Extended symmetry in a 4-leg spin tube

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Outline

1. N-leg ladders, cylinders and spin gap
2. Symmetry enlargement in one dimension
3. 4 leg tube and conformal field theory
4. 4 leg tube and abelian bosonization
5. conclusion

Haldane gap

Haldane (1983)

Antiferromagnetic spin S chains

$$H = J \sum_j \vec{S}_j \vec{S}_{j+1}$$

- have a gap above the ground state when $2S$ is even
- have gapless excitations described by abelian bosonization when $2S$ is odd

Haldane gap and topology

Semiclassical Lagrangian

$$S = \int dx d\tau \left[u(\partial_x \vec{n})^2 + \frac{(\partial_\tau \vec{n})^2}{u} \right] + i\frac{S}{2} \int dx d\tau \vec{n} \cdot (\partial_x \vec{n} \times \partial_\tau \vec{n})$$

$$Z = \int D\vec{n}(x,\tau) e^S$$

$S\vec{n}$ = staggered magnetization in the spin chain

- $2S$ even: no contribution from the topological term
- $2S$ odd: equivalent to the Lagrangian of the spin-1/2 chain

Analogy with N-leg ladders

Hamiltonian of the N-leg ladder

$$H = J_{\|} \sum_{j,n} \vec{S}_{j,n} \cdot \vec{S}_{j+1,n} + J_{\perp} \sum_{j,n} \vec{S}_{j,n} \cdot \vec{S}_{j,n+1}$$

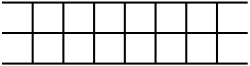
topological term with N coupled spin-1/2 chains

$$\frac{i}{4} \sum_{j=1}^{N} \int dx d\tau \vec{n}_j \cdot (\partial_x \vec{n}_j \times \partial_\tau \vec{n}_j)$$

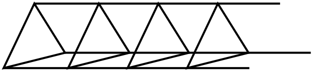
[G. Sierra J. Phys. A **29**, 3299 (1996)]

- Ferromagnetic interchain coupling: $\vec{n}_j = \vec{n}_1 \Rightarrow$ spin $N/2$ case
- Antiferromagnetic interchain coupling $\vec{n}_j = (-)^{j-1} \vec{n}_1 \Rightarrow$ cancellation if N is even, as in single chain if N is odd.

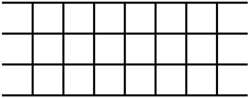
$N > 2$: Planar versus tube geometry



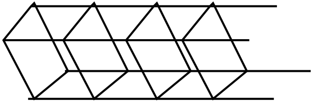
3 leg, planar



3 leg tube



4 leg, planar



4 leg tube

$$J_\perp \sum_{n=1}^{N-1} \vec{S}_{j,n} \cdot \vec{S}_{j,n+1} + J_\perp \vec{S}_{j,N} \cdot \vec{S}_{j,1}$$

The topological term does not distinguish the two cases

spin tubes

odd N

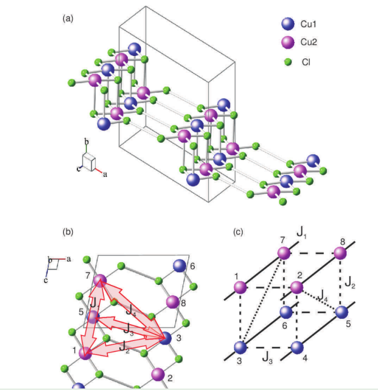
- The planar ladder is gapless
- the 3-leg tube is gapped as a result of frustration [Kawano & Takahashi J. Phys. Soc. Jpn. (1997)]

even N

The spin tube is gapped as the planar ladder

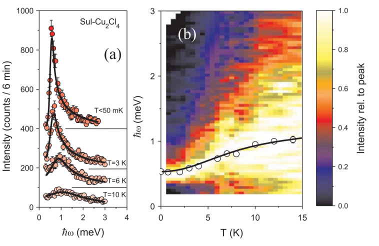
A frustrated four-leg spin tube

Sul-Cu_2Cl_4



Structure after Glazkov, JETP **131**, 46 (2020)

$\Delta = 0.55\text{meV} \quad u = 14\text{meV}$



Dispersion measured in neutron scattering after Zheludev *et al.* Phys. Rev. Lett. **100**, 157204 (2008)

The four leg spin tube

Hamiltonian of the spin-1/2 4-leg tube

$$
\begin{aligned}
H &= J_{\parallel} \sum_{n=1}^{4} \sum_{j} \vec{S}_{j,n} \cdot \vec{S}_{j+1,n} \\
&\quad + J_{\perp} \sum_{j} (\vec{S}_{j,1} \cdot \vec{S_{j,2}} + \vec{S}_{j,2} \cdot \vec{S_{j,3}} + \vec{S}_{j,3} \cdot \vec{S_{j,4}} + \vec{S}_{j,4} \cdot \vec{S_{j,1}}), \\
&= J_{\parallel} \sum_{n=1}^{4} \sum_{j} \vec{S}_{j,n} \cdot \vec{S}_{j+1,n} \\
&\quad + J_{\perp} \sum_{j} (\vec{S}_{j,1} + \vec{S}_{j,3}) \cdot (\vec{S}_{j,2} + \vec{S}_{j,4})
\end{aligned}
$$

Non-abelian bosonization approach

Low-energy Hamiltonian

$$
\begin{aligned}
\vec{S}_{j,p} &= a(\vec{J}_{R,p} + \vec{J}_{L,p})(ja) + (-)^j \lambda \vec{n}_p(ja) \\
\mathcal{H} &= \sum_{p=1}^{4} \frac{2\pi u}{3} \int dx (\vec{J}_{R,p} \cdot \vec{J}_{R,p} + \vec{J}_{L,p} \cdot \vec{J}_{L,p}) \\
&\quad + \frac{J_\perp \lambda^2}{a} \int dx (\vec{n}_1 + \vec{n}_3) \cdot (\vec{n}_2 + \vec{n}_4) \\
&\quad + \frac{J_\perp}{a} \int dx \sum_{\nu,\nu'} (\vec{J}_{\nu,1} + \vec{J}_{\nu,3}) \cdot (\vec{J}_{\nu',2} + \vec{J}_{\nu',4}) \\
[J^a_\nu(x), J^b_\nu(x')] &= ik\delta_{ab}\partial_x[\delta(x-x')] + i\delta(x-x')\epsilon_{abc}J^c_\nu(x')
\end{aligned}
$$

Conformal embeddings

Odd/Even modes

$$
\begin{aligned}
SU(2)_1 \times SU(2)_1 &\sim SU(2)_2 \times \mathrm{Ising} \\
\vec{J}_{\nu,1} + \vec{J}_{\nu,3} &= \vec{I}_{\nu,odd} \\
\vec{n}_1 + \vec{n}_3 &\sim \mu_{odd} \times \vec{N}_{odd}, \\
\vec{J}_{\nu,2} + \vec{J}_{\nu,4} &= \vec{I}_{\nu,even} \\
\vec{n}_2 + \vec{n}_4 &\sim \mu_{even} \vec{N}_{even},
\end{aligned}
$$

Magnetic modes [Georges and Sengupta PRL**74**, 2808 (1995)]

$$
SU(2)_{2,even} \times SU(2)_{2,odd} \sim \frac{SU(2)_2 \times SU(2)_2}{SU(2)_4} \times SU(2)_4,
$$

The coset $A(2,2) = \frac{SU(2)_2 \times SU(2)_2}{SU(2)_4}$

$\mathcal{N} = 1$ supersymmetric minimal series

cosets $\frac{SU(2)_k \times SU(2)_2}{SU(2)_{k+2}}$ [Di Francesco *et al. Conformal Field Theory*]

$c = \frac{3k}{k+2} + \frac{3}{2} - \frac{3(k+2)}{k+4} = \frac{3}{2}\left[1 - \frac{8}{(k+2)(k+4)}\right]$

for $k = 2$ we find $A(2,2)$ with $c = 1$

Primary operators Φ_{rs} with weights in the Kac table

Bosonized representation by E. B. Kiritzis J. Phys. A **21**, 297 (1998)

s \ r	1	2	3
1	0	$\frac{3}{8}$	1
2	$\frac{1}{16}$	$\frac{1}{16}$	$\frac{9}{16}$
3	$\frac{1}{6}$	$\frac{1}{24}$	$\star$

Rewriting the interactions

With Odd/Even representation

$$\frac{J_\perp}{a}\sum_{\nu,\nu'}(\vec{J}_{\nu,1}+\vec{J}_{\nu,3})\cdot(\vec{J}_{\nu',2}+\vec{J}_{\nu',3})=\frac{J_\perp}{a}\sum_{\nu,\nu'}\vec{I}_{\nu,odd}\cdot\vec{I}_{\nu,even}$$

$$\frac{J_\perp\lambda^2}{a}(\vec{n}_1+\vec{n}_3)\cdot(\vec{n}_2+\vec{n}_4)\sim\frac{J_\perp\lambda^2}{a}\mu_{odd}\mu_{even}\vec{N}_{odd}\cdot\vec{N}_{even}$$

With coset representation of magnetic modes

$$\vec{N}_{odd}\cdot\vec{N}_{even}\sim\Phi_{(21)}+\Phi_{(23)}\Phi^{(1)}_{SU(2)_4},$$

$\Phi^{(1)}_{SU(2)_4}$ = spin-1 primary fields. With Kiritzis (1998)

$$\vec{N}_{odd}\cdot\vec{N}_{even}\sim\cos(\sqrt{3}\phi_c)+\cos\left(\frac{\phi_c}{\sqrt{3}}\right)\Phi^{(1)}_{SU(2)_4},$$

The embedding $SU(3)_1 \subset SU(2)_4$

$SU(2)_4$ primaries and currents

- 3 currents $J_\nu^{x,y,z}$
- $2s+1$ primaries having spin $s \leq 2$
- conformal weight of spin s primary $h_s = \frac{s(s+1)}{4+2}$
    - $s = 1/2 : h_{1/2} = 1/8$ (2 fields)
    - $s = 1 : h_1 = 1/3$ (3 fields)
    - $s = 3/2 : h_{3/2} = 5/8$ (4 fields)
    - $s = 2 : h_2 = 1$ (5 fields)

$SU(3)_1$ primaries and currents

- 8 currents $J_\nu^{x,y,z}$
- 3 primary fields of conformal weight $1/3$
    - $s = 1 : h_1 = 1/3$ in $SU(2)_4 \to SU(3)_1$ primaries
    - $s = 2 : h_2 = 1$ in $SU(2)_4 \to SU(3)_1$ currents

$$\Phi^{(1)}_{SU(2)_4} \equiv \mathrm{Tr}(g_{SU(3)_1})$$

Final form of interaction

Relevant term is $SU(3)$ singlet

$$(\vec{n}_1 + \vec{n}_3) \cdot (\vec{n}_2 + \vec{n}_4) \sim \left[\cos(\sqrt{3}\phi_c) + \cos\left(\frac{\phi_c}{\sqrt{3}}\right) \mathrm{Tr}(g_{SU(3)_1})\right] \times \mu_{even}\mu_{odd}.$$

Marginal term is $SU(2)$ singlet

$$\sum_{\nu,\nu'} (\vec{J}_{\nu,1} + \vec{J}_{\nu,3}) \cdot (\vec{J}_{\nu',2} + \vec{J}_{\nu',4}) = \sum_{\nu,\nu'} \vec{I}_{\nu,odd} \cdot \vec{I}_{\nu',even}.$$

Up to logarithmic corrections, the spectrum should be formed of $SU(3)$ multiplets

Contrast with symmetry restoration

[Lin, Balents, Fisher Phys. Rev. B 58 1794 (1998)]

In a 2-leg fermion ladder at half-filling, under RG the Hamiltonian flows to the $SO(8)$ Gross-Neveu model.

Symmetry restoration

- Only marginal operators
- flow of coupling constants towards an asymptotic line of high symmetry

4-leg cylinder

- Both relevant and marginal operators
- relevant operators preserve exactly $SU(3)$.
- the marginal operators lower to $SU(2)$

Correlation functions revealing the $SU(3)$ symmetry

Decomposition of staggered magnetization

$$\vec{N}_{odd/even} \sim [\alpha_{odd/even}\Phi_{(1,2)} + \beta_{odd/even}\Phi_{(2,2)}]\mathrm{Tr}(g^{(1/2)}_{SU(2)_4}\vec{\sigma})$$

$\mathrm{Tr}(g^{(1/2)}_{SU(2)_4}\vec{\sigma})$ is not a primary operator of $SU(3)_1$.
$\Rightarrow$ spin-spin correlations cannot reveal the $SU(3)$ symmetry.
$\Rightarrow$ Need to consider tensor products

Abelian bosonization approach

Hamiltonian of decoupled chains

$$\mathcal{H}_0 = \sum_{j=1}^{4} \int \frac{dx}{2\pi} u \left[(\pi \Pi_j)^2 + (\partial_x \phi_j)^2 \right],$$

$$[\phi_j(x), \Pi_k(x')] = i\delta_{jk}\delta(x - x')$$

Bosonized staggered operators

$$\begin{aligned}
\partial_x \theta_p &= \pi \Pi_p \\
n_p^+(x) &= (n_p^x + i n_p^y)(x) = e^{-i\sqrt{2}\theta_p(x)}, \\
n_p^z(x) &= \sin \sqrt{2}\phi_p(x), \\
\epsilon_p(x) &= \cos \sqrt{2}\phi_p(x),
\end{aligned}$$

Rotation of the boson fields

[Strong and Millis Phys. Rev. Lett. **69**, 2419 (1992)]

$$\theta_{o,\pm} = \frac{1}{\sqrt{2}}(\theta_1 \pm \theta_3)\ \phi_{o,\pm} = \frac{1}{\sqrt{2}}(\phi_1 \pm \phi_3)$$
$$\theta_{e,\pm} = \frac{1}{\sqrt{2}}(\theta_2 \pm \theta_4)\ \phi_{e,r} = \frac{1}{\sqrt{2}}(\phi_2 \pm \phi_4)$$

Rewriting the staggered operators

$$n_1^+ + n_3^+ = 2e^{i\theta_{o,+}} \cos\theta_{o,-},$$
$$n_1^3 + n_3^3 = 2\sin\phi_{o,+} \cos\phi_{o,-},$$
$$n_2^+ + n_4^+ = 2e^{i\theta_{e,+}} \cos\theta_{e,-},$$
$$n_2^3 + n_4^3 = 2\sin\phi_{e,+} \cos\phi_{e,-},$$

Majorana fermion representation [Shelton, Nersesyan, and Tsvelik, Phys. Rev. B 53, 8521 (1996)]

Majorana fermion operators

$$
\begin{aligned}
\frac{1}{\sqrt{2}}(\zeta_{R/L,\nu,1} + i\zeta_{R/L,\nu,2}) &= \frac{e^{i(\theta_{\nu+} \mp \phi_{\nu+})}}{\sqrt{2\pi\alpha}}, \\
\frac{1}{\sqrt{2}}(\zeta_{R/L,\nu,3} + i\zeta_{R/L,\nu,0}) &= \frac{e^{i(\theta_{\nu-} \mp \phi_{\nu-})}}{\sqrt{2\pi\alpha}},
\end{aligned}
$$

Hamiltonian of decoupled chains

$$
\mathcal{H}_0 = -i\frac{u}{2}\int dx \sum_{\substack{\nu=e,o \\ j=0,1,2,3}} (\zeta_{R,\nu,j}\partial_x\zeta_{R,\nu,j} - \zeta_{L,\nu,j}\partial_x\zeta_{L,\nu,j}),
$$

Ising order/disorder parameter representation [Shelton, Nersesyan, and Tsvelik, Phys. Rev. B 53, 8521 (1996)]

Staggered magnetization operators

$$
\begin{aligned}
(n_1+n_3)^1 &= \mu_{o,1}\sigma_{o,2}\sigma_{o,3}\mu_{o,0} \; (n_2+n_4)^1 = \mu_{e,1}\sigma_{e,2}\sigma_{e,3}\mu_{e,0} \\
(n_1+n_3)^2 &= \sigma_{o,1}\mu_{o,2}\sigma_{o,3}\mu_{o,0} \; (n_2+n_4)^2 = \sigma_{e,1}\mu_{e,2}\sigma_{e,3}\mu_{e,0} \\
(n_1+n_3)^3 &= \sigma_{o,1}\sigma_{o,2}\mu_{o,3}\mu_{o,0} \; (n_2+n_4)^3 = \sigma_{e,1}\sigma_{e,2}\mu_{e,3}\mu_{e,0}
\end{aligned}
$$

Relevant interaction

$$
\frac{J_\perp \lambda^2}{a} \int dx \mu_{e,0}\mu_{o,0} \left[\mu_{o,1}\mu_{e,1}\sigma_{o,2}\sigma_{e,2}\sigma_{o,3}\sigma_{e,3} \right.
$$
$$
\left. + \mu_{o,2}\mu_{e,2}\sigma_{o,3}\sigma_{e,3}\sigma_{o,1}\sigma_{e,1} + \mu_{o,3}\mu_{e,3}\sigma_{o,1}\sigma_{e,1}\sigma_{o,2}\sigma_{e,2}\right].
$$

Pairing the odd/even Majorana fermions

New fermion fields and bosonization

$$\psi_{R,j} = \frac{1}{\sqrt{2}}(\zeta_{R,e,j} + i\zeta_{R,o,j}) = \frac{e^{i\vartheta_j - i\varphi_j}}{\sqrt{2\pi\alpha}}\eta_j,$$

$$\psi_{L,j} = \frac{1}{\sqrt{2}}(\zeta_{L,e,j} + i\zeta_{L,o,j}) = \frac{e^{i\vartheta_j + i\varphi_j}}{\sqrt{2\pi\alpha}}\eta_j.$$

Order/disorder Ising operators

$$\mu_{e,j}\mu_{o,j} \sim \cos\varphi_j \; \sigma_{e,j}\sigma_{o,j} \sim \sin\varphi_j$$

$$\sigma_{e,j}\mu_{o,j} \sim \cos\vartheta_j \; \mu_{e,j}\sigma_{o,j} \sim \sin\vartheta_j$$

Bosonized interchain coupling

Relevant interaction

$$(\vec{n}_1 + \vec{n}_3) \cdot (\vec{n}_2 + \vec{n}_4) = \cos\varphi_0 \left[-3\cos(\varphi_1 + \varphi_2 + \varphi_3) + \cos(\varphi_3 + \varphi_1 - \varphi_2) + \cos(\varphi_2 + \varphi_3 - \varphi_1) \cos(\varphi_1 + \varphi_2 - \varphi_3) \right],$$

Linear change of variables

$$\begin{pmatrix} \varphi_c \\ \varphi_a \\ \varphi_b \end{pmatrix} = \begin{pmatrix} \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \\ \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{6}} & -\frac{2}{\sqrt{6}} \end{pmatrix} \begin{pmatrix} \varphi_1 \\ \varphi_2 \\ \varphi_3 \end{pmatrix}$$

Resulting bosonized Hamiltonian

Including only the most relevant term

$$\mathcal{H} = \sum_{\nu=0,a,b,c} \int \frac{dx}{2\pi} u \left[(\partial_x \vartheta_\nu)^2 + (\partial_x \varphi_\nu)^2 \right] + \frac{J_\perp \lambda^2}{a} \cos\varphi_0 \left[2\cos\left(\frac{\varphi_c}{\sqrt{3}} - \sqrt{\frac{2}{3}} \varphi_b \right) \cos\sqrt{2}\varphi_a + \cos\left(\frac{\varphi_c}{\sqrt{3}} + 2\sqrt{\frac{2}{3}} \varphi_b \right) - 3\cos\sqrt{3}\varphi_c \right],$$

Making the $SU(3)$ symmetry apparent

Fermionized form of interchain interaction

$$\left[2\cos\left(\frac{\varphi_c}{\sqrt{3}} - \sqrt{\frac{2}{3}}\varphi_b\right)\cos\sqrt{2}\varphi_a + \cos\left(\frac{\varphi_c}{\sqrt{3}} + 2\sqrt{\frac{2}{3}}\varphi_b\right)\right]$$

$$= \sum_{j=1}^{3}(e^{i\sqrt{3}\varphi_c}e^{-2i\varphi_j} + \mathrm{H.c.}) \propto e^{i\sqrt{3}\varphi_c}\sum_{j=1}^{3} i\psi^\dagger_{R,j}\psi_{L,j} - \mathrm{H.c.}$$

$$-\frac{\sqrt{3}}{\pi}\partial_x\varphi_c = \sum_j(\psi^\dagger_{R,j}\psi_{R,j} + \psi^\dagger_{L,j}\psi_{L,j})$$

Invariant under $\psi_{R/L,j} = U_{jj'}\tilde{\psi}_{R/L,j'}$ with $U \in SU(3)$.

Observables: nematic order parameter

Nematic order parameter (symmetric combination)

$$
\begin{aligned}
2Q_{++}^{ab} &= (n_1^a + n_3^a)(n_2^b + n_4^b) + (n_1^b + n_3^b)(n_2^a + n_4^a) \\
&\quad -\delta_{ab}(\vec{n}_1 + \vec{n}_3) \cdot (\vec{n}_2 + \vec{n}_4)/3 \\
Q_{++}^{12} &\sim i \cos\varphi_0 \left[e^{-i\sqrt{3}\varphi_c} \Psi_R^\dagger \Lambda_1 \Psi_L - e^{i\sqrt{3}\varphi_c} \Psi_L^\dagger \Lambda_1 \Psi_R \right], \\
Q_{++}^{23} &\sim i \cos\varphi_0 \left[e^{-i\sqrt{3}\varphi_c} \Psi_R^\dagger \Lambda_6 \Psi_L - e^{i\sqrt{3}\varphi_c} \Psi_L^\dagger \Lambda_6 \Psi_R \right], \\
Q_{++}^{13} &\sim i \cos\varphi_0 \left[e^{-i\sqrt{3}\varphi_c} \Psi_R^\dagger \Lambda_4 \Psi_L - e^{i\sqrt{3}\varphi_c} \Psi_L^\dagger \Lambda_4 \Psi_R \right], \\
Q_{++}^{11} - Q_{++}^{12} &\sim i \cos\varphi_0 \left[e^{-i\sqrt{3}\varphi_c} \Psi_R^\dagger \Lambda_3 \Psi_L - e^{i\sqrt{3}\varphi_c} \Psi_L^\dagger \Lambda_3 \Psi_R \right], \\
Q_{++}^{33} &\sim i \cos\varphi_0 \left[e^{-i\sqrt{3}\varphi_c} \Psi_R^\dagger \Lambda_8 \Psi_L - e^{i\sqrt{3}\varphi_c} \Psi_L^\dagger \Lambda_8 \Psi_R \right],
\end{aligned}
$$

Observables: completing the octet

dimerization × staggered magnetization

$$(n_2+n_4)^1(\epsilon_1+\epsilon_3)+(n_1+n_3)^1(\epsilon_2+\epsilon_4)$$
$$\sim \cos\varphi_0\left[e^{-i\sqrt{3}\varphi_c}\Psi_R^\dagger\Lambda_7\Psi_L+e^{i\sqrt{3}\varphi_c}\Psi_L^\dagger\Lambda_7\Psi_R\right],$$
$$(n_2+n_4)^2(\epsilon_1+\epsilon_3)+(n_1+n_3)^2(\epsilon_2+\epsilon_4)$$
$$\sim \cos\varphi_0\left[e^{-i\sqrt{3}\varphi_c}\Psi_R^\dagger\Lambda_5\Psi_L+e^{i\sqrt{3}\varphi_c}\Psi_L^\dagger\Lambda_5\Psi_R\right],$$
$$(n_2+n_4)^3(\epsilon_1+\epsilon_3)+(n_1+n_3)^3(\epsilon_2+\epsilon_4)$$
$$\sim \cos\varphi_0\left[e^{-i\sqrt{3}\varphi_c}\Psi_R^\dagger\Lambda_2\Psi_L+e^{i\sqrt{3}\varphi_c}\Psi_L^\dagger\Lambda_2\Psi_R\right],$$

Combined with the nematic order parameter, these operators belong the the 8 representation of $SU(3)$

Observables: Nematic order parameter

Nematic order parameter (asymmetric combination)

$$Q^{ab}_{-+} = (n_1 - n_3)^a (n_2 + n_4)^b + (n_1 - n_3)^b (n_2 + n_4)^a$$

$$Q^{12}_{-+} \sim \sin\vartheta_0 \left[e^{-i\sqrt{3}\theta_c} \Psi_R \Lambda_1 \Psi_L + \mathrm{H.c.} \right],$$

$$Q^{23}_{-+} \sim \sin\vartheta_0 \left[e^{-i\sqrt{3}\theta_c} \Psi_R \Lambda_6 \Psi_L + \mathrm{H.c.} \right],$$

$$Q^{31}_{-+} \sim \sin\vartheta_0 \left[e^{-i\sqrt{3}\theta_c} \Psi_R \Lambda_4 \Psi_L + \mathrm{H.c.} \right],$$

$$Q^{11}_{-+} - Q^{22}_{-+} \sim \sin\vartheta_0 \left[e^{-i\sqrt{3}\theta_c} \Psi_R \Lambda_3 \Psi_L + \mathrm{H.c.} \right],$$

$$Q^{11}_{-+} + Q^{22}_{-+} - 2Q^{33}_{-+} \sim \sin\vartheta_0 \left[e^{-i\sqrt{3}\theta_c} \Psi_R \Lambda_8 \Psi_L + \mathrm{H.c.} \right],$$

$$(\vec{n}_1 - \vec{n}_3) \cdot (\vec{n}_2 + \vec{n}_4) \sim \sin\vartheta_0 \left[e^{-i\sqrt{3}\theta_c} \Psi_R \Psi_L + \mathrm{H.c.} \right],$$

Observables: $SU(2)_4$ Currents

Magnetization current and density

$$\sum_{n=1}^{4} J^x_{\nu,n} = \sum_{j,k} \psi^\dagger_{\nu,j} \Lambda^7_{jk} \psi_{\nu,k},$$

$$\sum_{n=1}^{4} J^y_{\nu,n} = -\sum_{j,k} \psi^\dagger_{\nu,j} \Lambda^5_{jk} \psi_{\nu,k},$$

$$\sum_{n=1}^{4} J^z_{\nu,n} = \sum_{j,k} \psi^\dagger_{\nu,j} \Lambda^2_{jk} \psi_{\nu,k}$$

$\Lambda^{2,5,7}$ are 3×3 Gell-Mann matrices generating a $SU(2)$ subalgebra of $SU(3)$.

$SU(3)_1$ currents

The 5 remaining $SU(3)$ currents

$$T_\nu^{1,3,4,6,8} = \sum_{j,k} \psi^\dagger_{\nu,j} \Lambda^{1,3,4,6,8}_{jk} \psi_{\nu,j},$$

- The currents T^a_ν contain products $\zeta_{\nu,e,j}\zeta_{\nu,o,j}$ and are **nonlocal** in terms of the original spin operators.
- correspond to the spin-2 primaries of $SU(2)_4$
- $\int dx[(T^a_R + T^a_L)(x), H] = 0$

Basis change

An adequate $SU(3)$ rotation

$$\begin{pmatrix} \psi_{\nu,1} \\ \psi_{\nu,2} \\ \psi_{\nu,3} \end{pmatrix} = e^{i\frac{\pi}{4}\Lambda^1} e^{i\frac{\pi}{4}(\Lambda^3 - \sqrt{3}\Lambda^8)} \begin{pmatrix} \bar{\psi}_{\nu,1} \\ \bar{\psi}_{\nu,-1} \\ \bar{\psi}_{\nu,0} \end{pmatrix},$$

diagonalizes the magnetization

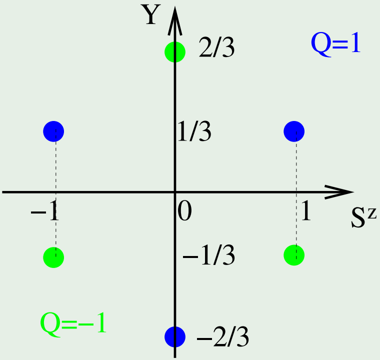
$$\sum_{n=1}^{4} J^z_{\nu,n} = (\bar{\psi}^\dagger_{\nu,1}\bar{\psi}_{\nu,1} - \bar{\psi}^\dagger_{\nu,-1}\bar{\psi}_{\nu,-1})$$

relates the bosonized field $\bar{\varphi}_a$ to original spin fields

$$\frac{1}{2}\sum_{p=1}^{4} \phi_p = \bar{\varphi}_a,$$

Excitations of the model: solitons and antisolitons

Classical limit



$$\Delta\varphi_0 = \pi$$

$$Q = -\frac{\sqrt{3}}{\pi}\Delta\varphi_c, S^z = -\frac{\sqrt{2}}{\pi}\Delta\bar{\varphi}_a, Y = -\frac{\sqrt{2}}{\pi\sqrt{3}}\Delta\bar{\varphi}_b.$$

Summary

- Hidden $SU(3)$ symmetry in the 4 leg spin tube
- Generators of the extended symmetry are non-local in the original spin variables
- $SU(3)$ symmetry manifests itself in degeneracy of the spectrum
- $SU(3)$ symmetry in the nematic order parameters

Open questions

- Are there soliton/antisoliton bound states ?
- Do we have a Valence Bond Solid order parameter ?
- Edge states of the semi-infinite spin tube ?
- Doped 4-leg tube (t-J or Hubbard) ?

Hidden order and Valence Bond Solid order parameter

[Nishiyama, Hatano, Suzuki J. Phys. Soc. Jpn. **65** 560 (1996)]

$$O_{VBS} = \lim_{|j-k|\to+\infty} \langle \left(\sum_{n=1}^{4} S^z_{j,n}\right) \exp\left[i\frac{\pi}{2} \sum_{j\leq l<k} \sum_{n=1}^{4} \left(\sum_{n=1}^{4} S^z_{l,k}\right)\right] \left(\sum_{n=1}^{4} S^z_{j,n}\right)$$

- S odd integer $\Rightarrow O_{VBS} \neq 0$