Good-for-Games Automata versus Deterministic Automata.

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Introduction

Deterministic automata are a central tool in automata theory:

- Polynomial algorithms for inclusion, complementation.
- Safe composition with games, trees.
- Solutions of the synthesis problem (verification).
- Easily implemented.

Problems :

- exponential state blow-up
- technical constructions (Safra)

Can we weaken the notion of determinism while preserving some good properties?

Good-for-Games automata

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Introduced independently in

- symbolic representation (Henzinger, Piterman '06)
  $\rightarrow$ simplification
- qualitative models (Colcombet '09) $\rightarrow$ replace determinism

Applications

- synthesis
- branching time verification
- tree languages (Boker, K, Kupferman, S '12)

Evaluating a game

Synthesis : design a system responding to environment, while satisfying a constraint φ.

Environment: I_1
System:

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System wins iff $(I_1, O_1), (I_2, O_2), (I_3, O_3), \ldots \models \varphi$.

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Classical approach: $\varphi \rightsquigarrow \mathcal{A}_{det}$ then solve game on $\mathcal{A}_{det}$.

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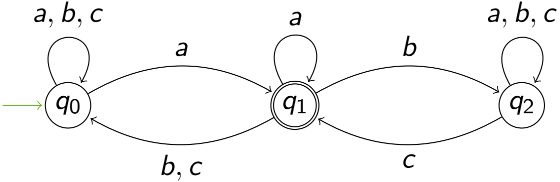
Wrong approach: $\varphi \rightsquigarrow \mathcal{A}_{non-det}$: no player can guess the future.
New approach: $\varphi \rightsquigarrow \mathcal{A}_{GFG}$.

Definition via a game

$\mathcal{A}$ automaton on finite or infinite words.

Refuter plays letters:

Prover: controls transitions



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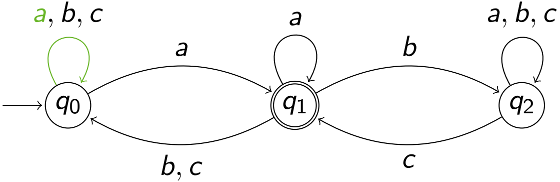

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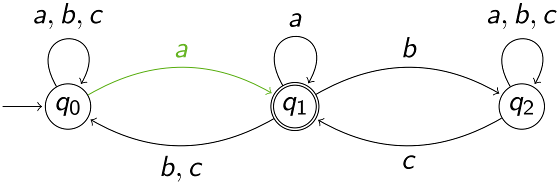

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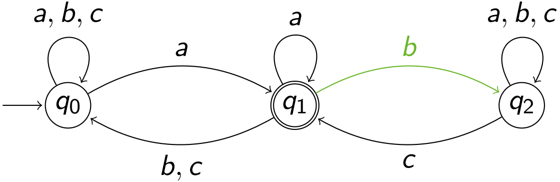

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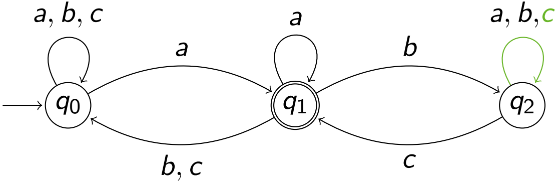

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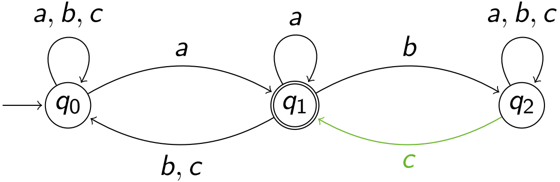

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Refuter plays letters: $a \;\; a \;\; b \;\; c \;\; c \; \dots \quad = w$

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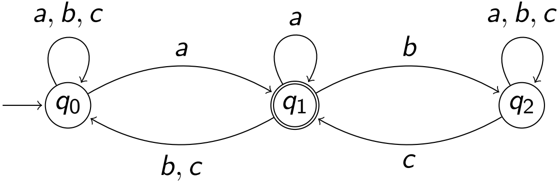

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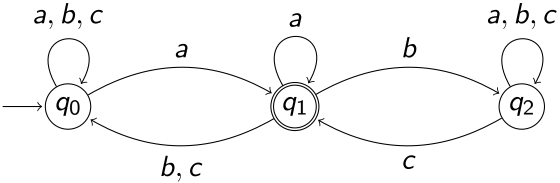

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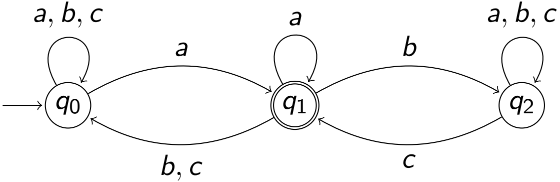

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How close is this to determinism?

Some properties of GFG automata

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Theorem (Boker, K, Kupferman, Skrzypczak, ICALP '12)

If $\mathcal{A}$ and $\mathcal{B}$ are **GFG** *for L and $\overline{L}$, there is a deterministic automaton for L of size $|\mathcal{A}| \cdot |\mathcal{B}|$.*

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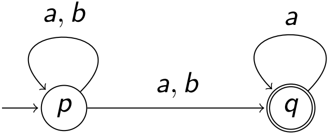
Theorem (Boker, K, Kupferman, Skrzypczak, ICALP '12)

Let $\mathcal{A}$ be an automaton for $L \subseteq A^\omega$. Then the tree version of $\mathcal{A}$ recognizes $\{t : \text{all branches of } t \text{ are in } L\}$ if and only if $\mathcal{A}$ is **GFG***.*

An automaton that is not GFG

This automaton for $L = (a+b)^* a^\omega$ is not **GFG**:



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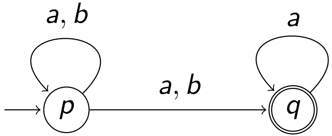

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Fact

GFG *automata with condition C have same expressivity as deterministic automata with condition C.*

Therefore, **GFG** could improve succinctness but not expressivity.

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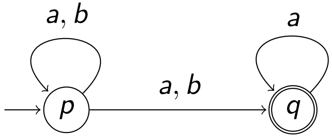

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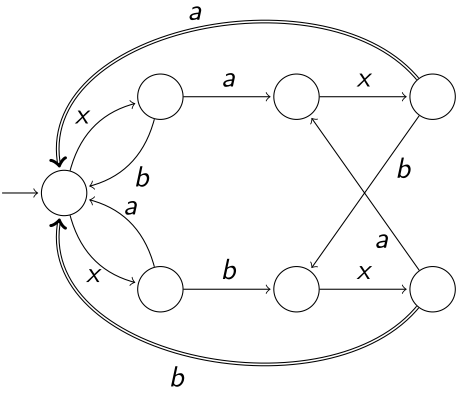
Therefore, **GFG** could improve succinctness but not expressivity.
But **GFG** on finite words $\iff$ deterministic (+useless transitions).
What about infinite words ?

A GFG Büchi example

Büchi condition: Run is accepting if infinitely many Büchi transitions are seen.



Language: $[(xa + xb)^*(xaxa + xbxb)]^\omega$

Determinization of Büchi GFG

Theorem

Let $\mathcal{A}$ a **GFG** *Büchi automaton. There exists a deterministic Büchi automaton $\mathcal{B}$ with $L(\mathcal{B}) = L(\mathcal{A})$ and $|\mathcal{B}| \leq |\mathcal{A}|^2$.*

Proof scheme:

- Use brutal powerset determinisation,
- rank signatures of Walukiewicz
- iterative normalization of $\mathcal{A}$
- dependency graph over the automaton

Conclusion: the automaton can use itself as memory structure $\Rightarrow$ quadratic blow-up only.

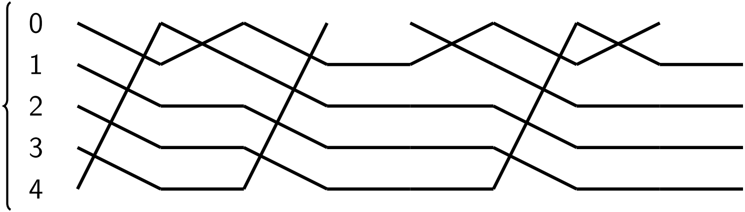
Is it true for all ω-regular conditions?

CoBüchi counter-example : the language L_n

n paths: σ, π permute paths, $\sharp$ cuts the current 0-path.
Here for $n = 5$:

α: σ π σ $\sharp$ π σ π $\sharp$

DAG:



time: 0 1 2 3 4 5 6 7 8

The word α is in L_n if it contains an infinite path.

Automaton for L_n

GFG coBüchi automaton for L_n with n states:

- letters σ and π permute states deterministically.
- letter $\sharp$:
  - state $0 \longrightarrow$ go anywhere but pay a coBüchi (must be finitely many times)
  - states $1, \ldots, n$: do nothing

Strategy σ: try paths one after the other. Uses memory 2^n, to ensure that all paths are visited.

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Theorem

Any deterministic automaton for L_n has $\Omega(2^n)$ states.

CoBüchi (and parity) **GFG** automata can provide both succinctness and sound behaviour with respect to games.

Question: Can we effectively use them?

Recognizing GFG automata

Question: Given an automaton $\mathcal{A}$, is it **GFG**?

Theorem

The complexity of deciding **GFG***-ness is in*

- **NP** *for Büchi automata*
- **P** *for coBüchi automata (involved proof)*
- *at least as hard as solving parity games (***NP** $\cap$ *co***NP***) for parity automata.*

Open Problems

- Is it in **P** for any fixed acceptance condition?
- Is it equivalent to parity games in the general case?

Summary and conclusion

Results

- **GFG** automata capture good properties of deterministic automata.
- Inclusion is in **P**, but Complementation $\sim$ Determinisation.
- Conditions Büchi and lower: **GFG** $\approx$ Deterministic.
- Conditions coBüchi and higher: exponential succinctness.
- Recognizing **GFG** coBüchi is in **P**.

Open Problems

- Can we build small **GFG** automata in a systematic way?
- Complexity of deciding **GFG**-ness for parity automata?
- Exact cost of Büchi **GFG** determinisation?