Formation and fate of quantum droplets in a quasi-1D dipolar Bose gas

Edmond Orignac

CNRS, Ens de Lyon, Laboratoire de Physique, F-69342 Lyon, France

May 9, 2022

Collaboration

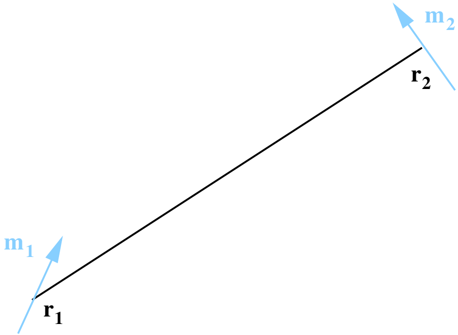
Coworkers

- Roberta Citro (Università degli Studi di Salerno)
- Stefania De Palo (Università degli Studi di Trieste)
- Maria Luisa Chiofalo (Università di Pisa)

Outline

1. Introduction
2. Variational method and ground state energy density
3. Instability and Maxwell construction: droplet formation
4. Generalized Gross-Pitaevskii equation: droplet profiles
5. Dynamics and droplet stability

Dipolar interaction



$$U_{\text{dipoles}} = \frac{\mu_0}{4\pi}\left[\frac{\mathbf{m}_1 \cdot \mathbf{m}_2}{|\mathbf{r}_1 - \mathbf{r}_2|^3} - 3\frac{\mathbf{m_1} \cdot (\mathbf{r}_1 - \mathbf{r}_2)\mathbf{m_1} \cdot (\mathbf{r}_1 - \mathbf{r}_2)}{|\mathbf{r}_1 - \mathbf{r}_2|^5}\right]$$

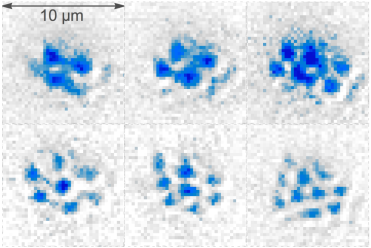
Dipolar bosonic cold atoms

[Chomaz et al. arXiv:2201.02762]

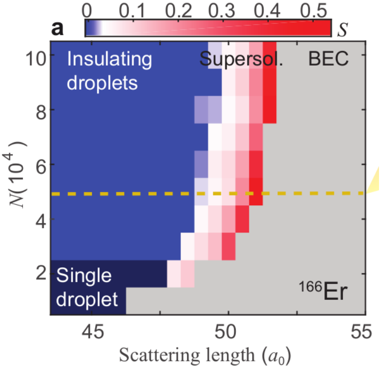
- Chromium (^{52}Cr, $\mu_{Cr} = 6\mu_B$)
- Erbium (^{168}Er, $\mu_{Er} = 7\mu_B$)
- **Dysprosium** (^{162}Dy $\mu_{Dy} = 6\mu_B$)

A strong enough magnetic field can be used to polarize dipoles.

Droplets and supersolidity in 2D/3D

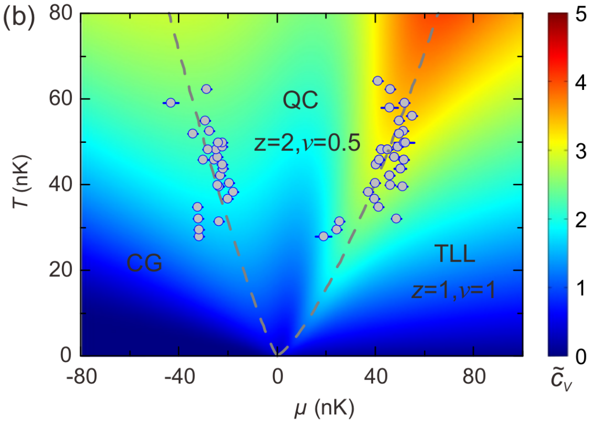


Kadau *et al.* Nature **530**, 194 (2016)



Chomaz *et al.* Phys. Rev. X **9**, 021012 (2019)

Cold bosonic atoms in one dimension



From B. Yang *et al.* Phys. Rev. Lett. **119**, 165701 (2017)

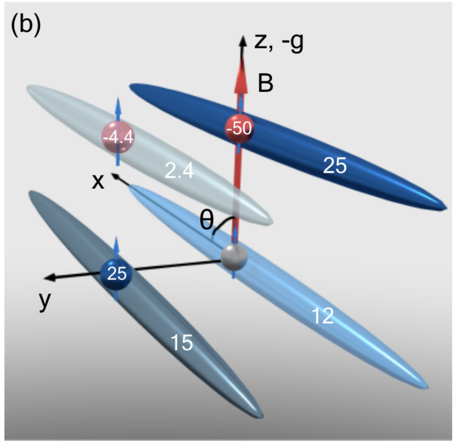
Tomonaga-Luttinger liquid

Main properties [FDM Haldane PRL 47, 1840 (1981)]

- $E_{GS} = Le(N/L)$
- velocity of excitations: $u^2 = \frac{ne''(n)}{m}$
- Tomonaga-Luttinger exponent: $K^2 = \frac{\pi^2\hbar^2 n}{me''(n)}$
- $\langle\psi^\dagger(x)\psi(0)\rangle_{GS} \sim x^{-1/(2K)}$
- $\langle n(x)n(0)\rangle_{GS} \sim n^2 + Ax^{-2} + B\cos(2\pi nx)x^{-2K}$

One dimensional dipolar boson gas

Realizations of dipolar boson gas



- ^{162}Dy in optical lattice
- $\omega_\perp \gg \omega_x$
- dipoles aligned by a magnetic field

From Tang *et al.* Phys. Rev. X **8**, 021030 (2018).

Tightly trapped gas in the transverse direction

Single Mode Approximation [Sinha& Santos PRL **99**, 140406 (2007); Reimann *et al.* PRA **87** 039903 (2013)]

$$V_{dd}(x-x') = V(\theta)\left[V_{DDI}^{1D}\left(\frac{x-x'}{l_\perp}\right) - \frac{8}{3}\delta\left(\frac{x-x'}{l_\perp}\right)\right],$$

$$V(\theta) = \frac{\mu_0\mu_D^2}{4\pi}\frac{1-3\cos^2\theta}{4l_\perp^3},$$

$$V_{DDI}^{1D}(u) = -2|u| + \sqrt{2\pi}\left[1+u^2\right]e^{\frac{u^2}{2}}\mathrm{erfc}\left(\frac{|u|}{\sqrt{2}}\right),$$

$$V_{DDI}^{1D}(|u|\to+\infty) = O(|u|^{-3})\ V_{DDI}^{1D}(|u|\to 0) = O(1)$$

Quasi-1D interacting dipolar atoms

Repulsive contact and dipolar: short ranged interactions

$$
\begin{aligned}
H_{Q1D} &= -\frac{\hbar^2}{2m}\sum_i \frac{\partial^2}{\partial x_i^2} + \sum_{i<j} V_{dd}(x_i - x_j) \\
&\quad + g_{1D}\sum_{i<j}\delta(x_i - x_j) \\
g_{1D} &= \frac{2\hbar^2}{m|a_{1D}|}
\end{aligned}
$$

Variational method

Variational principle

$$\begin{aligned}
&H = H_0 + V \\
&(H_0 + gU)|\psi_0(g)\rangle = E_0(g)|\psi_0(g)\rangle \\
&\langle\psi_0(g)|H|\psi_0(g)\rangle \geq E_{GS}(H) \\
&\Leftrightarrow E_0(g) - g\frac{dE_0}{dg} + \langle\psi_0(g)|V|\psi_0(g)\rangle \geq E_{GS}(H)
\end{aligned}$$

Our variational ansatz

H_0 = Kinetic energy
V= Potential energy
$H_0 + gU$ = Lieb-Liniger Hamiltonian
g = Variational parameter

The Lieb-Liniger model [Phys. Rev. 130, 1605 (1963)]

Hamiltonian

$$H = -\frac{1}{2m}\sum_j \frac{\partial^2}{\partial x_j} + g\sum_{j<k}\delta(x_j - x_k)$$

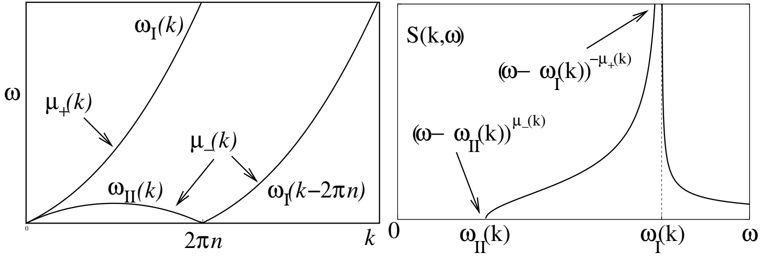
Exact ground state energy from integrability

$$2\pi\rho(k) = 1 + \int_{-q_0}^{q_0} \frac{2mg}{(mg)^2 + (k-k')^2}\rho(k')dk'$$

$$\frac{E}{L} = \int_{-q_0}^{q_0} dk\rho(k)\frac{k^2}{2m} = \frac{n^3}{2m}\epsilon(\gamma)$$

$$n = \int_{-q_0}^{q_0} dk\rho(k)\ \gamma = \frac{mg}{n}$$

Dynamical structure factor of the Lieb-Liniger model



J. S. Caux and P. Calabrese Phys. Rev A **74**, 031605 (2006)
Khodas *et al.* Phys. Rev. Lett. **99**, 110405 (2007)

The Cherny-Brand ansatz for the structure factor (I)

Dynamical Structure factor [Cherny & Brand Phys. Rev. A 79 043607 (2009)]

$$S(k,\omega) \simeq C \frac{[\omega^\alpha - \omega_{II}^\alpha(k)]^{\mu_-}}{[\omega_I^\alpha(k) - \omega^\alpha]^{\mu_+}} \Theta(\omega - \omega_{II}(k))\Theta(\omega_I(k) - \omega)$$

$$\alpha = 1 + \frac{1}{\sqrt{K}}$$

$$\int_0^{+\infty} d\omega \omega S(k,\omega) = N\frac{k^2}{2m}$$

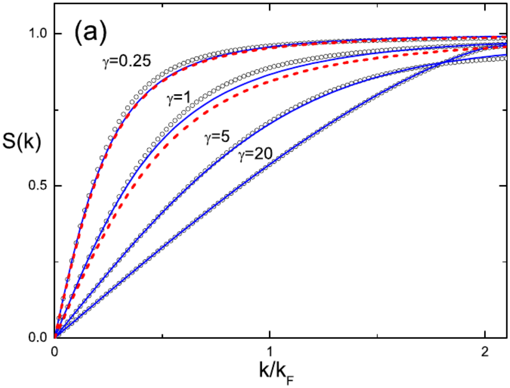
Cherny-Brand ansatz for the structure factor (II)

Static structure factor

$$S(k;g) \simeq \frac{k^2}{2m\omega_{II}} \times$$

$$\frac{{}_2F_1\left(1+\frac{\sqrt{K}}{1+\sqrt{K}}+\mu_-+\mu_+, 1+\mu_-; 2+\mu_- -\mu_+, 1-\left(\frac{\omega_{II}}{\omega_I}\right)^2\right)}{{}_2F_1\left(1+\frac{2\sqrt{K}}{1+\sqrt{K}}+\mu_-+\mu_+, 1+\mu_-; 2+\mu_- -\mu_+, 1-\left(\frac{\omega_{II}}{\omega_I}\right)^2\right)}$$

Comparison with the exact calculation of Caux and Calabrese



From Cherny and Brand Phys. Rev. A **79**, 043607 (2009)

Using the Lieb-Liniger Hamiltonian as variational Hamiltonian

Variational energy as a function of the structure factor

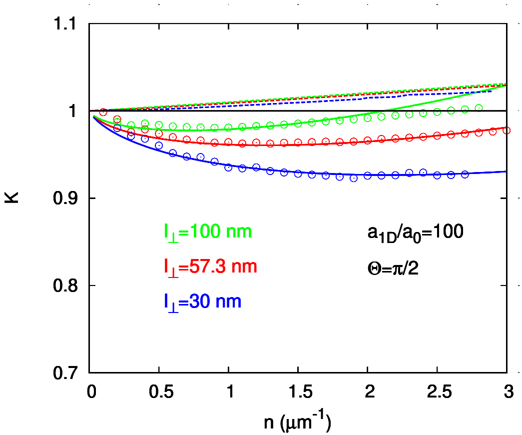
$$E_{var}(g) = E_0(g) - g\frac{\partial E_0(g)}{\partial g} + \frac{Nn}{2}\hat{v}(k=0)$$
$$+\frac{N}{2}\int_{-\infty}^{\infty}\frac{dk}{2\pi}\hat{v}(k)[S(k;g)-1].$$

Algorithm [PRB 101, 045102 (2020)]

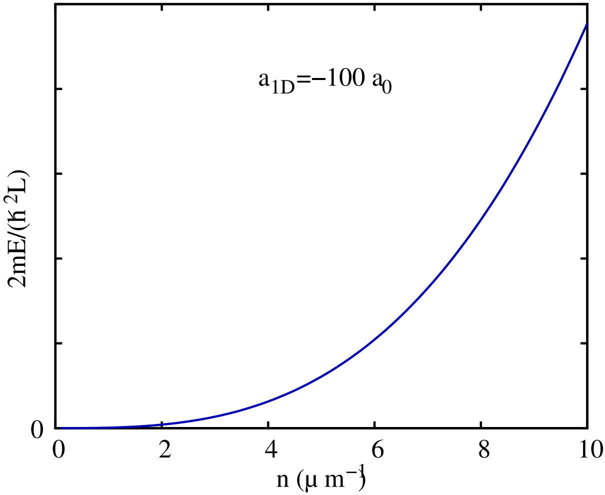
Description

1. For a given g, solve integrals equations for $\rho(k)$ and $F_B(\nu|\lambda)$.
2. Find $K, \omega_{I,II}(k), \mu_{\pm}(k)$ and deduce $S(k; g)$ from Cherny-Brand Ansatz.
3. Use $S(k; g)$ to obtain the variational energy by numerical integration.
4. Locate the minimum as a function of g.

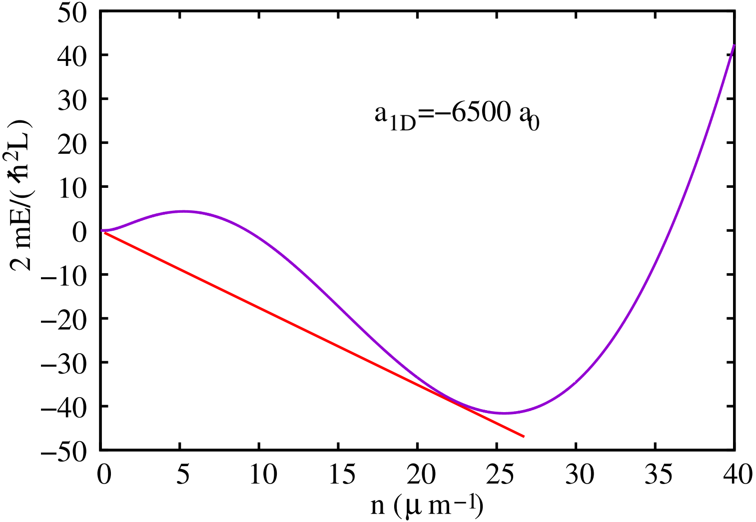
Repulsive dipolar interaction ($\theta = \pi/2$)



Attractive dipolar interaction ($\theta = 0$)



Instability with attractive interaction ($\theta = 0$)



Maxwell construction and droplets

Maxwell construction

$$\frac{E}{L} = e(N/L) \; n = \frac{N}{L}$$

$$\left(\frac{de}{dn}\right)_{n=n_1} = \frac{e(n_1)}{n_1}$$

$$0 < n < n_1 \Rightarrow \frac{E}{L} = n\frac{e(n_1)}{n_1}$$

A droplet of density n_1 and size Ln/n_1 is more stable than the homogeneous gas.

Generalized Gross-Pitaevskii equation

Generalized GPE [PRL 86, 5413 (2001); 89 240402 (2002)]

$$H_{\text{eff.}} = \int dx \left[\frac{\hbar^2}{2m} \left| \frac{\partial \psi}{\partial x} \right|^2 + e(|\psi|^2) \right]$$

$$i\hbar \frac{\partial \psi}{\partial t} = \frac{\delta H_{\text{eff.}}}{\delta \psi^*} = -\frac{\hbar^2}{2m} \frac{\partial^2 \psi}{\partial x^2} + e'(|\psi|^2)\psi$$

$e(|\psi|^2)$ obtained from previous variational calculation.

Static solutions: droplet profiles [arXiv:2202.12071]

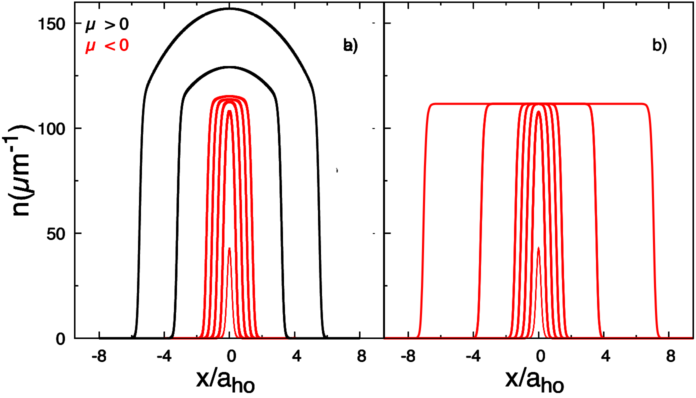


Figure: a) with longitudinal trapping and N=100,200,200,400 (red) N=1000,2000 (red)

b) no trapping N=100,200,200,400,1000;2000 ($|a_{1D}| = 7500 a_0$

Static solutions: small particle number

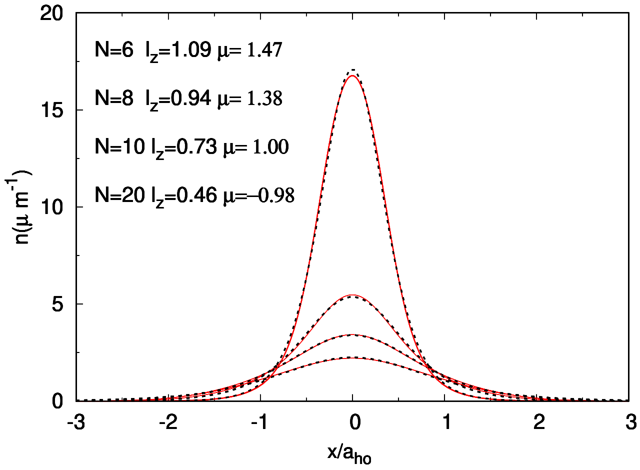


Figure: N=6,8,10,20 and $|a_{1D}| = 6500a_0$. Fitted to $n(x) = a/(4\cosh^2(x/l_z))$

Phase diagram

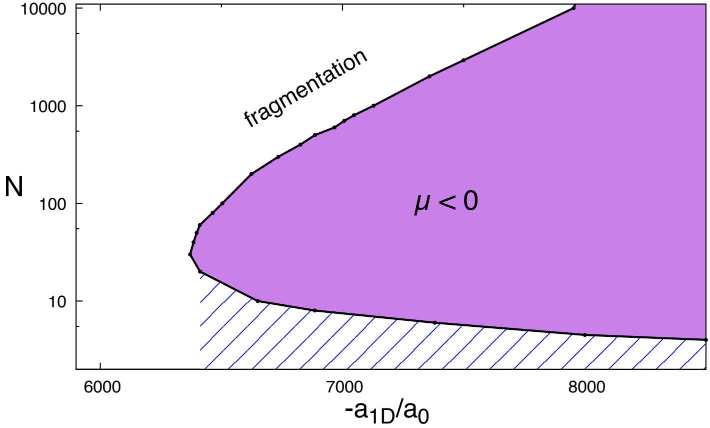


Figure: blue dashed region: $\mu > 0$ and sech^2 density profile. Violet region: $\mu < 0$ and droplet profile.

time dependent solutions: breathing modes in a trap [PRB 103 115109 (2021)]

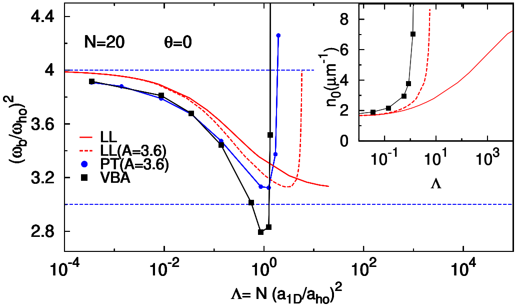


Figure: Breathing modes in a trap with attractive dipolar interaction

time dependent solutions: breathing modes in a trap [PRB 103 115109 (2021)]

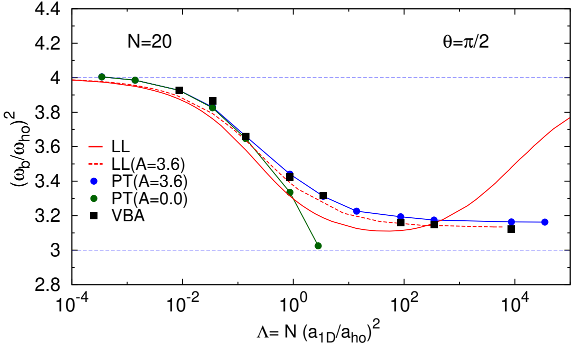


Figure: Breathing modes in a trap with repulsive dipolar interaction

Comparison with ^{162}Dy experiment

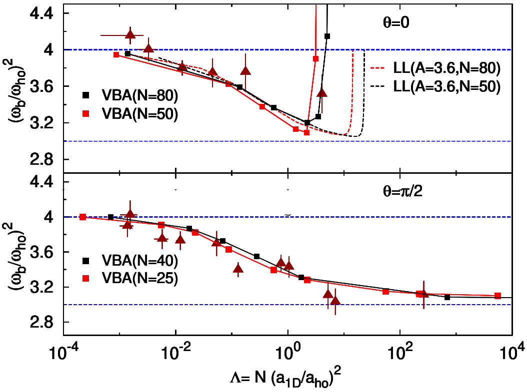


Figure: Comparison with Kao et al. [Science **371** 6526 (2021) and private communication]

time dependent solution: evolution after sudden release from trap

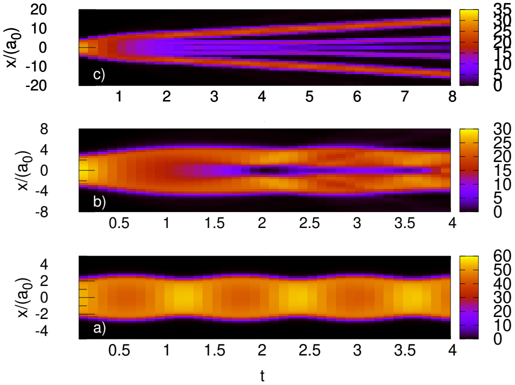


Figure: a) Oscillations of the droplet b) intermediate regime c) Fragmentation of the droplet.

time dependent solutions: closer look at fragmentation

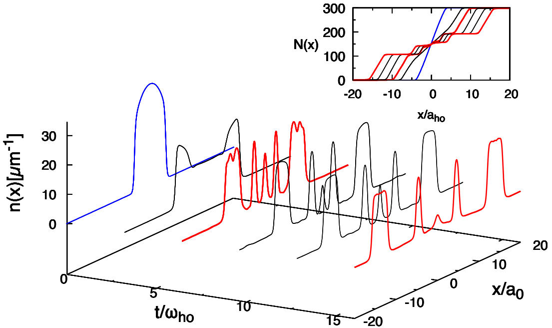


Figure: Density profiles as a function of time in a fragmented condensate. $N = 300$ and $a_{1D} = -6500a_0$.

Inset: $N(x) = \int_{-\infty}^{x} dx n(x)$

Conclusions

Summary

- Variational calculation of ground state energy of interacting bosons and Generalized GPE
- instability towards droplet formation for attractive dipolar interaction
- fragmentation of the droplets upon sudden release

Publications

1. Phys. Rev. B **101**, 045102 (2020) [variational method]
2. Phys. Rev. B **103**, 115109 (2021) [breathing mode]
3. `arXiv:2202.12071` [droplets]

Conclusions

Perspectives

- Other time dependent protocols (droplet collisions, . . .)
- More general ansatz allowing width variation in transverse direction
- Applications to other models with short range interactions