The multichannel Kondo-Heisenberg chain

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P. Azaria and P. Lecheminant cond-mat/9912406
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P. Azaria, P. Lecheminant and A. A. Nersesyan Phys. Rev. B**58** p. R8881 (1998)
E. Orignac, N. Andrei Phys. Rev. B **62** p. R3596 (2000)

The Kondo Effect - single channel

Dilute distribution of magnetic impurities in a metal.

$$H = \sum_{k,a} \epsilon(k) c^{\dagger}_{k,a} c_{k,a} + \lambda_K \sum_{a,b} c^{\dagger}_a(0) \vec{\sigma}_{a,b} c_b(0).\vec{S}$$

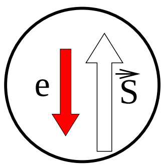
- Weak Coupling at high temperatures:

$$\begin{aligned} \rho(T) &\sim \ln \frac{D}{T} \\ \chi(T) &\sim \frac{\mu^2}{T}\left(1 - \frac{\lambda_K}{\ln \frac{D}{T}} + \cdots\right) \end{aligned}$$

- Strong Coupling at low temperatures:

$$\begin{aligned} \frac{\partial \lambda_K}{\partial l} &= {\lambda_K}^2 \\ \lambda_K(l) &\to \infty \end{aligned}$$

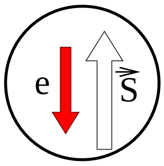
formation of a Kondo singlet: screening.



The Kondo Lattice

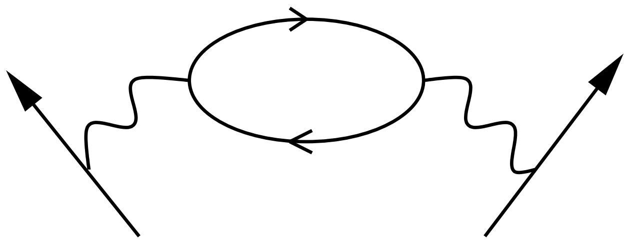
Two Competing Effects:

A. Formation of Kondo Singlets on every site



- Quenching of *local moments*.
- Fermi Iiquid
  - *no. electrons* $\neq$ *no. local moments*.
- Kondo Insulator
  - *otherwise*.

B. RKKY Effective Interaction



- Ordering of *local moments*.
- important for actinides, rare earth compounds.
- Interplay leads to many possibilities

Kondo Lattice in 1D

There are powerful analytical and numerical tools available:

Analytic Approaches

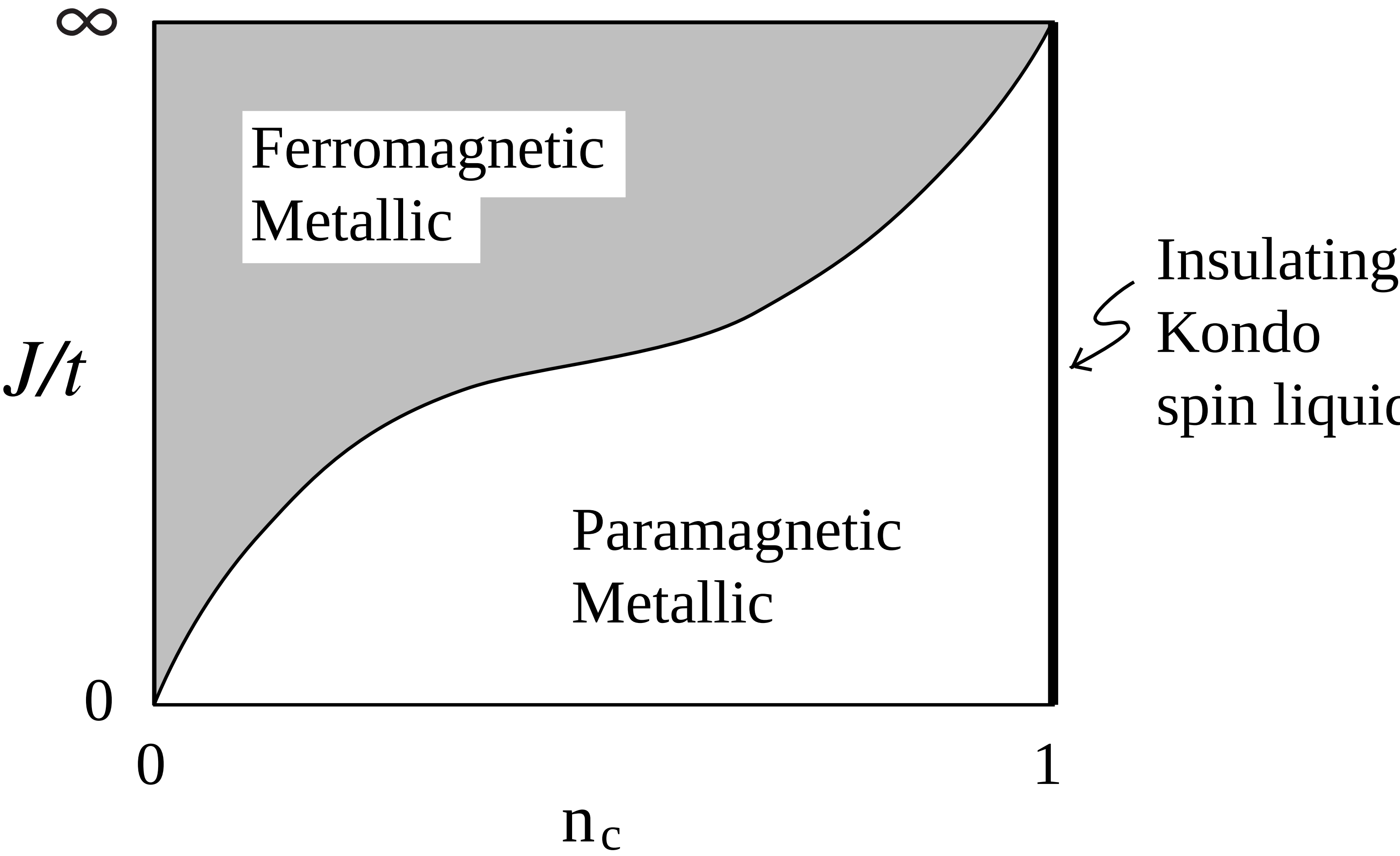
- bosonization - in the vicinity of half filling (Kawakami-Fujimoto '94)
  - a spin gap at half filling
  - a charge gap at half filling (Kondo insulator)
  - away from half filling:
    - large Fermi surface
    - Luttinger Liquid
- large N approach (Tsvelik '99)

  - a paramagnetic metal.

Numerical Approaches

- DMRG
- Exact Diagonalization of clusters

Phase diagram
of the one dimensional
Kondo Lattice



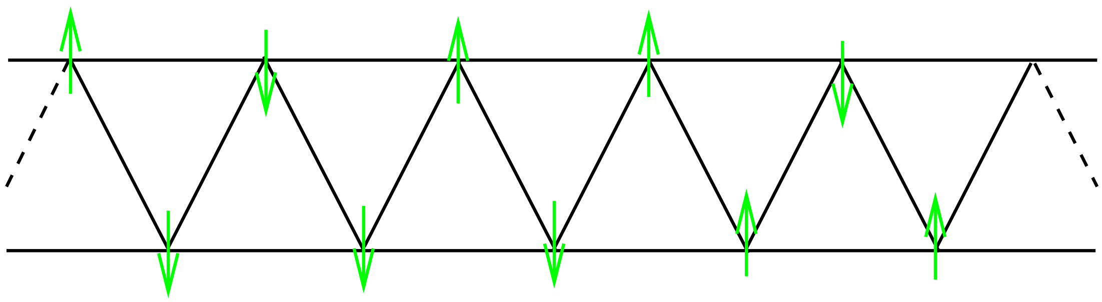
From Shibata and Ueda J. Phys. Cond. Matt. **11** R1 (1999)

Kondo-Heisenberg Lattice

The Hamiltonian

$$H = \sum_{i,a} \left(c^{\dagger}_{i,a} c_{i+1,a} + H.c. \right) + \lambda_H \sum_i \vec{S}_i d \vec{S}_{i+1}$$
$$+ \lambda_K \sum_{i,a,b} c^{\dagger}_{i,a} \vec{\sigma}_{a,b} c_{i,b} . \vec{S}$$

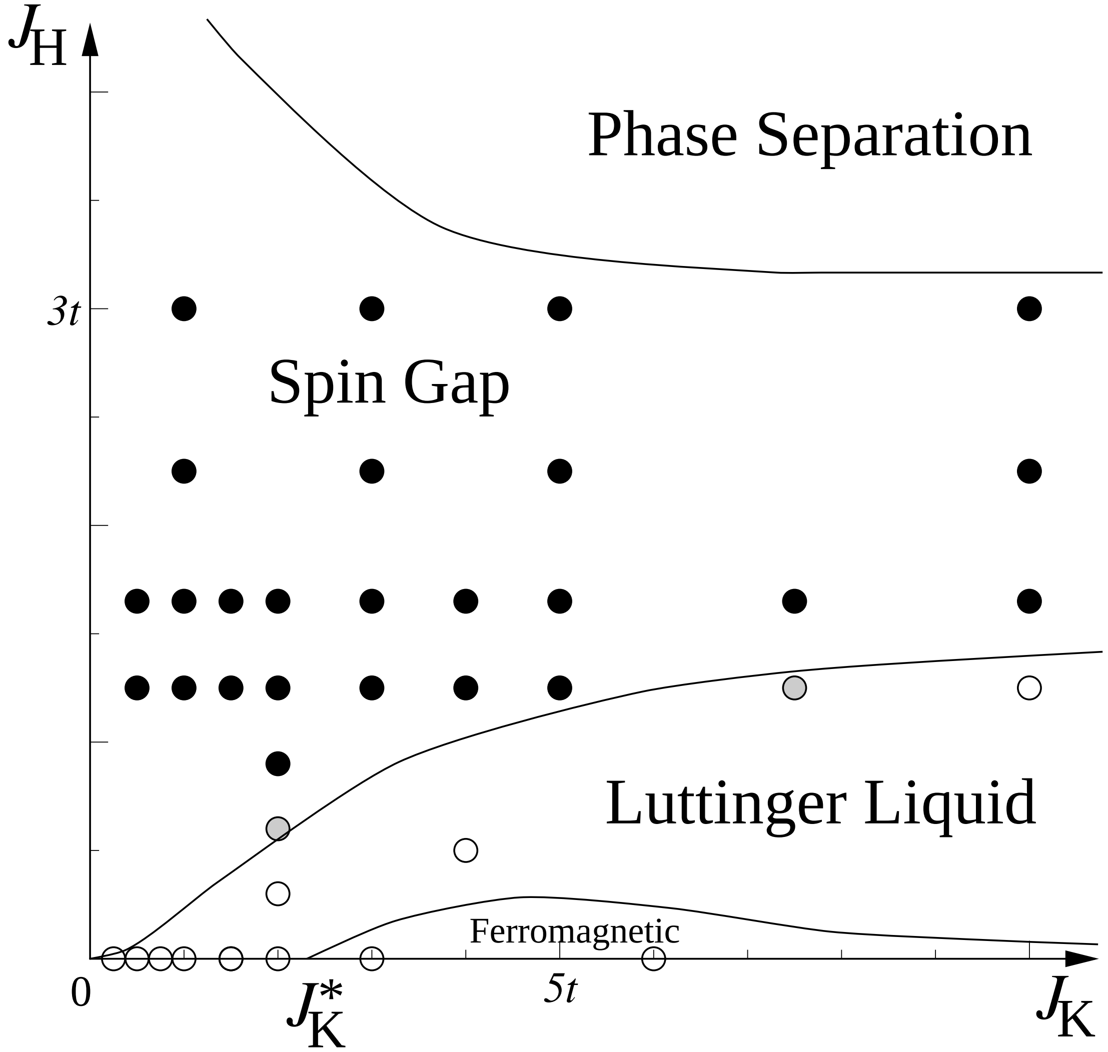
Away from half filling the spin sector is described by a zig-zag spin ladder.



Frustrated system $\rightarrow$ develops a spin gap

- Metal with a spin gap!
- Displays non-conventional pairing. (Affleck, White 96, Sikkema, Affleck, White 97)

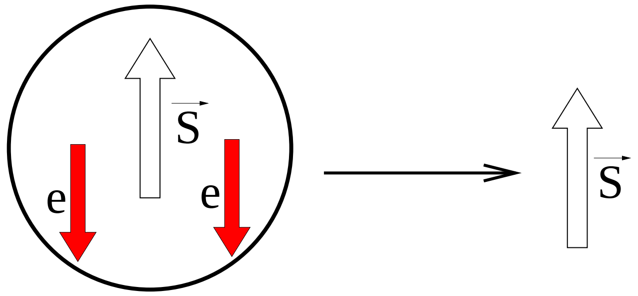
Phase diagram
of the one dimensional
Kondo-Heisenberg Lattice



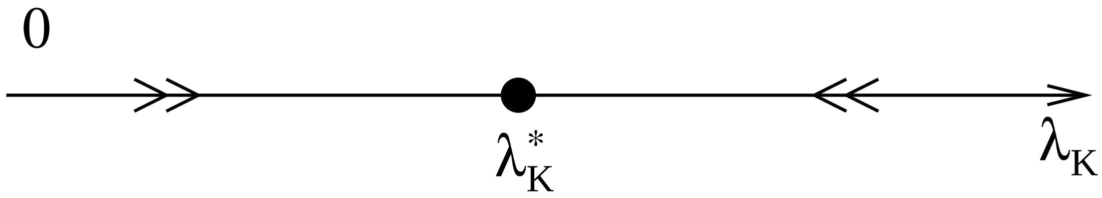
From A. E. Sikkema et al. Phys. Rev. Lett **79** p. 929 (1997)

The multichannel Kondo problem

Consider $N > 1$ bands of conduction electrons coupled to a single impurity of spin $S = \frac{1}{2}$



The $\lambda_K = \infty$ fixed point is unstable



- The multichannel lattice?
- Potential applications U^{4+}, Ce^{3+} compounds, (Cox)

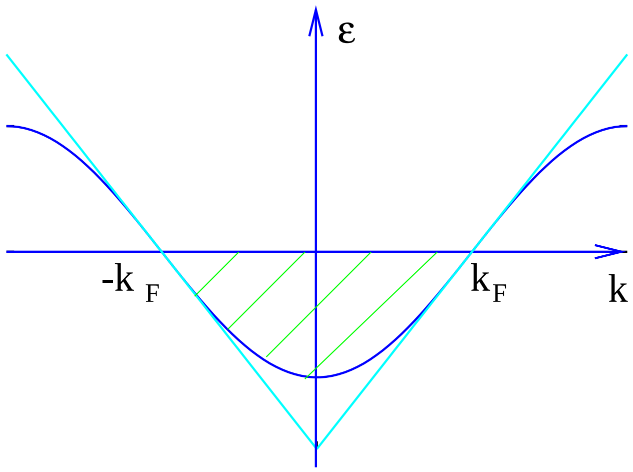
The multichannel 1D Kondo-Heisenberg Lattice

The Hamiltonian

$$H = \sum_{\substack{i,n,a \\ 1 \geq n \leq N}} \left(c^\dagger_{i,n,a} c_{i+1,n,a} + H.c. \right) + \lambda_H \sum_i \vec{S}_i . \vec{S}_{i+1}$$

$$+\lambda_K \sum_{i,n,a,b} c^\dagger_{i,n,a} \vec{\sigma}_{a,b} c_{i,n,b} . \vec{S}_i$$

The continuum limit



Keep low energy modes:

$$\frac{c_{i,n,a}}{\sqrt{a_0}} = \psi_{n,a}(x) = e^{\imath k_F x} \psi_{R,n,a}(x) + e^{-\imath k_F x} \psi_{L,n,a}(x)$$

The continuum Hamiltonian,

$$H^{fermion}_{cont} = -i v_F \sum_{n,a} \int dx \left(\psi^\dagger_{R,n,a} \partial_x \psi_{R,n,a} - \psi^\dagger_{L,n,a} \partial_x \psi_{L,n,a} \right)$$

Non abelian bosonization of the continuum Hamiltonian

The local moments

$$H_{latt}^{moments} = \lambda_H \sum_i \vec{S}_i . \vec{S}_{i+1}$$

becomes:

$$H_{cont}^{moments} = \frac{2\pi}{3} v_s \int dx \left(\vec{S_R} \vec{S_R}(x) + \vec{S_L} \vec{S_L}(x) \right)$$

with $v_s = \frac{\pi}{2} \lambda_H a$

and the spin currents satisfy,

$$[S_R^\alpha(x), S_R^\beta(y)] = \imath \epsilon^{\alpha\beta\gamma} S_R^\gamma(x) \delta(x-y) + \frac{k}{2\pi} \delta'(x-y), \qquad k = 1$$

$\rightarrow$ $SU(2)_1$ Kac-Moody Algebra, $\rightarrow$:
The $SU(2)_1$ Wess Zumino Novikov Witten (WZNW) Model.

Non abelian bosonization of the continuum Hamiltonian

Electrons The electron Hamiltonian

$$H^{fermion}_{cont} = -iv_F \sum_{n,a} \int dx \left(\psi^{\dagger}_{R,n,a} \partial_x \psi_{R,n,a} - \psi^{\dagger}_{L,n,a} \partial_x \psi_{L,n,a} \right)$$

can be expressed in terms of currents:

$U(1)$ charge currents:

$$J_R(x) = \psi^{\dagger}_{R,n,a} \psi_{R,n,a} \qquad J_L = \psi^{\dagger}_{L,n,a} \psi_{L,n,a}$$

$SU(N)_2$ channel currents:

$$T^{\alpha}_R(x) = \psi^{\dagger}_{R,n,a} (t^{\alpha})_{n,n'} \psi_{R,n',a} \qquad T^{\alpha}_L = \psi^{\dagger}_{L,n,a} (t^{\alpha})_{n,n'} \psi_{L,n',a}$$

$SU(2)_N$ spin currents:

$$\vec{\sigma}_R(x) = \psi^{\dagger}_{R,n,a} (\vec{\sigma})_{a,a'} \psi_{R,n,a'} \qquad \vec{\sigma}_L(x) = \psi^{\dagger}_{L,n,a} (\vec{\sigma}_{a,a'} \psi_{L,n,a'}$$

$$H^{fermion}_{cont} = v_F \int \frac{2\pi}{3} (J^2_R + J^2_L)$$

$$+v_F \int \frac{2\pi}{N+2} (\vec{\sigma}_R \vec{\sigma}_R + \vec{\sigma}_L \vec{\sigma}_L) + v_F \int \frac{2\pi}{N+2} (T^{\alpha}_R T^{\alpha}_R + T^{\alpha}_L T^{\alpha}_L)$$

The Kondo coupling

We now wish to couple the electrons and the local moments:
- Incommensurate filling: $2k_F \neq \frac{\pi}{a}$

Then:

- $\vec{S}_i = a\left(\vec{\sigma}_R(x) + \vec{\sigma}_L(x)\right) + e^{\imath\pi\frac{x}{a}}\vec{n}(x)$
- $\vec{\sigma}_i = c^{\dagger}_{i,b}\vec{\sigma}_{b,b'}c_{i,b'} =$
  $a\left(\vec{\sigma}_L(x) + \vec{\sigma}_R(x)\right) + e^{\imath 2k_F x}\vec{O}_{SDW}(x) + e^{-\imath 2k_F x}\vec{O}^{\dagger}_{SDW}(x)$

and the coupling between the electrons and the localized moments is:

$$H_K = \lambda_K a \int dx \left(\vec{\sigma}_L(x) + \vec{\sigma}_R(x)\right)\left(\vec{S}_L(x) + \vec{S}_R(x)\right)$$

It preserves *charge/spin/channel* separation.

$\rightarrow$ We can focus on the spin modes.

The Spin Hamiltonian

In the continuum,

$$
\begin{aligned}
H \quad = \quad & v_F \int \frac{2\pi}{N+2}[\vec{\sigma}_R(x)\vec{\sigma}_R(x) + \vec{\sigma}_L(x)\vec{\sigma}_L(x)] \\
& + v_s \int \frac{2\pi}{3}[\vec{S}_R(x)\vec{S}_R(x) + \vec{S}_L(x)\vec{S}_L(x)] \\
& + \lambda_k^f \int [\vec{S}_R(x)\vec{\sigma}_R(x) + \vec{S}_L(x)\vec{\sigma}_L(x)] \\
& + \lambda_k^b \int [\vec{S}_R(x)\vec{\sigma}_L(x) + \vec{S}_L(x)\vec{\sigma}_R(x)]
\end{aligned}
$$

Near the weak coupling fixed point,

- $\vec{S}_R(x)\vec{\sigma}_L(x)$, $\vec{S}_L(x)\vec{\sigma}_R(x)$ - relevant perturbations.
- $\vec{S}_R(x)\vec{\sigma}_R(x)$, $\vec{S}_L(x)\vec{S}_L(x)$ - marginal perturbations
- velocity difference - marginal perturbation

Strategy: drop marginal perturbations, identify fixed point and study relevance of neglected operators there.

The fixed point

The Hamiltonian to study:

$$
\begin{aligned}
H &= H_1 + H_2 \\
H_1 &= v \int [\frac{2\pi}{3} \vec{S}_R \vec{S}_R(x) + \frac{2\pi}{N+2} \vec{\sigma}_L \vec{\sigma}_L] + \lambda_K^b \int \vec{S}_R \vec{\sigma}_L \\
H_2 &= R \to L
\end{aligned}
$$

The λ_K^b perturbation is relevant - where does the hamiltonian flow?

Can find the fixed point via *chiral stabilization*:
The flow preserves $c_R - c_L$ and $k_R - k_L$, minimizes $c_R + c_L$.
(Andrei, Douglas, Jerez 98)

$\rightarrow$ The fixed point is:

$$M_N \otimes SU(2)_{N-1}$$

- The minimal model M_N, central charge $c = 1 - \frac{6}{(N+1)(N+2)}$ describes the localized moments .
- The WZNW $SU(2)_{N-1}$ describes the electrons (spin sector).

Consequences

- a *coset singlet* is formed in the ground state
- the remaining degrees of freedom are,

$$c_{spin} = 1 - \frac{6}{(N+1)(N+2)} + \frac{3(N-1)}{N+1}$$
$$c_{total} = 2N + 1 - \frac{12}{(N+1)(N+2)}$$

Thus, $\frac{6}{(N+1)(N+2)}$ local moments modes pair with the same number of electron modes.

- specific heat and susceptibility

$$C_V = \frac{\pi}{6} c_{total} T$$
$$\chi = \frac{1}{2\pi v}(N-1) = \frac{1}{2\pi v}(N+1) - \frac{1}{2\pi v}2$$

As if two modes have disappeared.

- There are not enough modes to carry spin if we remove $\frac{6}{(N+1)(N+2)}$ modes from $SU(2)_1$.

Consequences

- For $N = 2$,

$$c = 1 - \frac{6}{(N+1)(N+2)} = \frac{1}{2}, \qquad \to M_2 = Ising$$

- Spin excitations described by - $Ising \otimes SU(2)_1$.
- The local moments form a band of Majorana fermions.
- A triplet Majorana bands (electrons and moments) at the fixed point.
- Explicit solution is possible (Azaria, Lecheminant, Nersesyan 1998) and confirms our approach.

Consequences

- Electron Correlation functions

$$-i\langle T\psi(x,t)\psi^\dagger(0,0)\rangle$$

$$= \frac{e^{\imath k_F x}}{(x-vt)^{1+\delta_N/2}(x+vt)^{\delta_N/2}} + \frac{e^{-\imath k_F x}}{(x-vt)^{\delta_N/2}(x+vt)^{1+\delta_N/2}}$$

where

$$\delta_N = \frac{3}{(N+1)(N+2)}$$

$\rightarrow$ Non Fermi liquid

$\rightarrow$ Fermi liquid recovered for $N \rightarrow \infty$

- Conventional order parameters

$$\begin{aligned} O_{CDW} &= \psi^\dagger_{R,a,n}\psi_{L,a,n} \\ \vec{O}_{SDW} &= \psi^\dagger_{R,a,n}(\vec{\sigma})_{a,b}\psi_{L,b,n} \\ O_{SSC} &= \psi^\dagger_{R,a,n}(\sigma_y)_{a,b}\psi_{R,b,n} \\ \vec{O}_{TSC} &= \psi^\dagger_{R,a,n}(\vec{\sigma}\sigma_y)_{a,b}\psi_{R,b,n} \end{aligned}$$

We find:

$$\langle O_A(x,t)O_A(0,0)\rangle = \left(x^2-(vt)^2\right)^{(1+\delta_N)}$$

where: A= CDW, SDW, SSC, TSC.

Consequences

- Non conventional order parameters

$$\begin{aligned} O_{c-CDW} &= \vec{n}.\vec{O}_{SDW} \\ O_{c-SP} &= \vec{n}.\vec{O}_{TSC} \end{aligned}$$

- pair correlations with O_{c-SP} are <u>odd</u> in time :

- bound pairs of electrons + spin

$\rightarrow$ Odd frequency pairing (Berezinsky 79)

- The non-conventional correlators,

$$\langle O_B(x,t) O_B(0,0) \rangle = \left(x^2 - (vt)^2\right)^{3/2 - 3/(N+2)}$$

where: B= c-CDW, c-SP

- Two different regimes:

1. $\frac{3}{2} - \frac{3}{N+2} < 1 + \delta_N, \quad N \leq 4$
   - Non conventional order parameters have dominant correlations.
   - Electrons and moments are tightly bound
2. $\frac{3}{2} - \frac{3}{N+2} > 1 + \delta_N, \quad N \geq 6$
   - Conventional order parameters are dominant
   - Electrons and moments are loosely bound
   - For $N \rightarrow \infty$ recover free electrons.

The case $N = 5$: depends on the approach to the fixed point

Conclusions

1. The 1D Multichannel Kondo-Heisenberg lattice displays a Non Fermi-liquid phase of the *chiral liquid* type.
2. for N=2, its properties are known from Azaria et al. (1998) and our solution is in full agreement with the explicit solution.
3. For less than 5 channels odd frequency becomes the dominamt instability.
4. For more that 5 channels conventional order parameters dominate.

   Fermi liquid is recoved in the limit $N \to \infty$

Perspectives

1. Investigate integrability of the model:
   $\rightarrow$ the Kondo scales, the exhaustion problem.
2. Investigate the effect of interchain coupling
   - two channel models
   - mean field theory
3. What about the pure Kondo limit?
4. Hubbard interactions
5. Superconductivity and magnetism