The Mott Glass: A new disordered phase in 1d[a]

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[a] Phys. Rev. Lett. **83** p. 2378 (1999)

Plan

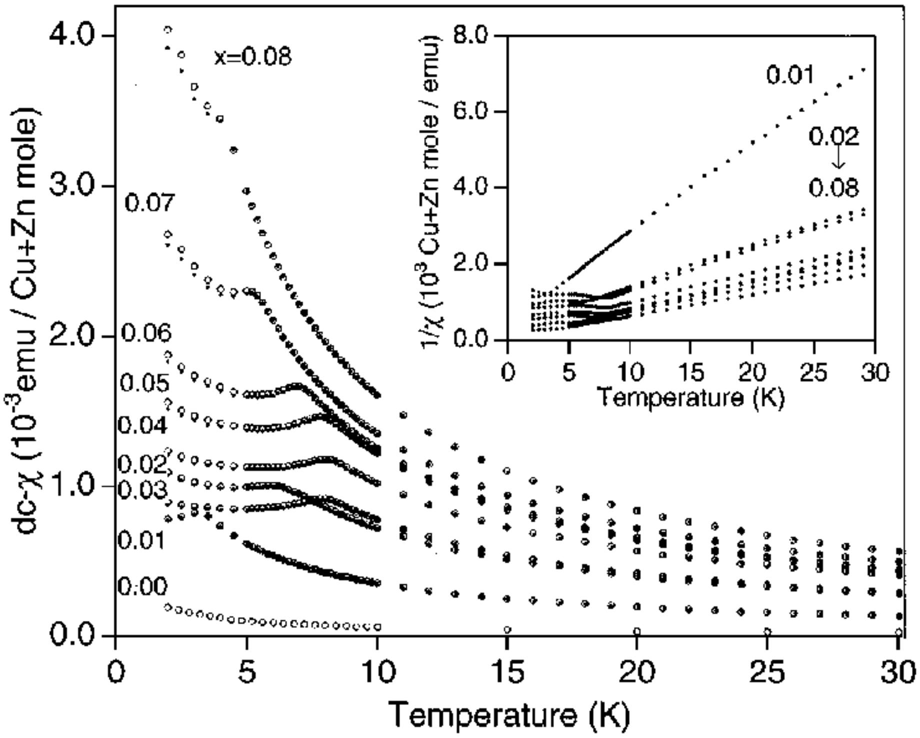
Gapped systems with disorder

- Disordered Mott Insulators
- Charge density wave with commensuration + disorder (Fukuyama 1978)
- In one dimension:
  - Random exchange in a spin 1 chain ($MgVOBO_3$)
  - Disordered dimerized/spin-Peierls systems ($CuGe_{1-x}Si_xO_3$,$Cu_{1-x}Zn_xGeO_3$)
  - Disordered spin-ladders ($SrCu_{2-x}Zn_xO_3$)

Non Magnetic impurities in a spin ladder

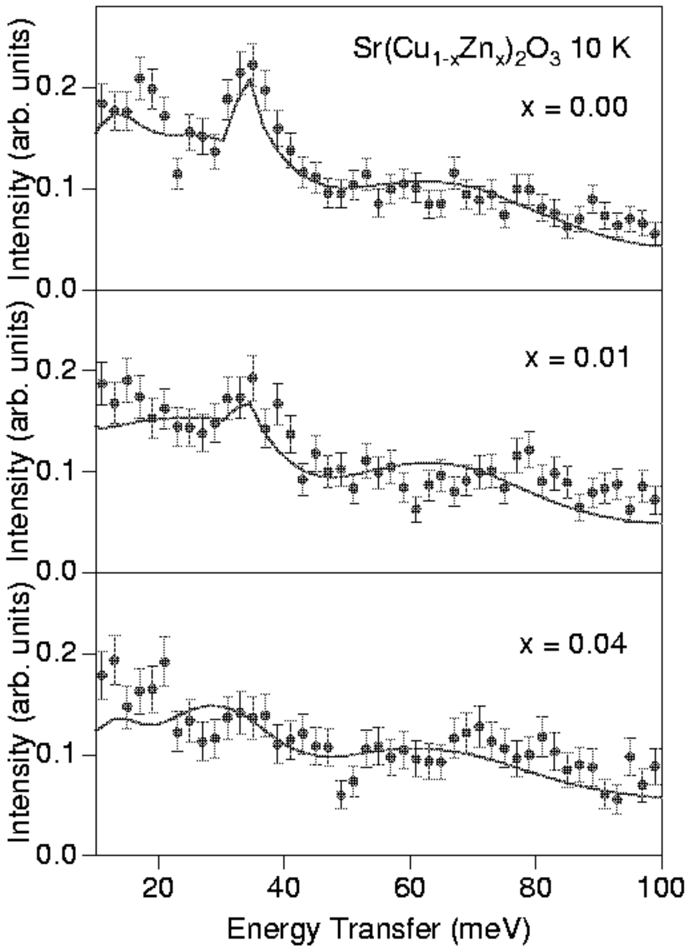
Azuma *et al.* Phys. Rev. B **55** R8658 (1997)

$Sr(Cu_{1-x}Zn_x)_2O_3$



- Undoped = spin ladder, spin gap $\sim$ 36 meV
- Low doping $x \simeq 0.01 \Rightarrow$ Antiferromagnetic ordering below 10 K
- Magnetic Susceptibility follows a Curie-Weiss Law $\chi = \chi_0 + \frac{C}{T+\theta}$

Neutron Scattering on $Sr(Cu_{1-x}Zn_x)_2O_3$



- spin gap at 33 meV independant of x
- the peak at 33 meV is broadened
- Reduction of weight in the peak $\Rightarrow$ States below the gap

(Azuma *et al.* J. Phys. Soc. Jpn. **67** 740 (1998))

Random exchange in spin 1 chains

RSRG (strong disorder)

(Boechat *et al.* 1996)

(Hyman *et al.* 1996)

(Monthus *et al.* 1997)

$\downarrow$

random singlet

or

Griffiths phase

Bosonization (weak disorder)

(Orignac, Giamarchi 1998)

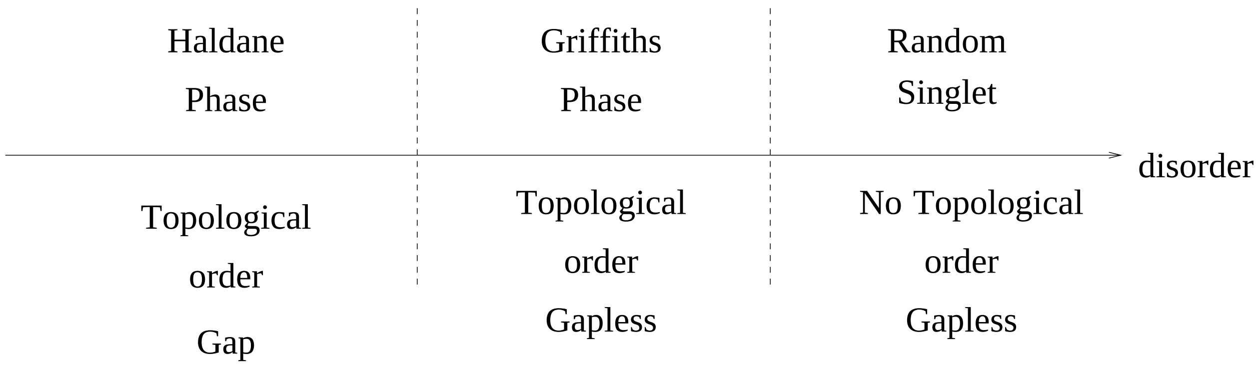
(Brunel , Jolicoeur 1998)

$\downarrow$

Nothing at weak disorder

$\Rightarrow$ Griffiths phase

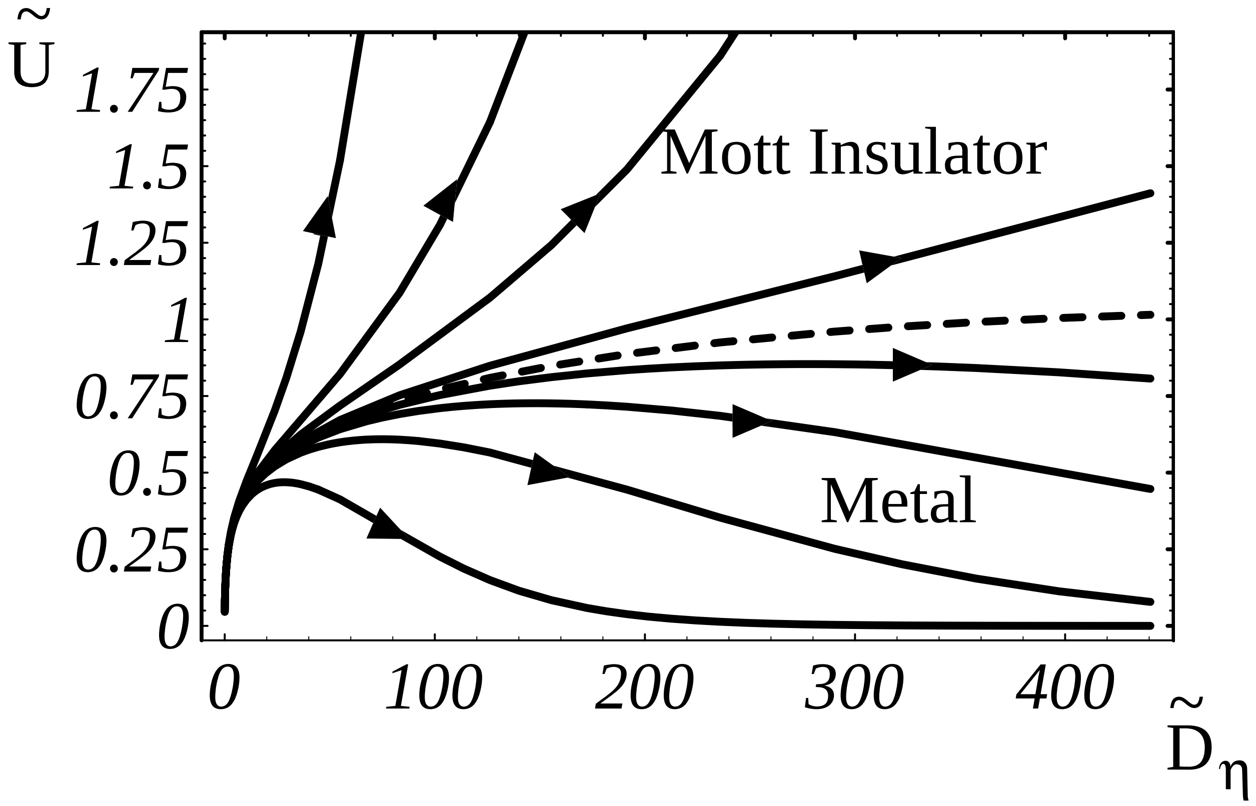
needs finite disorder



Griffiths phase from the bosonization approach ?

Competition of Mott and Anderson localization in 1D

Fujimoto, Kawakami Physical Review B 54, R11018 (1996)



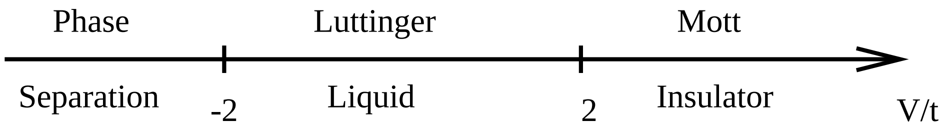
- Prototype of the disorder vs. gap formation problem in 1D
- Phase transition as Mott correlation length $\sim$ localization length

A one dimensional spinless Mott insulator

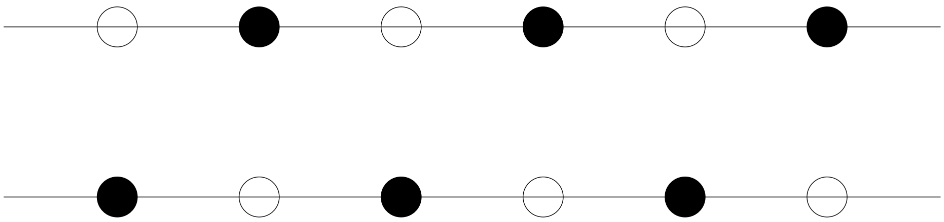
$$H = -t \sum_i (c^\dagger_{i+1} c_i + c^\dagger_i c_{i+1}) + V n_i n_{i+1}$$

$$n_i = c^\dagger_i c_i$$

Half filling: $\langle n_i \rangle = \frac{1}{2}$



Mott Insulating state $V \gg t$:



Degenarate, Non Uniform density

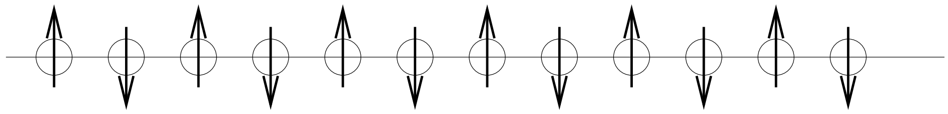
$V \gg t \Rightarrow$ Charge Gap $\Delta \sim V$.

One-dimensional Mott-Hubbard Insulator

$$H = -t \sum_{i,\sigma} (c^\dagger_{i,\sigma} c_{i+1,\sigma} + c^\dagger_{i+1,\sigma} c_{i,\sigma}) + U \sum_i n_{i,\uparrow} n_{i,\downarrow}$$

$$U > 0$$

At Half-Filling, $\langle n_{i,\sigma} \rangle = \frac{1}{2} \Rightarrow$ Insulating state.



Uniform electronic density.

$U \gg t \Rightarrow$ Charge Gap $\Delta \sim U$.

Bosonization of 1D Mott Insulators – Sine Gordon Model

$$S = \int dx d\tau \left[\frac{1}{2\pi K} \left(v(\partial_x \phi)^2 + \frac{1}{v}(\partial_\tau \phi)^2 \right) - \frac{2g}{(\pi a)^2} \cos \beta\phi \right]$$

$$\rho(x,\tau) = -\frac{\partial_x \phi(x,\tau)}{\pi}$$
$$j(x,\tau) = \frac{\partial_\tau \phi(x,\tau)}{\pi}$$

$\phi =$ Charge Excitations

- $K < 1$: Repulsive interactions.
- $K > 1$: Attractive interactions.
- $g = 0$: Luttinger liquid
- Spinless Fermions: $\beta = 4, g \sim Va$
- Fermions with spin: $\beta = \sqrt{8}, g \sim Ua$

Renormalization Group treatment of the Sine Gordon Model

$$\frac{dK}{dl} = -\frac{K^2}{2}\left(\frac{g}{\pi v}\right)^2$$
$$\frac{dg}{dl} = (2 - \frac{\beta^2}{4}K)g$$

- $K < \frac{8}{\beta^2}$:
  - $g \to \infty \Rightarrow \langle \phi \rangle = 0, \langle \phi^2 \rangle < \infty$
  - Characteristic lengthscale: $d \sim a\left(\frac{\pi v}{g}\right)^{1/(2-K)}$
  - Energy gap: $\Delta_{\mathrm{Mott}} \sim \frac{v}{a}\left(\frac{g}{\pi v}\right)^{1/(2-K)}$
- $K > \frac{8}{\beta^2}$:
  - $g \to 0 \Rightarrow \langle \phi^2 \rangle = \infty$
  - Power law correlations
  - Gapless spectrum $\Rightarrow$ Luttinger Liquid

Self Consistent Harmonic Approximation

Trial action:

$$S_0 = \int \frac{dx d\tau}{2\pi K}\left[v(\partial_x \phi)^2 + \frac{1}{v}(\partial_\tau \phi)^2\right] + \frac{m^2}{2}\int dx d\tau \phi^2$$

$$F = -\ln \int D\phi e^{-S[\phi]}; F_0 = -\ln \int D\phi e^{-S_0[\phi]}$$

Variational Principle:

$$F \leq F_{var} = F_0 + \frac{1}{\beta\hbar}\langle S - S_0\rangle_{S_0}$$

$$\frac{\partial F_{var}}{\partial_m} = 0 \Rightarrow m \sim g^{\frac{1}{2-\frac{\beta^2}{4}K}}$$

Recover RG prediction.

Disorder in 1D

$$V_{dis.} = \sum_i \epsilon_i n_i$$

$$\overline{\epsilon_i \epsilon_j} = W \delta_{i,j}$$

- spinless fermions: $n_i = c_i^\dagger c_i$
- fermions with spin: $n_i = \sum_\sigma c_{i,\sigma}^\dagger c_{i,\sigma}$

Without interaction:

- An infinitesimal amount of disorder will cause Anderson localization.
- $l_{\mathrm{loc.}} \sim \frac{1}{W}$
- $\sigma(\omega) \sim \omega^2 (\ln \omega)^2$

With interactions: Disorder in a Luttinger liquid.

Focus on spinless fermions.

Bosonization

$\downarrow$

$$S = \int dx d\tau \left[\frac{1}{2\pi K} \left(v(\partial_x \phi)^2 + \frac{1}{v} (\partial_\tau \phi)^2 \right) \right.$$
$$\left. -\eta(x) \frac{\partial_x \phi(x,\tau)}{\pi} - \xi(x) \frac{e^{2\imath\phi(x)}}{2\pi a} - \xi^*(x) \frac{e^{-2\imath\phi(x)}}{2\pi a} \right]$$

$$\overline{\eta(x)\eta(x')} = D_{\text{forward}} \delta(x - x')$$
$$\overline{\xi(x)\xi^*(x)} = D_{\text{backward}} \delta(x - x')$$

$\phi(x) \to \phi(x) - \frac{2K}{v} \int^x dx' \eta(x') \Rightarrow$ gauge out $\eta(x)$.

Renormalization Group approach to disorder and interactions in one dimension

(Giamarchi, Schulz 1987)

$$\frac{dK}{dl} = -K^2 \frac{D_b a}{\pi v^2}$$

$$\frac{dD}{dl} = (3 - 2K) D_b$$

- $K > 3/2 \Rightarrow D \to 0$: Luttinger liquid
- $K < 3/2 \Rightarrow D \to \infty$: Anderson Insulator Ground state
- $l_{\text{loc.}} = a \left(\frac{\pi v^2}{D_b a} \right)^{\frac{1}{3-2K}}$
- $T \gg T_{\text{loc.}} = \frac{\hbar v}{l_{\text{loc.}}} \Rightarrow \sigma(T) \sim T^{2-2K}$
- $\omega \gg \omega_{\text{loc.}} = \frac{v}{l_{\text{loc.}}} \Rightarrow \sigma(\omega) \sim \omega^{2K-4}$

Replica Approach

(Giamarchi, Le Doussal 1995)

$$-\beta F = \overline{\ln Z} = \lim_{n\to 0} \frac{\overline{Z^n} - 1}{n}$$

$$\frac{S_{\text{rep.}}}{\hbar} = \sum_a \left[\int \frac{dx d\tau}{2\pi K} \left(v(\partial_x \phi_a)^2 + \frac{(\partial_\tau \phi_a)^2}{v} \right) \right]$$

$$- \frac{D_b}{(2\pi\alpha\hbar)^2} \sum_{a,b} \int dx \int_0^\beta d\tau \int_0^\beta d\tau' \cos\left(2(\phi_a(x,\tau) - \right.$$

$$\left. - \phi_b(x,\tau'))\right)$$

Disorder $\Rightarrow$ Non Local in time interactions, $n \to 0$ limit.

How do we solve this action ?

Self Consistent Harmonic Approximation in replica space

(Mézard, Parisi 1991)
(Giamarchi, Le Doussal 1995)

$$S_0 = \frac{1}{2\beta\hbar}\sum_{\omega_n}\int \frac{dq}{2\pi}\phi_a(q,\omega_n)G^{-1}_{ab}(q,\omega_n)\phi_b(q,\omega_n)$$

$$vG^{-1}_{ab}(q,\omega) = \frac{((vq)^2+\omega^2)}{\pi K}\delta_{ab} - \Sigma_{ab}(q,\omega)$$

$$G_c = G_{aa} + \sum_{b\neq a} G_{ab}$$

$$\sigma(\omega) = \lim_{q\to 0}\omega G_c(q,\omega)$$

$$\chi = \lim_{q\to 0} q^2 G_c(q,\omega=0)$$

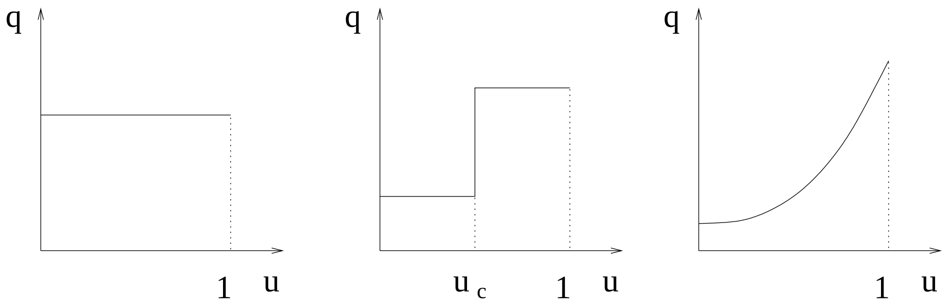
Apply the Variational Principle !
How to take $n \to 0$?

Parisi's Ansatz

(Parisi, 1980) $\Sigma = n \times n$ matrix. the $n \to 0$ limit is defined by:

$$\begin{cases} \Sigma_{aa} \to \tilde{\Sigma} \\ \Sigma_{a \neq b} \to \Sigma(u) \; u \in [0,1] \end{cases}$$

Addition and Multiplication rules follow from the definition.



Replica Symmetric (RS)

Broken Replica Symmetry (1 step RSB)

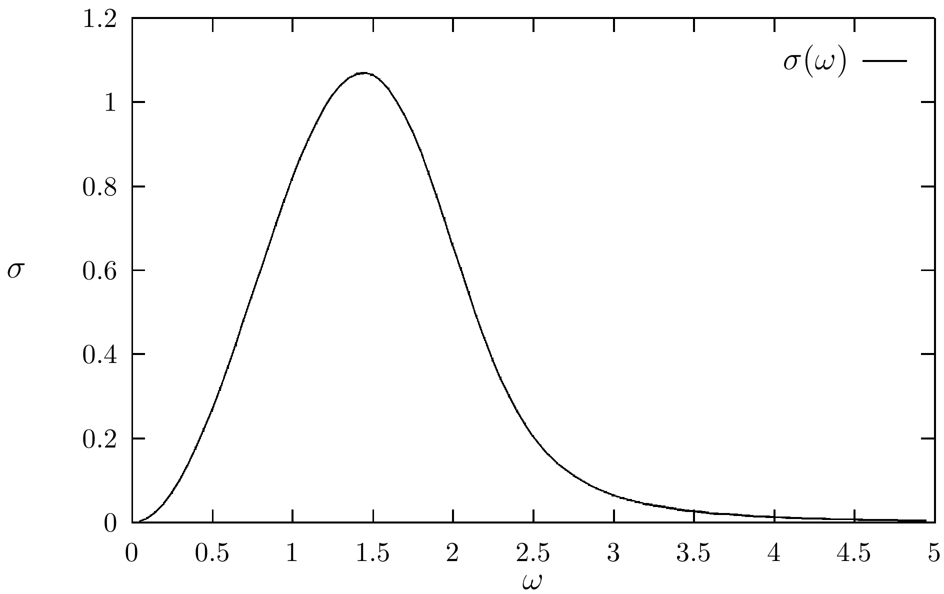
Broken Replica Symmetry (Full RSB)

Broken Replica Symmetry $\Rightarrow$ Many metastable states, slow (glassy) dynamics, lack of self averaging ...

The Anderson Insulator

(Giamarchi, Le Doussal 1995)

Solution of the saddle point equations for $K, \hbar \to 0$, $\frac{K}{\hbar}$ fixed.



- Finite Compressibility: $\chi = \frac{K}{\hbar \pi^2 v}$
- $\sigma(\omega) \sim \omega^2$
- localization length : $l_0 = \left(\frac{\hbar^2 v^2 a^2}{16 K^2 W} \right)^{1/3}$
- Glassy state with one step of RSB

What makes gap + disorder difficult

Relevant Umklapp process = gaps

+

Disorder = relevant perturbation

=

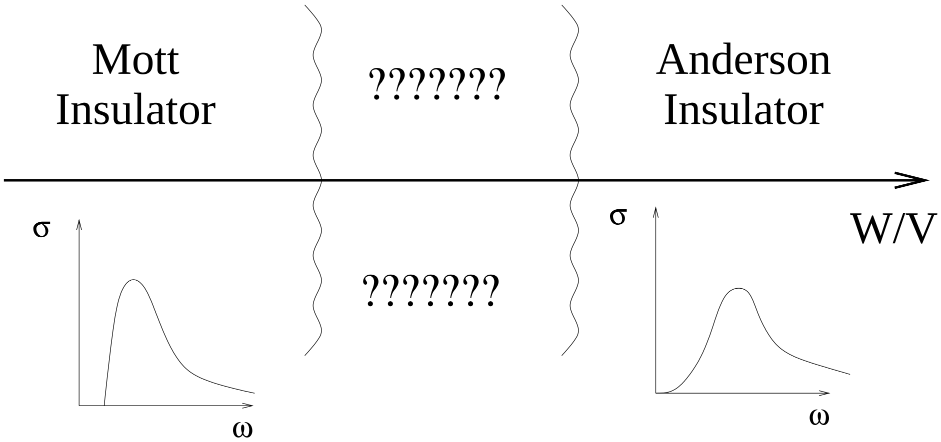
RG with two relevant parameters

$\Rightarrow$ No perturbative fixed point

$\Rightarrow$ Necessity to find a simple solvable model of competition disorder/gap.

What about a spinless fermions problem ?

Guess at a phase diagram of spinless Mott insulator with disorder



$d = \left(\frac{\hbar v \alpha}{4gK}\right)^{\frac{1}{2-4K}}$: Correlation length in the Mott Insulator.

$l_0 = \left(\frac{(\hbar v \alpha)^2}{16WK^2}\right)^{\frac{1}{3-2K}}$: Localization length in the Anderson Insulator.

Is that correct ?

Effect of infinitesimal disorder in the spinless Mott insulator

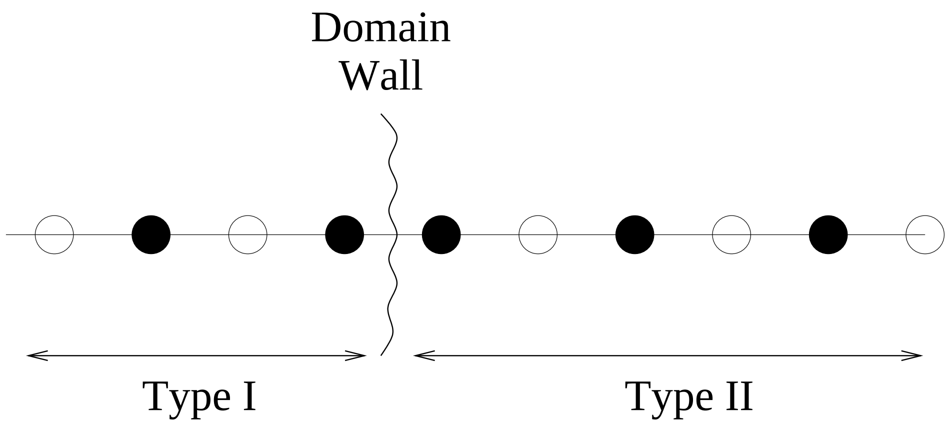
$$V_{dis.} = \sum_i \epsilon_i n_i$$

$$\overline{\epsilon_i \epsilon_j} = W \delta_{i,j}$$

$\Rightarrow$ Imry-Ma effect

Consider $t = 0$:

- Cost of a domain wall: $2V$
- Typical gain of a domain of size L: $\sim \sqrt{WL}$



An infinitesimal disorder suppresses the Mott Gap !

(Shankar, 1990)

Disorder in the One-dimensional Hubbard Insulator

$$V = \sum_i \epsilon_i (n_{i,\uparrow} + n_{i,\downarrow})$$

$$\overline{\epsilon_i \epsilon_j} = W \delta_{i,j}$$

No Imry-Ma Effect at $t = 0$.

- $W < W_c = \frac{U}{4}$ the gap persists
- $W > W_c$ the gap is suppressed

How to simulate this with spinless fermions ? $\Rightarrow$ Find a system with non-degenerate gapped ground state.

Spinless Band Insulator in 1D

$$H = \sum_k \epsilon(k) c_k^\dagger c_k + V \sum_k c_{k+2k_F}^\dagger c_k + V^* \sum_k c_{k-2k_F}^\dagger c_k$$

$$\epsilon(k) = -2t\cos(ka)$$

Add an interaction $V \sum_i n_i n_{i+1}$.

Bosonization:

$$S = \int dx d\tau \left[\frac{1}{2\pi K} \left(v(\partial_x \phi)^2 + \frac{1}{v}(\partial_\tau \phi)^2 \right) - \frac{2g}{(\pi a)^2} \cos 2\phi \right]$$

Difference with the spinless Mott insulator:
$\beta = 4 \Rightarrow \beta = 2$.

$\Rightarrow$ No degeneracy ! $\Rightarrow$ There should exist a critical disorder to close the gap.

The replicated action

$$
\begin{aligned}
\frac{S_{\text{rep.}}}{\hbar} = \sum_a \Bigg[\int \frac{dx d\tau}{2\pi K} \left(v(\partial_x \phi_a)^2 + \frac{(\partial_\tau \phi_a)^2}{v} \right) \\
- \frac{g}{\pi \alpha \hbar} \int dx d\tau \cos 2\phi_a \Bigg] \\
- \frac{W}{(2\pi\alpha\hbar)^2} \sum_{a,b} \int dx \int_0^\beta d\tau \int_0^\beta d\tau' \\
\cos\left(2(\phi_a(x,\tau) - \phi_b(x,\tau'))\right)
\end{aligned}
$$

Similar action in:

- spin ladder with random exchange ($K = 1$)
- dimerized chain with random exchange ($K = 1/2$)
- Mott Hubbard Insulator with disorder and $U \gg t$
- classical CDW with commensurability + disorder (“$K = 0$”)

A classical analog of the Fermion system

Consider the following XY model with $p-$fold anisotropy:

$$H = \sum_{\langle i,j \rangle} \cos(\phi_i - \phi_j) - h_p \sum_i \cos(p\phi_i) + V_p \sum_i \cos(p(\phi_i - \zeta_{i_x}))$$

- ζ_{i_x} uniformly distributed in $[-\pi, \pi]$
- $i = (i_x, i_y) \in Z^2$.
- $p > 4$

This model has the same replicated action with:

$$K = \frac{p^2 T}{4\pi}$$
$$h_p \sim g$$
$$W \sim \frac{V_p^2}{a}$$

The Saddle point equations

$$vG_c^{-1}(q,\omega_n) = \frac{1}{\pi \overline{K}}((vq)^2 + \omega_n^2) + m^2 + \Sigma_1(1-\delta_{n,0}) + I(\omega_n)$$

$$I(\omega_n) = \frac{2Wv}{(\pi\alpha)^2\hbar}\int_0^{\beta\hbar}\left[e^{-4\hbar\tilde{B}(\tau)} - e^{-4\hbar B(u>u_c)}\right]\times (1-\cos(\omega_n\tau))d\tau$$

$$\Sigma_1 = u_c(\sigma(u>u_c) - \sigma(u<u_c)) = [\sigma](u>u_c)$$

$$\sigma(u) = \frac{2Wv}{(\pi\alpha)^2}e^{-\hbar 4B(u)}\beta\delta_{n,0}$$

$$m^2 = \frac{4gv}{\pi\alpha}e^{-2\hbar\tilde{G}(0)}$$

$$\tilde{B}(\tau) = \tilde{G}(0) - \tilde{G}(\tau)$$

$$B(u) = \tilde{G}(0) - G(u)$$

The solutions

(Giamarchi, Le Doussal, Orignac 1999)

Only in the limit $K \to 0$ = Strongly Repulsive Interactions

$$\downarrow$$

Three Regimes

1. RS with $m \neq 0$: the dirty "Mott" Insulator
2. One step RSB with $m \neq 0$: the Mott Glass
3. One step RSB with $m = 0$: the Anderson Insulator

Characteristic lengthscales

- ξ = Correlation length in the dirty Mott Insulator

$$\boxed{\xi^2 = \frac{v^2}{(\pi \overline{K} m^2)}}$$

- d = Correlation length in the clean Mott Insulator

$$\boxed{\frac{1}{d^2} = \frac{4g\overline{K}}{(\alpha v)}}$$

- l_0 = Correlation length in the Anderson Insulator

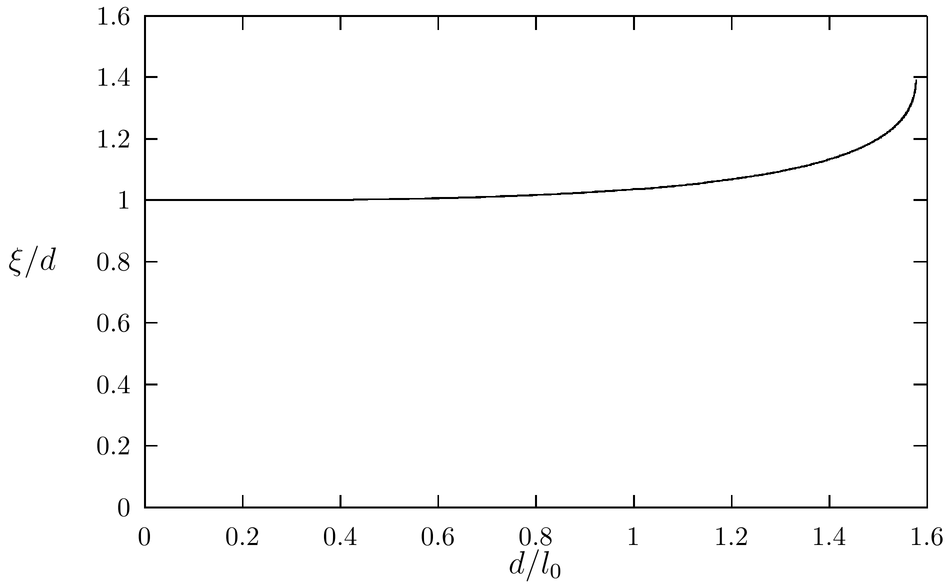
$$\boxed{\frac{1}{l_0^3} = \frac{16W\overline{K}^2}{(\alpha v)^2}}$$

The dirty Mott Insulator

- Replica Symmetric solution
- finite correlation length
- Conductivity gap
- Zero Compressibility due to $m \neq 0$

The behavior of the correlation length in the dirty Mott insulator

$$\left(\frac{l_0}{\xi}\right)^2 \exp\left[\frac{1}{16}\left(\frac{\xi}{l_0}\right)^3\right] = \left(\frac{l_0}{d}\right)^2$$



Solutions only for

$$d/l_0 < 2\left(\frac{4}{3e}\right)^{1/3} \simeq 1.577\ldots$$

The behavior of the conductivity

$$\omega^* = \frac{v}{\xi}$$

$$I(\omega_n) = \frac{(\omega^*)^2}{\pi \overline{K}} f\left(\frac{\omega_n}{\omega^*}\right)$$

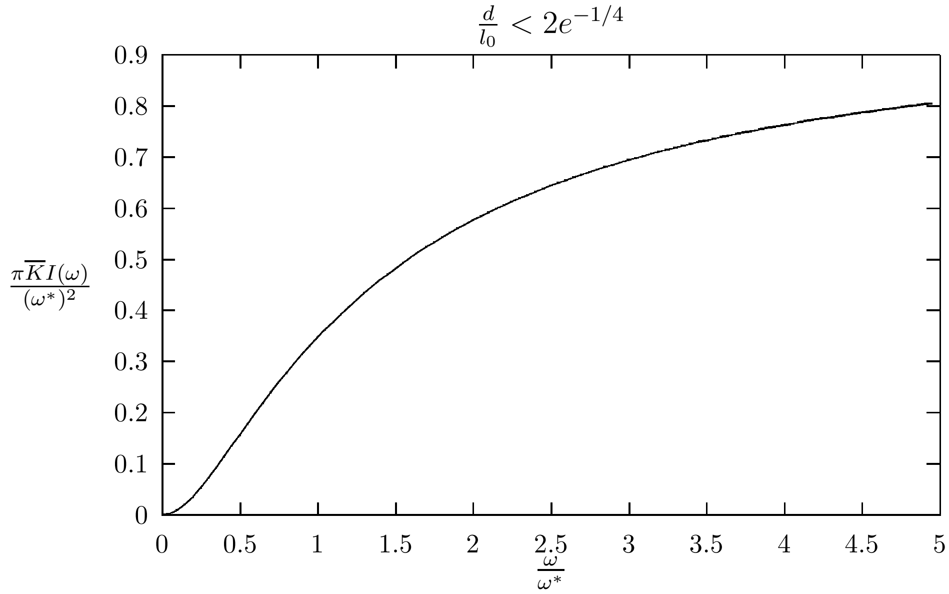
$$f(x) = \frac{1}{4}\left(\frac{\xi}{l_0}\right)^3 \left[1 - \frac{1}{\sqrt{1 + x^2 + f(x)}}\right]$$

$$\sigma(\omega) = \frac{v\overline{K}}{\pi(\omega^*)^2} \frac{\imath\omega}{\left(1 + f(\imath\frac{\omega}{\omega^*}) - \left(\frac{\omega}{\omega^*}\right)^2\right)}$$

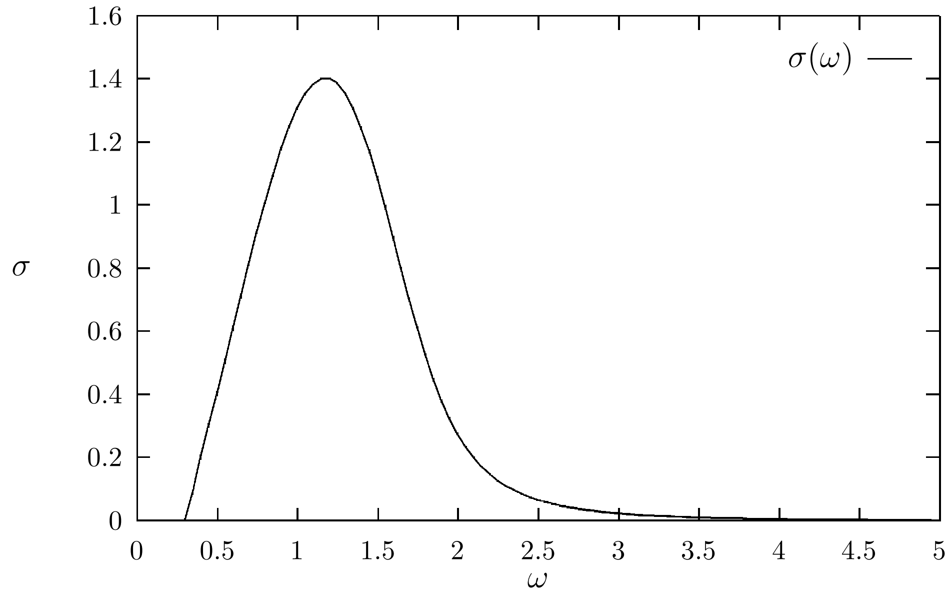
Two Regimes:

- $d/l_0 < 2e^{-1/4} \simeq 1.557\ldots$ Weak disorder
- $d/l_0 > 2e^{-1/4}$ Strong disorder

The Regime $d/l_0 < 2e^{-1/4}$



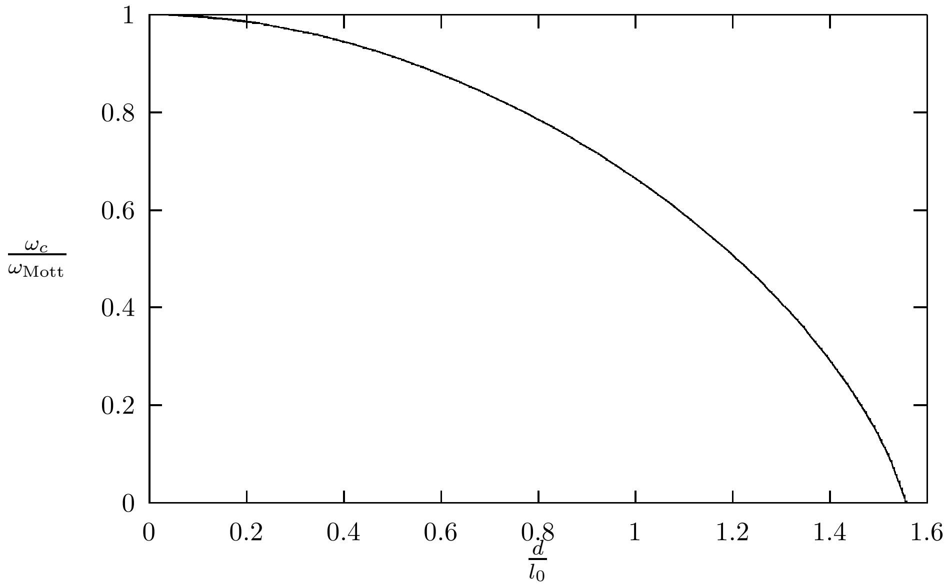
$$I(\omega) \propto \omega^2 \Rightarrow \text{Conductivity Gap}$$



$$\omega \to \omega_c + 0 \Rightarrow \mathrm{Re}\sigma(\omega) \sim \sqrt{\omega - \omega_c}$$

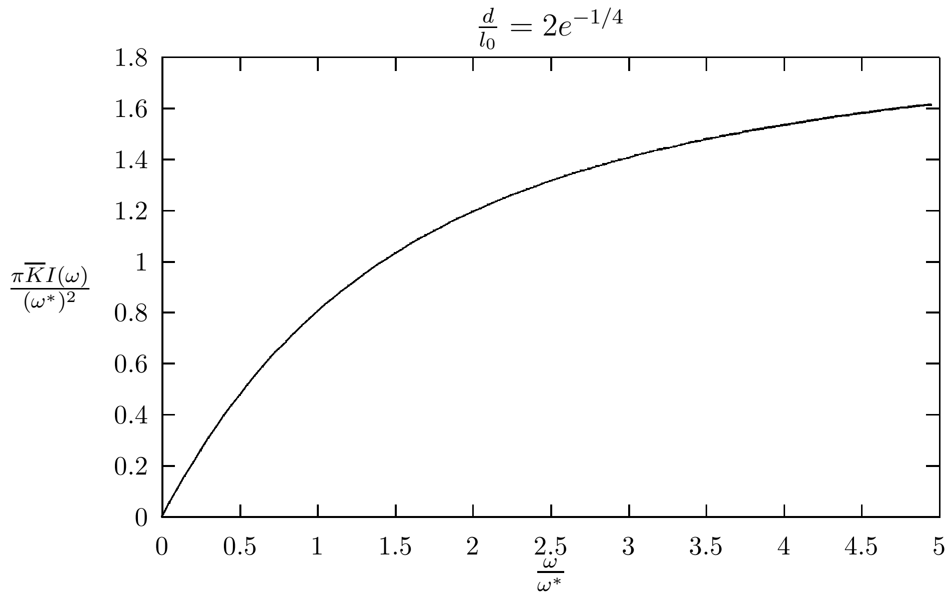
Behavior of the conductivity gap as a function of disorder

$$\boxed{\mathrm{Re}\sigma(\omega) = 0 \leftrightarrow \omega < \omega_c}$$

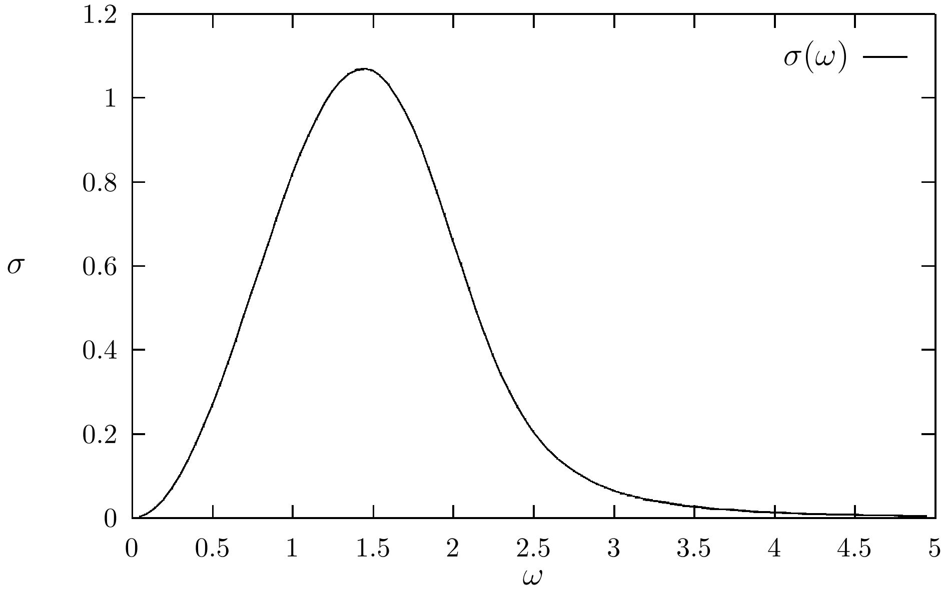


- The conductivity Gap ω_c goes to zero at $\frac{d}{l_0} = 2e^{-1/4}$
- $\omega_c \propto \left(2e^{-1/4} - \frac{d}{l_0}\right)^{\nu}$, $\boxed{\nu = 1}$
- Instability of the RS solution ($I(\omega_n) \to -\infty$) for $d/l_0 > 2e^{-1/4}$)

The critical point $\frac{d}{l_0} = 2e^{-1/4}$



$I(\omega) \propto |\omega| \Rightarrow \mathrm{Re}\sigma(\omega) \propto \omega^2$



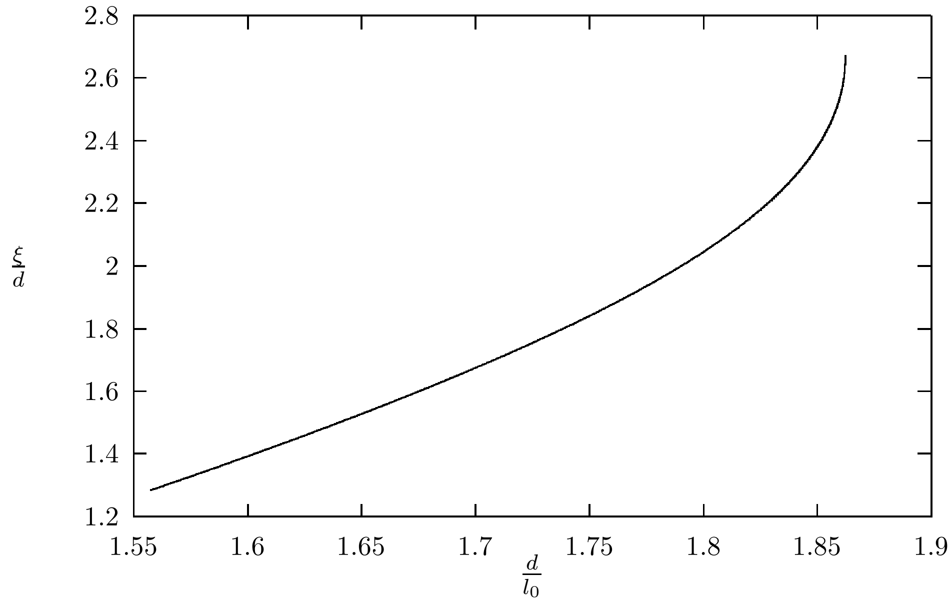
However, Incompressible !

For $d/l_0 > 2e^{-1/4}$, $\Rightarrow$ Development of RSB.

The Mott Glass, $\frac{d}{l_0} > 2e^{-1/4}$

- One step of replica symmetry breaking
- finite correlation length
- pseudogap in conductivity, $\sigma(\omega) \sim \omega^2$
- Incompressible due to $m \neq 0$

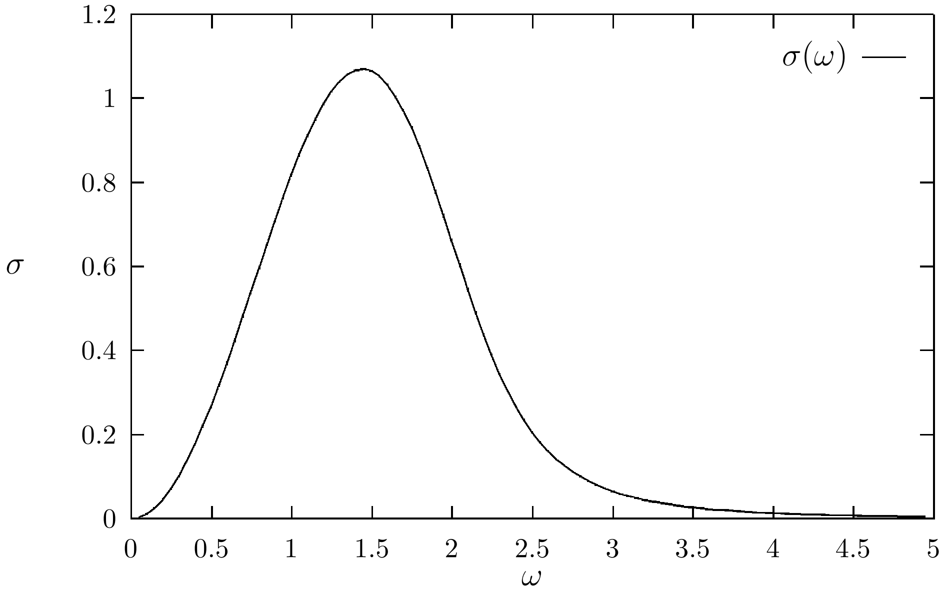
Correlation length in the Mott Glass



For $d/l_0 > 1.861\ldots$, the Mott Glass is unstable

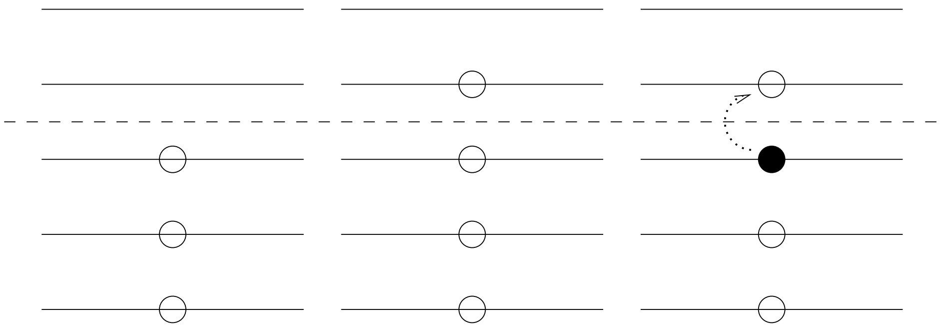
$\Rightarrow$ Anderson Insulator recovered !

Conductivity in the Mott Glass



- Conductivity pinned at $d/l_0 = 2e^{-1/4}$.
- pseudogap as in the Anderson Insulator $\sigma(\omega) \sim \omega^2$

How can we have a pseudogap in the conductivity in an incompressible system ?



Impossible for a non-interacting system

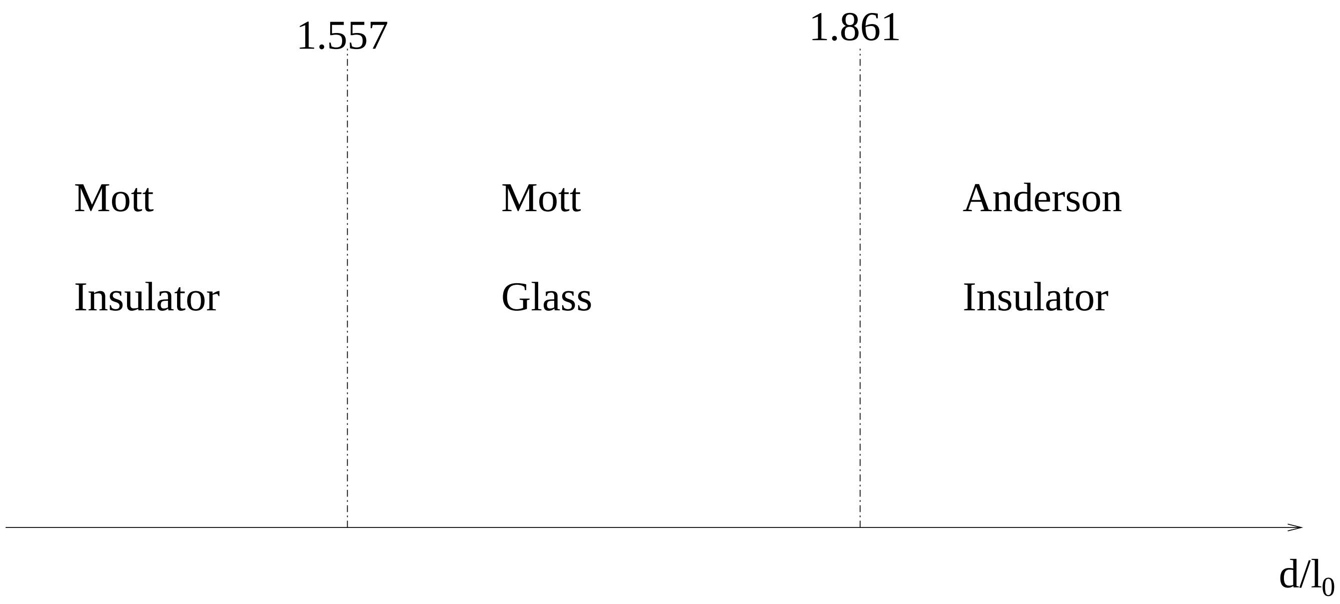
$\Rightarrow$ possible with long range interactions: 1D Wigner Crystal.

$$S = \int dx d\tau \left[\frac{(\partial_\tau \phi)^2 + v^2 (\partial_x \phi)^2}{2\pi v K} + \int \frac{dx'}{\pi^2} V(x-x') \partial_x \phi \partial_{x'} \phi \right]$$

$$\Rightarrow \begin{cases} \sigma(\omega) = \lim_{q\to 0} \frac{\imath(\omega+\imath 0)}{q^2 V(q) - (\omega+\imath 0)^2} \propto \delta(\omega) \\ \chi = \lim_{q\to 0} \lim_{\omega \to 0} \frac{q^2}{q^2 V(q) - \omega^2} = 0 \end{cases}$$

$\Rightarrow$ interactions can depress the particle-hole excitation gap by attraction in the final state.

Phase Diagram



* Conductivity Gap
* Incompressible

* Pseudogap in conductivity
* Incompressible

* Pseudogap in conductivity
* Finite Compressibilty

Conclusions

1. A model of disorder + gap tractable by a variational approach.
2. Description of a disordered Mott Insulator
3. New phase between the Mott and the Anderson Insulator
   - incompressible
   - pseudogap in the conductivity
4. “Excitonic” interpretation
5. Full phase diagram

Perspectives

1. Dynamics in the Mott Glass phase ?
2. finite temperature ?
3. interaction strength needed to suppress the MG ?
4. Transition Mott Glass Anderson Insulator ?
5. Application of a chemical potential on the Mott Glass ?
6. Mott Glass phase in other systems than the one we considered ?