The spin tube in a magnetic field *

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Yamanaka, Oshikawa, Affleck (1997):

If $NS - m$ *is integer there can be a gap above a non degenerate ground state.*

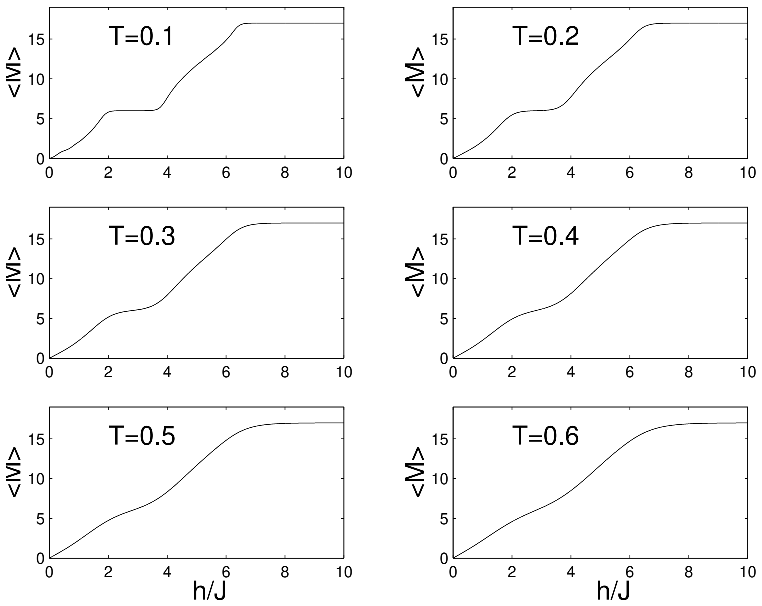
$N =$ Number of coupled chains, $S =$ Spin on each site.

$m =$ Magnetization per site

$\rightarrow$ Gap in magnetic excitations of 3 coupled spin 1/2 chains for $m = 1/2$.

$\rightsquigarrow$ Magnetization plateau at $m = 1/2$

Magnetization process of three coplanar spin 1/2 chains.



36 sites, $J_{\perp} = 3J$

From Tandon *et al.* Phys. Rev. B **59** p. 396 (1999).

Effective Hamiltonian for three coplanar chains for $1/2 < m < 3/2$:

$\rightarrow$ derived for $J_\perp / J \gg 1$.

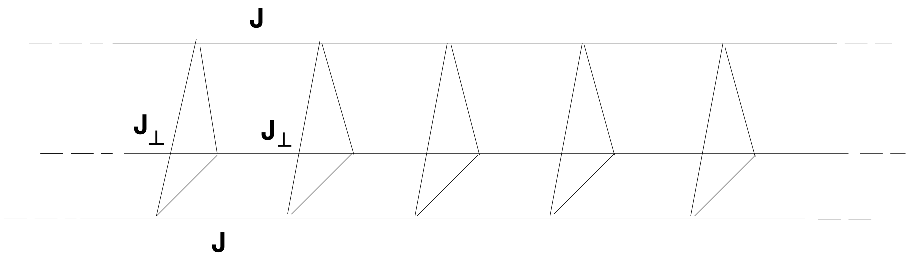
$$H_{eff} = J \sum_i \left(\sigma_i^x \sigma_{i+1}^x + \sigma_i^y \sigma_{i+1}^y + \frac{1}{2} \sigma_i^z \sigma_{i+1}^z \right) - \left(\frac{h}{2} - \frac{3J_\perp}{2} - \frac{J}{2} \right) \sum_i \sigma_i^z$$

Magnetization per Rung:

$$S_i^z = 1 + \sigma_i^z$$

Effective spin 1/2 XXZ Chain
$\rightsquigarrow$ One Component Luttinger Liquid
for $1/2 < m < 2/3$.

The spin tube model

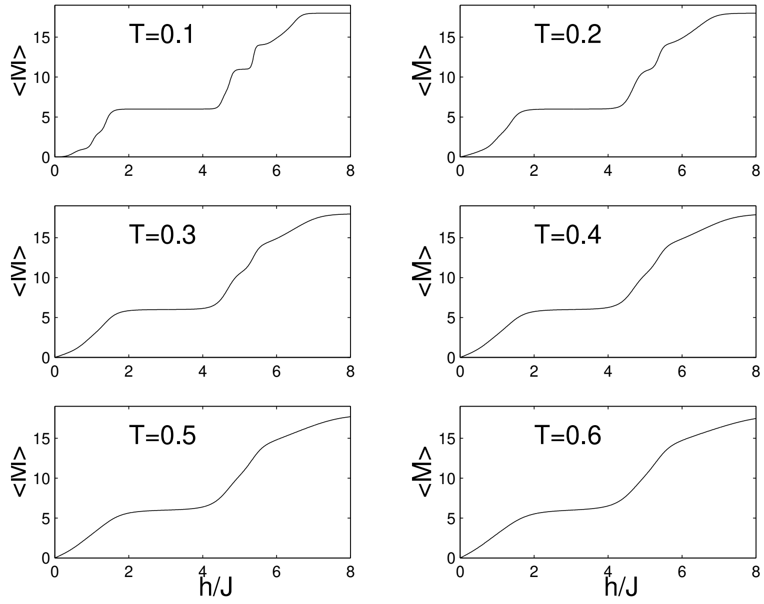


$$
\begin{aligned}
H &= J \sum_{i=1}^{N} \sum_{p=1}^{3} \vec{S}_{i,p} \vec{S}_{i+1,p} \\
&+ J_{\perp} \sum_{i=1}^{N} \sum_{p=1}^{3} \vec{S}_{i,p} \vec{S}_{i,p+1} \\
&- \vec{h} \sum_{i=1}^{N} \sum_{p=1}^{3} \vec{S}_{i,p}
\end{aligned}
$$

$\rightarrow$ frustrated model.

$\neq$ Three chains in a plane.

Magnetization process of the spin tube.



36 sites, $J_\perp = 3J$

From Tandon *et al.* Phys. Rev. B **59** p. 396 (1999).

QUESTIONS

- What is the effective model describing the transition from $m = 1/2$ to $m = 3/2$ in the spin tube ?

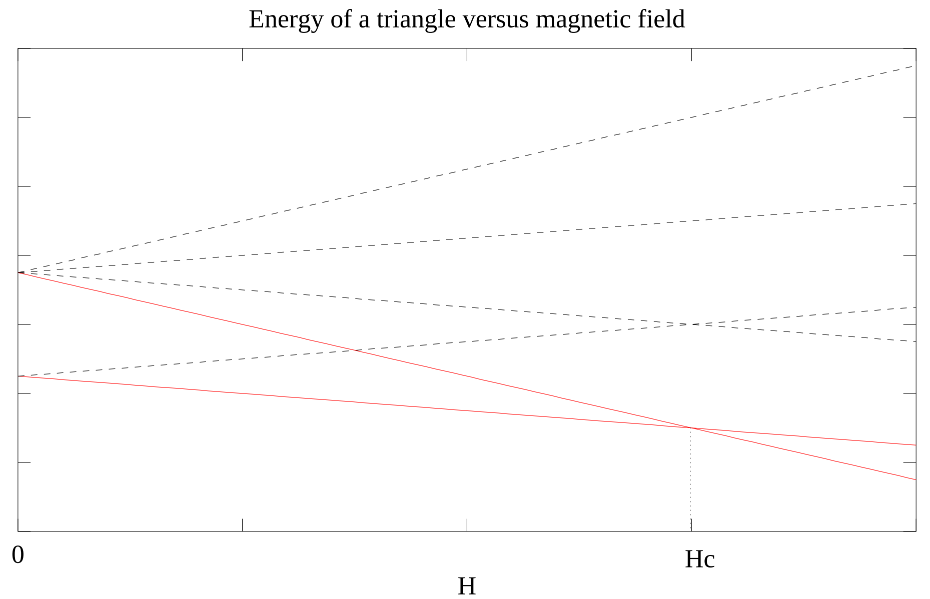
- Is it an effective XXZ chain or a more complicated model ?

- Is there a real plateau at m=1 ?

- Can such plateau be obtained from the effective model ?

Isolated Triangle

$$
\begin{aligned}
H &= \frac{J_\perp}{2}\left((\vec{S}_1+\vec{S}_2+\vec{S}_3)^2-\frac{9}{4}\right) \\
&- \vec{h}.(\vec{S}_1+\vec{S}_2+\vec{S}_3)
\end{aligned}
$$



Strong Magnetic field Retain 3 states:

- $|1\rangle = |S = 3/2, S^z = 3/2\rangle$
- $|2\rangle = |S = 1/2, S^z = 1/2, +\rangle$
- $|3\rangle = |S = 1/2, S^z = 1/2, -\rangle$

3 states per site $\rightsquigarrow$ Gell'Mann Matrices λ^p, $p = 1, \ldots, 8$.

(3 $\times$ 3 traceless Hermitian matrices $\rightarrow$ generators of SU(3))

spins as a function of SU(3) spin operators
λ^p, $p = 1, \ldots, 8$.

$$
\begin{aligned}
S^{+}_{i,p} &= \frac{1}{2\sqrt{3}}[j^{p-1}(\lambda_i^6 + i\lambda_i^7) + j^{2(p-1)}(\lambda_i^4 - i\lambda_i^5)] \\
S^{z}_{i,p} &= \frac{1}{3}[\frac{5}{6}I - \frac{\lambda_i^8}{\sqrt{3}} - j^{2(p-1)}(\lambda_i^1 + i\lambda_i^2) \\
&- j^{(p-1)}(\lambda_i^1 - i\lambda_i^2)]
\end{aligned}
$$

Magnetization per triangle:

$$S_i^z = 5/6 - \lambda_i^8/\sqrt{3}$$

$\rightarrow$ Replace in the effective Hamiltonian.

Effective Hamiltonian for $J_\perp \gg J$:

$$
\begin{aligned}
H_{\text{eff.}} &= \tilde{H}_0 + \tilde{H}_I, \\
\tilde{H}_0 &= \frac{J}{4}\sum_{i=1}^{N}\sum_{\alpha=1}^{8} \lambda_i^\alpha \lambda_{i+1}^\alpha \\
\tilde{H}_I &= -\frac{J}{12}\sum_{i=1}^{N}[\lambda_i^1\lambda_{i+1}^1 + \lambda_i^2\lambda_{i+1}^2] \\
&- \frac{J}{4}\sum_{i=1}^{N}\lambda_i^3\lambda_{i+1}^3 - \frac{5J}{36}\sum_{i=1}^{N}\lambda_i^8\lambda_{i+1}^8 \\
&+ \frac{h - h_c - 5J/9}{\sqrt{3}}\sum_{i=1}^{N}\lambda_i^8
\end{aligned}
$$

NOT INTEGRABLE ! $\rightsquigarrow$ Approximate methods.

Bosonized Hamiltonian for the anisotropic SU(3) spin chain.

$$
\begin{aligned}
H &= \int \frac{dx}{2\pi}\left[u_a K_a (\pi \Pi_a)^2 + \frac{u_a}{K_a}(\partial_x \phi_a)^2\right] \\
&+ \int \frac{dx}{2\pi}\left[u_b K_b (\pi \Pi_b)^2 + \frac{u_b}{K_b}(\partial_x \phi_b)^2\right] \\
&+ \sum_{i=1}^{3} \frac{2g_i}{(2\pi a)^2} \int dx \cos(\sqrt{8}\vec{\alpha}_i . \vec{\phi})
\end{aligned}
$$

$$\vec{\phi} = (\phi_a, \phi_b).$$

ϕ_a = Chirality excitations,

ϕ_b = Magnetic Excitations.

$\vec{\alpha}_i$'s = root vectors of SU(3).

$[\phi_\alpha(x), \Pi_\beta(x')] = \imath \delta_{\alpha,\beta} \delta(x - x')$

Expression of spin operators

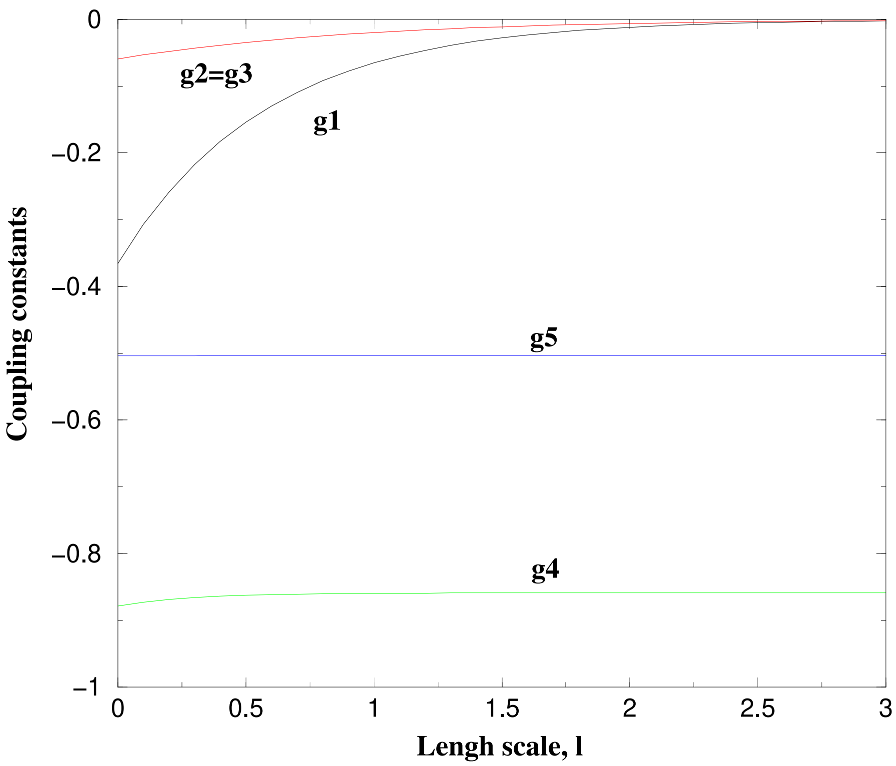
$$
\begin{aligned}
\Lambda^1 &+ \imath\Lambda^2 = \frac{2e^{\imath\sqrt{2}\theta_a}}{\pi a}\left[\cos\sqrt{2}\phi_a + \cos(\frac{2\pi}{3a}x - \sqrt{\frac{2}{3}}\phi_b)\right] \\
\Lambda^3 &= -\frac{\sqrt{2}}{\pi}\partial_x\phi_a + \frac{\cos\left(2\frac{\phi_b}{\sqrt{6}} - \frac{2\pi}{3a}x\right)\sin\sqrt{2}\phi_a}{\pi a} \\
\Lambda^4 &+ \imath\Lambda^5 = \frac{2e^{\imath\frac{\theta_a+\sqrt{3}\theta_b}{\sqrt{2}}}}{\pi a}\left[\cos\frac{\phi_a+\sqrt{3}\phi_b}{\sqrt{2}}\right. \\
+ &\quad \left.\cos\left(\frac{\sqrt{3}\phi_a - \phi_b}{\sqrt{6}} + \frac{2\pi}{3a}x\right)\right] \\
\Lambda^6 &+ \imath\Lambda^7 = \frac{2e^{\imath\frac{\theta_a-\sqrt{3}\theta_b}{\sqrt{2}}}}{\pi a}\left[\cos\frac{\phi_a-\sqrt{3}\phi_b}{\sqrt{2}}\right. \\
+ &\quad \left.\cos\left(\frac{\sqrt{3}\phi_a + \phi_b}{\sqrt{6}} + \frac{2\pi}{3a}x\right)\right] \\
\Lambda^8 &= -\frac{\sqrt{2}}{\pi}\partial_x\phi_b + \frac{e^{\imath\frac{2\pi}{3a}x}}{\pi\sqrt{3}a}\left[e^{-\imath\sqrt{\frac{2}{3}}\phi_b}\cos\sqrt{2}\phi_a - e^{\imath\sqrt{\frac{8}{3}}\phi_b}\right] + \text{H.c.}
\end{aligned}
$$

Renormalization Group Equations

$$
\begin{aligned}
\frac{dy_1}{dl} &= (2-2K_a)y_1 - \frac{y_2 y_3}{2} \\
\frac{dy_2}{dl} &= \left(2 - \frac{K_a}{2} - 3\frac{K_b}{2}\right) y_2 - \frac{y_1 y_3}{2} \\
\frac{dy_3}{dl} &= \left(2 - \frac{K_a}{2} - 3\frac{K_b}{2}\right) y_3 - \frac{y_1 y_2}{2} \\
\frac{d}{dl}\left(\frac{1}{K_a}\right) &= \frac{y_1^2}{2} + \frac{y_2^2}{8} + \frac{y_3^2}{8} \\
\frac{d}{dl}\left(\frac{1}{K_b}\right) &= 3\frac{y_2^2}{8} + 3\frac{y_3^2}{8}
\end{aligned}
$$

$$
y_i = \frac{g_i}{\pi u}
$$

RG flow



$\rightarrow$ Gapless behavior

$\rightarrow$ Two Component Luttinger Liquid

$\rightarrow$ All correlation functions can be obtained as a function of K_a^*,K_b^*.

$\rightarrow$ straightforward generalization to $\langle S^z \rangle \neq 5/6$.

Is there a plateau at $m = 1$?

What could cause a breakdown of the preceding picture ?
$\rightarrow$ Special commensurate filling that cause a fermion Umklapp.

At $m = 1$:

$$H_{\text{Umklapp}} = \frac{2g_U}{(2\pi a)^2} \cos(4\sqrt{\frac{2}{3}}\phi_b)$$

If this Umklapp is relevant, it can cause a Magnetization Plateau at $m = 1$.

In the following, we assume it is relevant and explore the consequences.

Bosonization description of the $m = 1$ plateau.

We have $\langle \phi_b \rangle = \ldots.$
As a result:

$$\langle T_\tau \Lambda^\alpha(r) \Lambda^\alpha(0,0) \rangle \sim e^{-r/\xi}$$

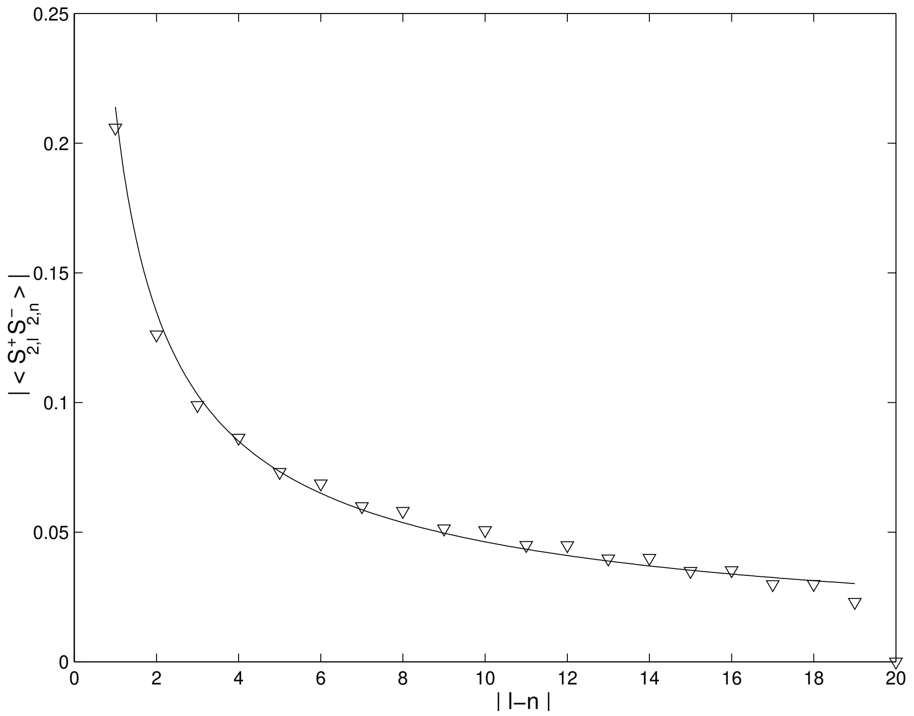
For $\alpha = 4, 5, 6, 7$.
Also, $\Lambda_{1,2,3,8}$ show power law correlations.
This leads to:

$$\langle S^+_{n,p} S^-_{n',p'} \rangle \sim e^{-|n-n'|/\xi}$$

$$\langle S^z_{n,p} S^z_{n',p'} \rangle \sim \frac{1}{|n-n'|^\eta}$$

How does this compare with the results of Sen et al. for correlation functions ?

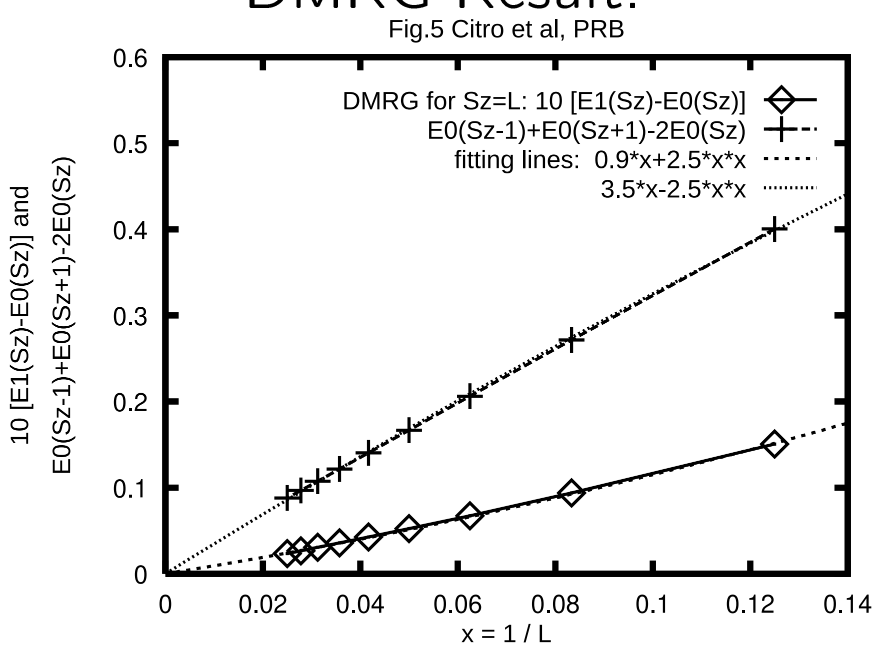
Comparison with the results of Tandon et al.



$$\langle S^+_{n,p} S^-_{n',p} \rangle \sim \frac{1}{|n-n'|^\alpha} \neq e^{-|n-n'|/\xi}$$

Predictions of bosonization $\neq$ Numerics.
Too small system in numerics ?

DMRG Result:



$\rightarrow$ No Gap for $L \rightarrow \infty$.

Conclusions

- A frustrated three chain spin ladder has been analyzed.

- A strong coupling Hamiltonian in the form of an anisotropic SU(3) spin chain has been proposed to describe the regime $1/2 < \langle S^z \rangle < 3/2$.

- This model has been shown to lead to a two component Luttinger Liquid behavior.

- The presence of a plateau at $m = 1$ has been ruled out.

- This has been confirmed by DMRG.

Perspectives

- Analyze the anisotropic version of this model and search for a $m = 1$ plateau.

- Describe the $0 < m < 1/2$ regime.

- Map K_a, K_b, u_a, u_b by numerical techniques.

- Look at the consequences for the t-J of Hubbard version of this model.