcal{A}$ ND automaton on finite or infinite words.

Letter game of $\mathcal{A}$:
Adam plays letters: a a b c

Eve: resolves non-deterministic choices for transitions

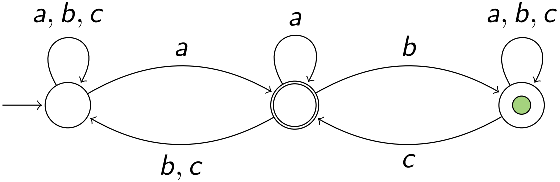


Definition of HD via a game

$\mathcal{A}$ ND automaton on finite or infinite words.

<u>**Letter game of $\mathcal{A}$**:</u>
Adam plays letters: $a \;\; a \;\; b \;\; c \;\; c$

Eve: resolves non-deterministic choices for transitions

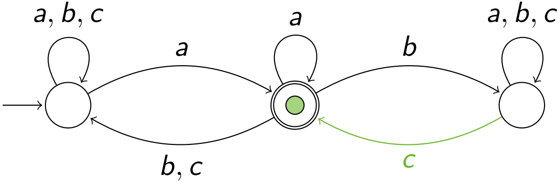


Definition of HD via a game

$\mathcal{A}$ ND automaton on finite or infinite words.

Letter game of $\mathcal{A}$:
Adam plays letters: $a \quad a \quad b \quad c \quad c$

Eve: resolves non-deterministic choices for transitions

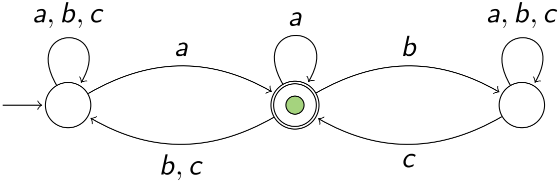


Definition of HD via a game

$\mathcal{A}$ ND automaton on finite or infinite words.

Letter game of $\mathcal{A}$:
Adam plays letters: $a \;\; a \;\; b \;\; c \;\; c \;\; \dots \quad = \; w$

Eve: resolves non-deterministic choices for transitions



Eve wins if: $w \in L(\mathcal{A}) \Rightarrow$ Run accepting.

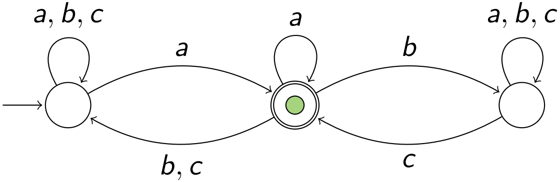
Definition of HD via a game

$\mathcal{A}$ ND automaton on finite or infinite words.

Letter game of $\mathcal{A}$:

Adam plays letters: $a \;\; a \;\; b \;\; c \;\; c \;\; \dots \quad = \; w$

Eve: resolves non-deterministic choices for transitions



Eve wins if: $w \in L(\mathcal{A}) \Rightarrow$ Run accepting.

$\mathcal{A}$ HD $\Leftrightarrow$ Eve wins the Letter game on $\mathcal{A}$

$\Leftrightarrow$ there is a strategy $\sigma_{\mathrm{HD}} : A^* \to Q$ accepting all words of $L(\mathcal{A})$.

Recognizing HD automata

Complexity of the HDness problem:
Input: A nondeterministic automaton $\mathcal{A}$
Output: Is $\mathcal{A}$ HD?

Recognizing HD automata

Complexity of the HDness problem:
Input: A nondeterministic automaton $\mathcal{A}$
Output: Is $\mathcal{A}$ HD?

- On finite words: PTime [Löding]

Recognizing HD automata

Complexity of the HDness problem:
Input: A nondeterministic automaton $\mathcal{A}$
Output: Is $\mathcal{A}$ HD?

- On finite words: PTime [Löding]
- On infinite words: Open problem !

Recognizing HD automata

Complexity of the HDness problem:
Input: A nondeterministic automaton $\mathcal{A}$
Output: Is $\mathcal{A}$ HD?

- On finite words: PTime [Löding]
- On infinite words: Open problem !
  - Upper bound: ExpTime [Henzinger, Piterman '06]

Recognizing HD automata

Complexity of the HDness problem:
Input: A nondeterministic automaton $\mathcal{A}$
Output: Is $\mathcal{A}$ HD?

- On finite words: PTime [Löding]
- On infinite words: Open problem !
  - Upper bound: ExpTime [Henzinger, Piterman '06]
  - PTime algorithm conjectured to be correct [Bagnol, K. '18]
    Proved correct for Büchi and CoBüchi conditions.

Recognizing HD automata

Complexity of the HDness problem:
Input: A nondeterministic automaton $\mathcal{A}$
Output: Is $\mathcal{A}$ HD?

- On finite words: PTIME [Löding]
- On infinite words: Open problem !
  - Upper bound: EXPTIME [Henzinger, Piterman '06]
  - PTIME algorithm conjectured to be correct [Bagnol, K. '18]
    Proved correct for Büchi and CoBüchi conditions.

What about building HD automata ?

Recognizing HD automata

Complexity of the HDness problem:
Input: A nondeterministic automaton $\mathcal{A}$
Output: Is $\mathcal{A}$ HD?

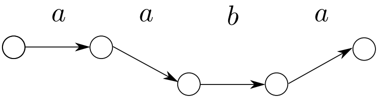
- On finite words: PTime [Löding]
- On infinite words: Open problem !
  - Upper bound: ExpTime [Henzinger, Piterman '06]
  - PTime algorithm conjectured to be correct [Bagnol, K. '18]
    Proved correct for Büchi and CoBüchi conditions.

What about building HD automata ?

To tackle these questions, we generalize the notion of HD ...

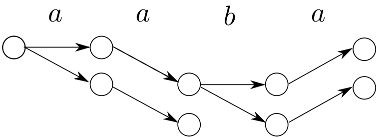
Allowing more runs

Idea: Allow to build several runs, at least one accepting.



1 state - 1 run

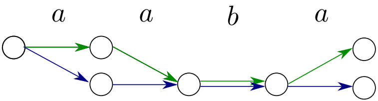
HD



2 states

Width 2

[K. + Majumdar '18 '19]



2 runs

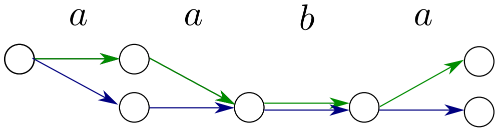
2**-Explorable**

[This paper]

Explorable Automata

***k*-explorability game**:

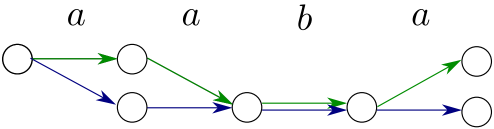
Adam plays letters, Eve moves k tokens



Eve wins if $w \in L(\mathcal{A}) \Rightarrow$ at least one token follows an accepting run.

Explorable Automata

k-**explorability game**:
Adam plays letters, Eve moves k tokens



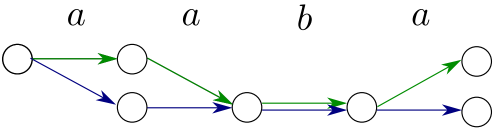
Eve wins if $w \in L(\mathcal{A}) \Rightarrow$ at least one token follows an accepting run.

$\mathcal{A}$ is k-explorable if Eve wins the k-explorability game.

Explorable Automata

***k*-explorability game**:

Adam plays letters, Eve moves k tokens



Eve wins if $w \in L(\mathcal{A}) \Rightarrow$ at least one token follows an accepting run.

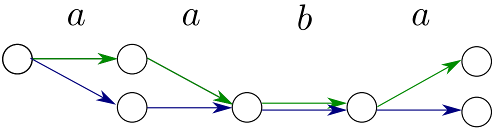
$\mathcal{A}$ is k-explorable if Eve wins the k-explorability game.

$\mathcal{A}$ is explorable if it is k-explorable for some k.

Explorable Automata

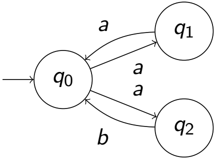
***k*-explorability game**:

Adam plays letters, Eve moves k tokens



Eve wins if $w \in L(\mathcal{A}) \Rightarrow$ at least one token follows an accepting run.

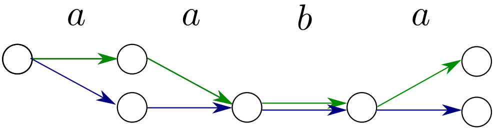
$\mathcal{A}$ is k-explorable if Eve wins the k-explorability game.
$\mathcal{A}$ is explorable if it is k-explorable for some k.



A ?-explorable safety NFA

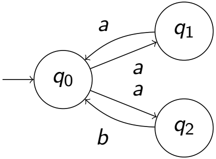
Explorable Automata

k-**explorability game**:
Adam plays letters, Eve moves k tokens



Eve wins if $w \in L(\mathcal{A}) \Rightarrow$ at least one token follows an accepting run.

$\mathcal{A}$ is k-explorable if Eve wins the k-explorability game.
$\mathcal{A}$ is explorable if it is k-explorable for some k.



A non-explorable safety NFA

First results

Theorem [K., Majumdar '18]:
Deciding $|Q|/2$-explorability is $\textsc{ExpTime}$-complete.

First results

Theorem [K., Majumdar '18]:
Deciding $|Q|/2$-explorability is $\textsc{ExpTime}$-complete.

Motivating Theorem [Hazard, K.]

The HDness problem is in $\textsc{PTime}$ for explorable automata.

First results

Theorem [K., Majumdar '18]:
Deciding $|Q|/2$-explorability is $\textsc{ExpTime}$-complete.

Motivating Theorem [Hazard, K.]

The HDness problem is in $\textsc{PTime}$ for explorable automata.

Can we decide explorability ? If yes, how efficiently ?

First results

Theorem [K., Majumdar '18]:
Deciding $|Q|/2$-explorability is $\textsc{ExpTime}$-complete.

Motivating Theorem [Hazard, K.]

The HDness problem is in $\textsc{PTime}$ for explorable automata.

Can we decide explorability ? If yes, how efficiently ?

If better than $\textsc{ExpTime}$: improve on general HDness !

First results

Theorem [K., Majumdar '18]:
Deciding $|Q|/2$-explorability is $\textsc{ExpTime}$-complete.

Motivating Theorem [Hazard, K.]

The HDness problem is in $\textsc{PTime}$ for explorable automata.

Can we decide explorability ? If yes, how efficiently ?

If better than $\textsc{ExpTime}$: improve on general HDness !

How many tokens might be needed in explorable automata ?

A related paper

Similar questions in [Betrand et al 2019: Controlling a population]

k-**population game**: Arena like *k*-explorability game on NFA,
Goal of Adam: bring all tokens to a sink state.

A related paper

Similar questions in [Betrand et al 2019: Controlling a population]

k-**population game**: Arena like *k*-explorability game on NFA,
Goal of Adam: bring all tokens to a sink state.

Population Control Problem (PCP): $\exists k$ s.t. Eve wins ?

A related paper

Similar questions in [Betrand et al 2019: Controlling a population]

k-**population game**: Arena like *k*-explorability game on NFA,
Goal of Adam: bring all tokens to a sink state.

Population Control Problem (PCP): $\exists k$ s.t. Eve wins ?

Results in [Bertrand, Dewaskar, Genest, Gimbert, Godbole]:

- The PCP is EXPTIME-complete
- Doubly exponentially many tokens might be needed.

A related paper

Similar questions in [Betrand et al 2019: Controlling a population]

k-**population game**: Arena like *k*-explorability game on NFA,
Goal of Adam: bring all tokens to a sink state.

Population Control Problem (PCP): $\exists k$ s.t. Eve wins ?

Results in [Bertrand, Dewaskar, Genest, Gimbert, Godbole]:

- The PCP is $\textsc{ExpTime}$-complete
- Doubly exponentially many tokens might be needed.

Our goal: Generalize to Explorability, but

- Game harder to solve: the input word has to be in $L(\mathcal{A})$
- Must deal with acceptance conditions on infinite words.

Results

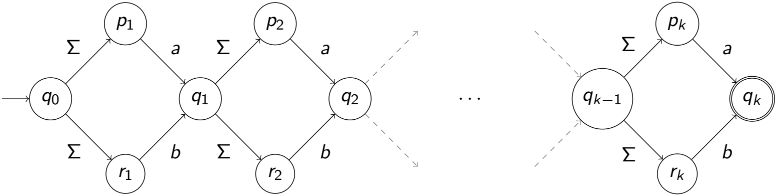
Theorems [Hazard, K.]

Explorability Problem is EXPTIME-complete for NFA, Büchi.
Doubly exponentially many tokens might be needed.

Results

Theorems [Hazard, K.]

Explorability Problem is ExpTime-complete for NFA, Büchi.
Doubly exponentially many tokens might be needed.



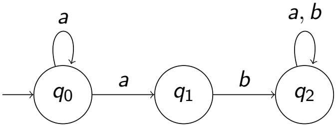
NFA needing exponentially many tokens.

ω-explorability

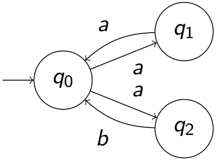
What happens if we allow a countable infinity of tokens ?

ω-explorability

What happens if we allow a countable infinity of tokens ?



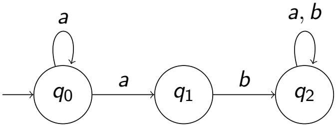
not explorable but
ω-explorable



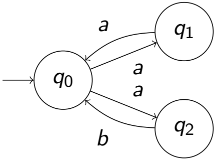
not ω-explorable

ω-explorability

What happens if we allow a countable infinity of tokens ?



not explorable but
ω-explorable



not ω-explorable

Intuition:
Non-explorable: Adam can kill a run **chosen by Eve**
Non-ω-explorable: Adam can kill a run **of its choice**

Results on ω-explorability

Facts:

- any NFA is ω-explorable,
- any automaton $\mathcal{A}$ with $L(\mathcal{A})$ countable is ω-explorable.
- any Reachability automaton is ω-explorable,

Results on ω-explorability

Facts:

- any NFA is ω-explorable,
- any automaton $\mathcal{A}$ with $L(\mathcal{A})$ countable is ω-explorable.
- any Reachability automaton is ω-explorable,

Theorem [Hazard, K.]

ω-explorability is $\textsc{ExpTime}$-complete for safety, coBüchi.

Results on ω-explorability

Facts:

- any NFA is ω-explorable,
- any automaton $\mathcal{A}$ with $L(\mathcal{A})$ countable is ω-explorable.
- any Reachability automaton is ω-explorable,

Theorem [Hazard, K.]

ω-explorability is $\textsc{ExpTime}$-complete for safety, coBüchi.

Decidability open for Büchi.

Current and future work

Internship with Olivier Idir:

- Expressivity of (ω-)expl. parity automata
- $\textsc{ExpTime}$ expl. algorithms for coBüchi, Parity $[0, 2]$
- Decidability open for Parity $[1, 3]$ (general case !)

Current and future work

Internship with Olivier Idir:

- Expressivity of (ω-)expl. parity automata
- $\textsc{ExpTime}$ expl. algorithms for coBüchi, Parity $[0, 2]$
- Decidability open for Parity $[1, 3]$ (general case !)

In the future...

- Open decidability: $[1, 3]$-expl., Büchi ω-expl.
- Complexity of k-expl. with k in binary?
- Studying HD and expl. models in other frameworks.
- Practical applications, experimental evaluations.
- $\textsc{PTime}$ HDness for parity automata.
- ...

Current and future work

Internship with Olivier Idir:

- Expressivity of (ω-)expl. parity automata
- ExpTime expl. algorithms for coBüchi, Parity $[0, 2]$
- Decidability open for Parity $[1, 3]$ (general case !)

In the future...

- Open decidability: $[1, 3]$-expl., Büchi ω-expl.
- Complexity of k-expl. with k in binary?
- Studying HD and expl. models in other frameworks.
- Practical applications, experimental evaluations.
- PTime HDness for parity automata.
- ...

Thanks for your attention!