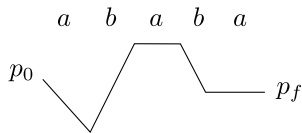# Explorable Automata

Emile Hazard, Olivier Idir, Denis Kuperberg
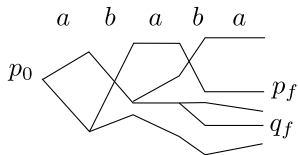
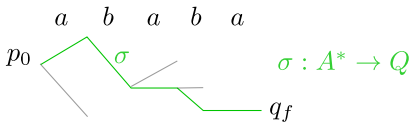Highlights 2024, Bordeaux, September 20th 2024

History-Deterministic Automata

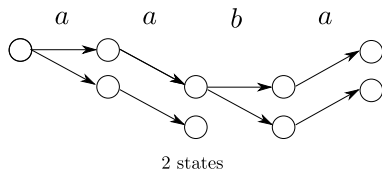
Deterministic

Non-deterministic

History-Deterministic
(a.k.a. Good-for-Games)

$\sigma : A^* \to Q$

Allowing more runs

Idea: Allow to build several runs, at least one accepting.

1 state - 1 run

HD

2 states
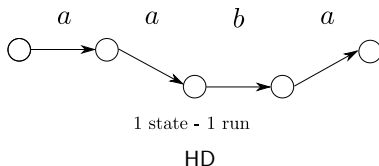
Width 2

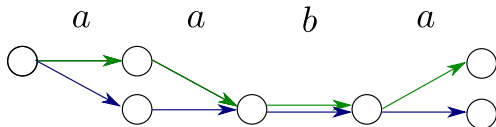[K. + Majumdar '18 '19]

2 runs
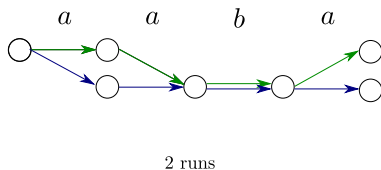
2-Explorable

[This work]

Explorable Automata

k-**explorability game**:
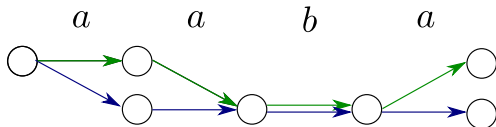Adam plays letters, Eve moves *k* tokens

Eve wins if $w \in L(\mathcal{A}) \Rightarrow$ at least one token follows an accepting run.

Explorable Automata

k-**explorability game**:
Adam plays letters, Eve moves k tokens

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$\mathcal{A}$ is k-explorable if Eve wins the k-explorability game.

Explorable Automata

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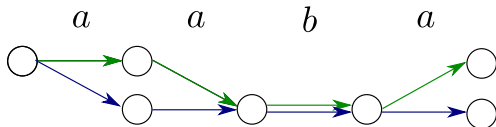Adam plays letters, Eve moves k tokens

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$\mathcal{A}$ is explorable if it is k-explorable for some k.

Explorable Automata

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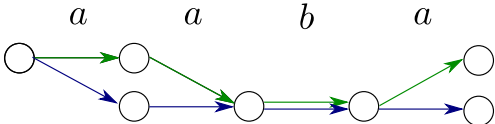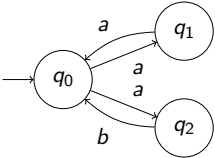
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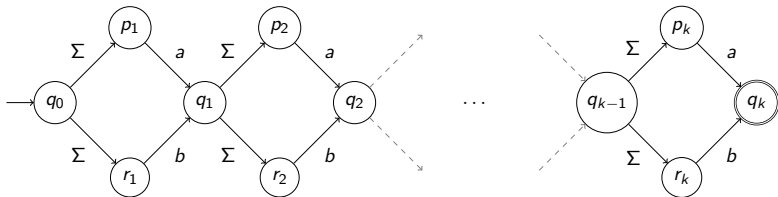A non-explorable safety NFA

Results

Theorems [Hazard, K. 2023]

Explorability is ExpTime-complete for NFA, Büchi.
Doubly exponentially many tokens might be needed.

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>
> Explorability is EXPTIME-complete for NFA, Büchi.
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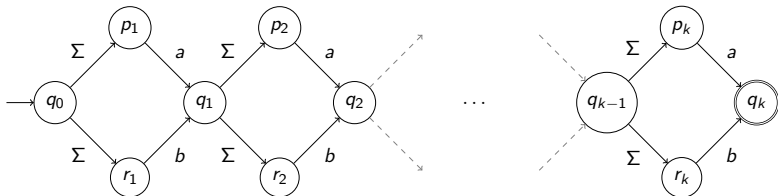NFA needing exponentially many tokens.

Results

<div style="border:1px solid green">

Theorems [Hazard, K. 2023]

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</div>

NFA needing exponentially many tokens.
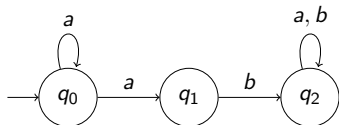
<div style="border:1px solid green">

Theorems [Idir, K.]

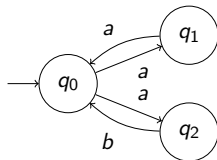Explorability is ExpTime for coBüchi, $[0, 2]$-Parity.

</div>

ω-**explorability**

What happens if we allow a countable infinity of tokens ?

ω-**explorability**
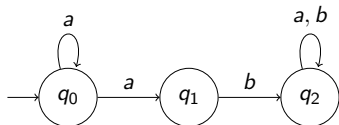
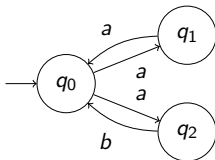What happens if we allow a countable infinity of tokens ?

not explorable but
ω-explorable

not ω-explorable

ω-**explorability**

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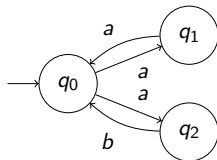**Intuition**:
Non-ω-explorable: Adam can always kill any run

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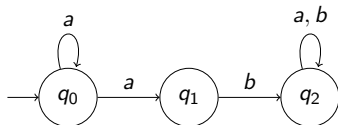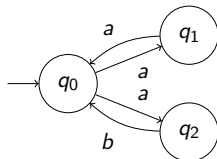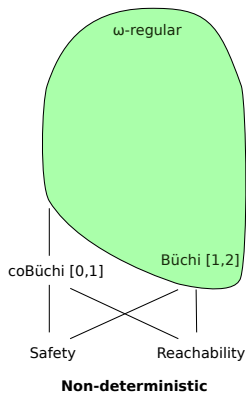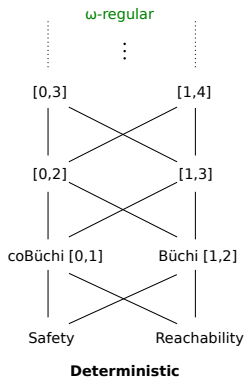
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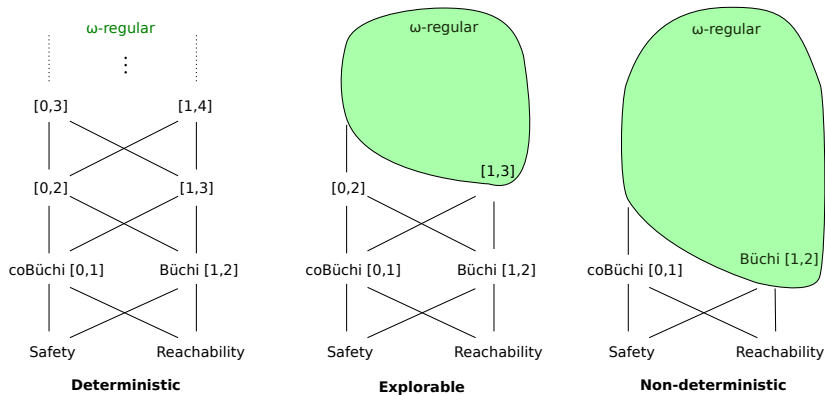
> Theorem [Hazard, K. 2023]
>
> ω-explorability is EXPTIME-complete for safety, coBüchi.

Decidability open for Büchi.

Expressivity of explorable automata
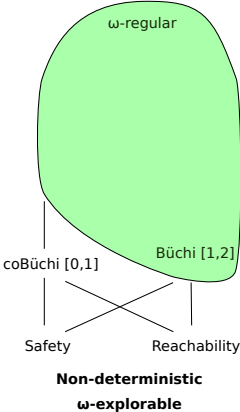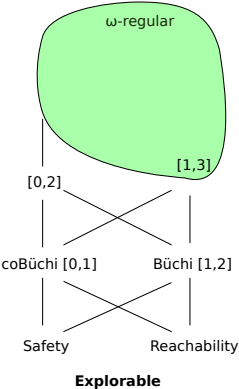

Deterministic

Non-deterministic

Expressivity of explorable automata

Expressivity of explorable automata

Deterministic

Explorable

Non-deterministic
ω-explorable

Expressivity of explorable automata

Deterministic   **Explorable**   **Non-deterministic** / **ω-explorable**
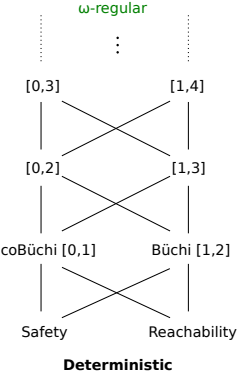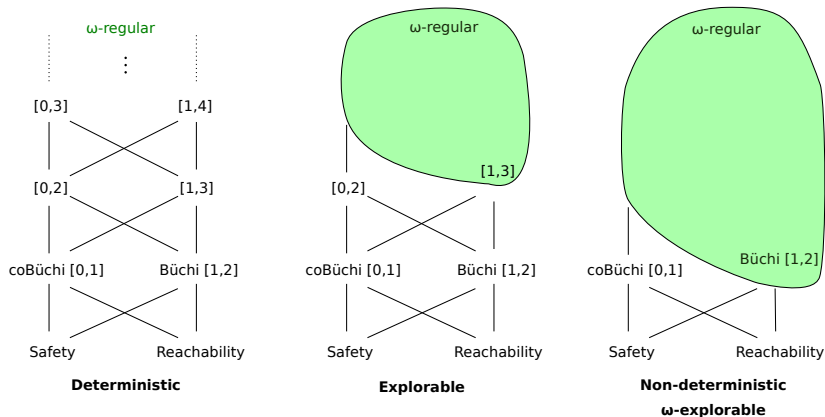
Theorem (Idir, K.)

$[1,3]$-*explorability decidable* $\Leftrightarrow$ *Parity explorability decidable*
Büchi ω-*explorability decidable* $\Leftrightarrow$ *Parity* ω-*explorability decidable*

Future work

- Open decidability: $[1,3]$-expl., Büchi ω-expl.
- Complexity of k-expl. with k in binary?
- Studying HD and expl. models in other frameworks.
- Practical applications, experimental evaluations.
- PTIME HDness for parity automata.
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Thanks for your attention!