# Hall Effect in Ladder Systems

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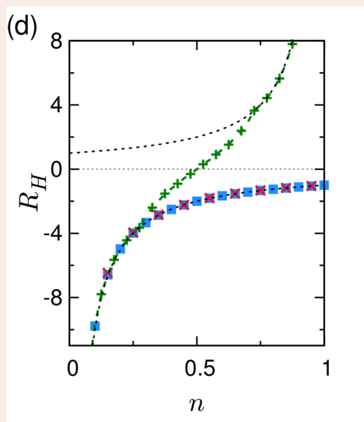
## Hall effect in the two leg ladder geometry

$R_H = -1/n$ dotted line

Fermions: green Bosons: blue and purple




Greschner et al. Phys. Rev. Lett. **122**, 083402 (2019)

## Tomonaga-Luttinger liquid in a single chain of interacting bosons

$$H = \sum_j -t(b_j^\dagger b_{j+1} + b_{j+1}^\dagger b_j) + \frac{U}{2} n_j(n_j - 1)$$

$$H_{TLL} = \int \frac{dx}{2\pi}\left[uK(\nabla\theta)^2 + \frac{u}{K}(\nabla\phi)^2\right]$$

$$\frac{n_j}{a} = \rho_0 - \frac{\partial_x \phi}{\pi} + \sum_{m\neq 0} A_m e^{i2m(\phi(x)-\pi\rho_0 x)}$$

$$b_j = e^{i\theta(x)} \sum_m B_m e^{i2m(\phi(x)-\pi\rho_0 x)}$$

$[\phi(x), \nabla\theta(y)] = i\pi\delta(x-y) (x = ja)$

## Non-perturbative definition of Tomonaga-Luttinger parameters

$$\text{stiffness}: \quad \mathcal{D} = uK$$

$$\text{compressibility}: \quad \chi = K/(\pi\rho_0^2 u)$$

F. D. M. Haldane Phys. Rev. Lett. **47**, 1840 (1981).

## Tomonaga-Luttinger liquid : Band curvature corrections

$$H_{\text{curv.}} = \int dx \left[\alpha(\partial_x\phi)^3 + \gamma(\nabla\theta)^2 \partial_x\phi\right]$$

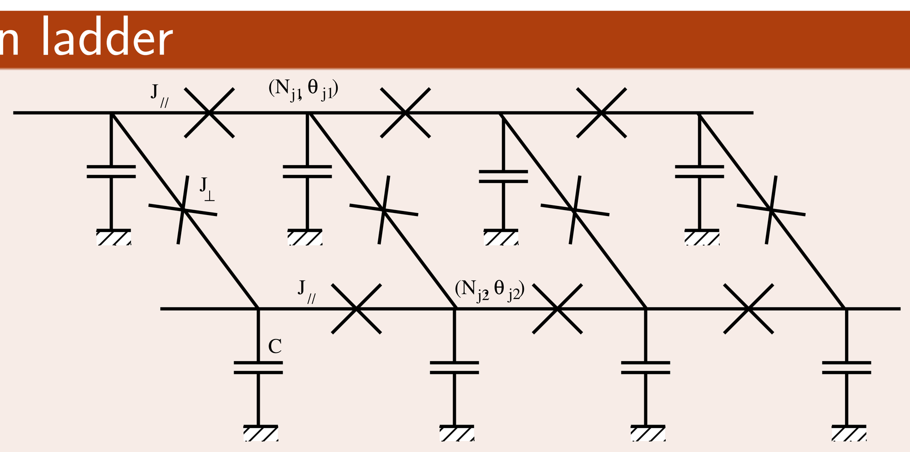
$$\alpha = -\frac{\partial}{\partial\rho_0}\left(\frac{u}{6\pi^2 K}\right)$$

$$\gamma = -\frac{\partial}{\partial\rho_0}\left(\frac{uK}{2\pi^2}\right).$$

Present unless there is particle-hole symmetry $(\phi \to -\phi, \theta \to -\theta)$.

Matveev JETP 117, 508 (2013)

## Josephson junction ladder




$$H = \sum_j \left\{\left[\sum_{p=1,2} \frac{(e^* N_{j,p})^2}{2C} - J_\parallel \cos(\theta_{j+1,p} - \theta_{j,p})\right] - J\cos(\theta_{j,1} - \theta_{j,2})\right\}$$

## In magnetic field: Peierls substitution

$$b_j^\dagger b_k \to e^{ie\int_j^k \vec{A}d\vec{l}} b_j^\dagger b_k$$

$$\cos(\theta_j - \theta_k) \to \cos\left[\theta_j - \theta_k - e\int_j^k \vec{A}d\vec{l}\right]$$

Landau gauge $A_x = -By$.

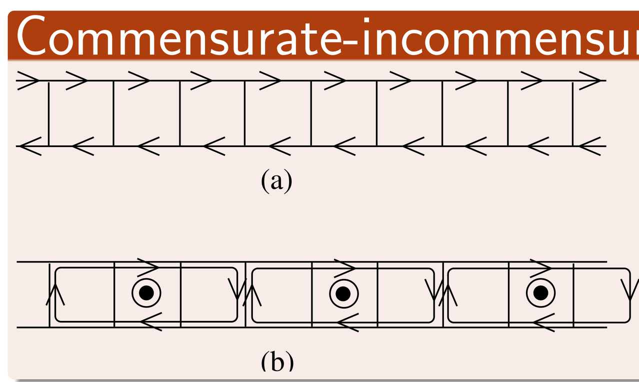
## Bosonized Hamiltonian with band curvature

$$H = \int \frac{dx}{2\pi}\left[uK(\nabla\theta_s)^2 + \frac{u}{K}(\partial_x\phi_s)^2\right]$$
$$+ \int \frac{dx}{2\pi}\left[uK\left(\nabla\theta_a - eBa/\sqrt{2}\right)^2 + \frac{u}{K}(\partial_x\phi_a)^2 - \frac{2t_\perp}{a_0}\cos(\sqrt{2}\theta_a)\right]$$
$$+ \int \frac{dx}{\sqrt{2}}\left[\gamma\partial_x\phi_s(\nabla\theta_s)^2 + \alpha(\partial_x\phi_s)^3\right]$$
$$+ \int \frac{dx}{\sqrt{2}}\left\{\partial_x\phi_s\left[\gamma\left(\nabla\theta_a - eBa/\sqrt{2}\right)^2 + 3\alpha(\partial_x\phi_a)^2\right]\right.$$
$$\left. + 2\gamma\left(\nabla\theta_a - eBa/\sqrt{2}\right)(\nabla\theta_s)\partial_x\phi_a\right\}.$$

## Hall imbalance from first order perturbation theory in $\alpha, \gamma$.

$$\langle P_H\rangle = -\frac{\sqrt{2}}{\pi}\int_{-\infty}^{+\infty}\langle\partial_x\phi_a\rangle$$
$$= \frac{2\gamma L\langle\nabla\theta_s\rangle}{\pi}\int d\tau dx \left\langle T_\tau\left[(\nabla\theta_a - eBa/\sqrt{2})\partial_x\phi_a\right](x,\tau)\partial_x\phi_a(0,0)\right\rangle + O(\gamma^2)$$

## Commensurate-incommensurate transition




(a) $B < B_c$ Meissner phase

$\langle\theta_a\rangle = 0$, gap

(b) $B > B_c$ Vortex phase

$\langle\theta_a\rangle = m(B)x$, gapless

## Hall imbalance in Meissner phase

$$\langle P_H\rangle^{(1)} = \frac{\pi Ba\gamma\langle j_s\rangle}{uK}\chi_{\rho_a\rho_a}(q=0, \omega_n=0)$$

Hall voltage obtained by cancelling $\langle P_H\rangle$

$$V_H = -\frac{\pi^2\gamma\langle j_s\rangle}{euK}Ba$$
$$= -\langle j_s\rangle\frac{Ba}{2e}\frac{1}{uK}\frac{\partial}{\partial\rho_0}(uK)$$
$$= -\langle j_s\rangle\frac{Ba}{2e}\frac{\partial(\ln\mathcal{D})}{\partial\rho_0}$$

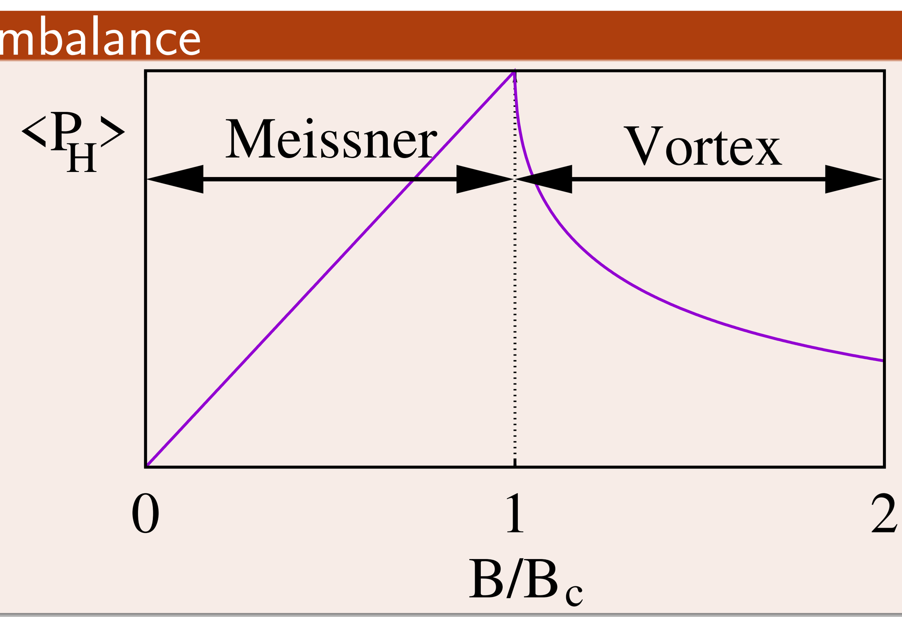
## Hall coefficient near the Metal-insulator transition

At $1/n$ filling $(n \in N^*)$

$$H_s = \int\frac{dx}{2\pi}\left[u_s K_s(\nabla\theta_\rho)^2 + \frac{u_s}{K_s}(\partial_x\phi_s)^2\right]$$
$$-\frac{2g_U}{(2\pi\alpha)^2}\int dx\cos(n\sqrt{2}\phi_s) + \frac{\sqrt{2}}{\pi}\mu\int dx\partial_x\phi_s,$$

$$u_s^* \sim (\rho_0 - \rho_{0,c}) \quad \& \quad K_s^* \sim \frac{2}{n^2} \Rightarrow R_H \sim \frac{1}{e(\rho_0 - \rho_{0,c})}$$

## Behavior of Hall imbalance




## Perspectives

- Beyond perturbation theory
- Systems of many coupled chains
- Quantum Hall phases
- Vortex lattice phases