Disordered Quantum Smectics

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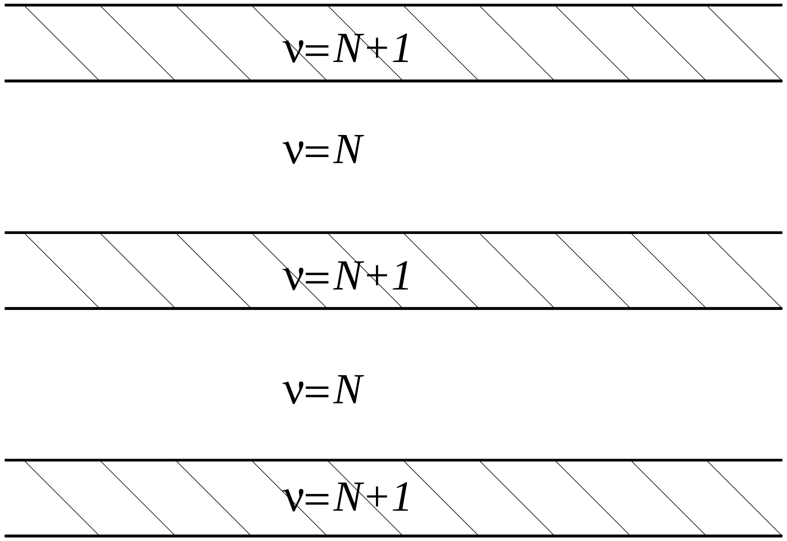
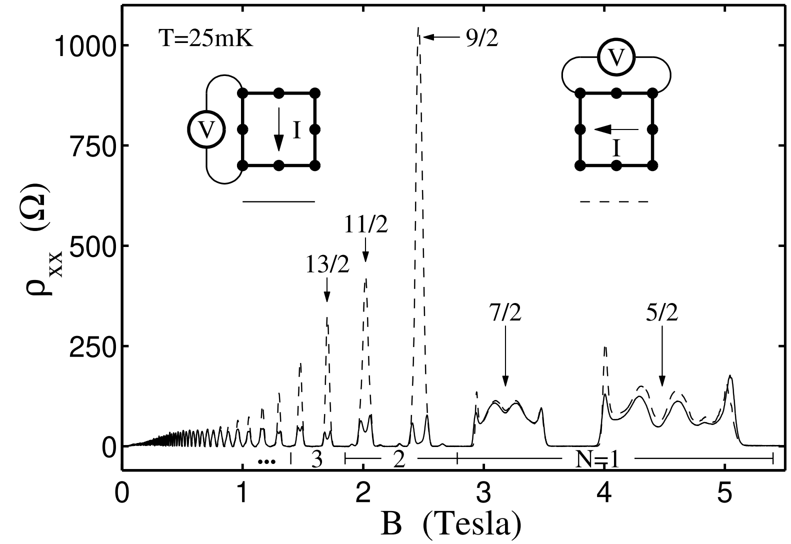
High Landau levels ($\nu = 9/2$)

Hartree Fock $\Rightarrow$ Formation of a stripe phase



(Platzman et al. 1979, Fogler et al. 1996, Moessner and Chalker 1996)

Experiments



M. P. Lilly et al. Phys. Rev. Lett. **82** p. 389 (1999)

$\rightarrow$ evidence for anisotropic phase

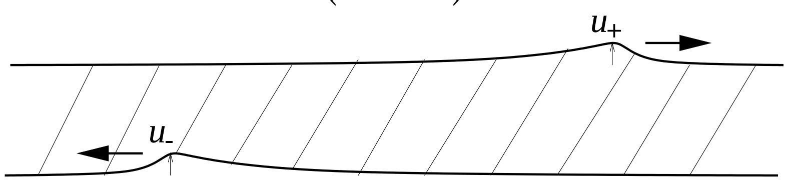
Stability of stripe phase to quantum fluctuations ?

NO !

$\rightarrow$ anisotropic phase $\equiv$ quantum smectic

E. Fradkin and S. Kivelson Phys. Rev. B **59** 8065 (1999)

Hamiltonian description of the quantum smectic
A. H. Mac Donald and M. P. A. Fisher Phys. Rev. B **61** 5724 (2000)



- $\rho_{n,\pm}(x) = \pm u_{n,\pm}(x)/l^2$
- Translation invariance $u_{n,\pm}(x) \to u_{n,\pm}(x) + a$
- Rotation invariance $u_{n,\pm}(x) \to u_{n,\pm}(x) + \tan\theta x$

$$
\begin{aligned}
H &= \int dx \Bigg[\frac{Q}{2} \sum_n (\nabla^2 (u_{n,+} + u_{n,-})(x))^2 \\
&+ \frac{1}{2} \sum_{n,n'} \int dx' \kappa_{n,n'}(x-x')(u_{n,+} - u_{n,-})(x)(u_{n',+} - u_{n',-})(x') \Bigg]
\end{aligned}
$$

Kăc-Moody commutation relations:

$$[\rho_r(x), \rho_{r'}(x')] = \tfrac{ir}{2\pi} \partial_x(\delta(x-x'))$$

$$\rho_r = \tfrac{1}{2}(r - \tfrac{1}{\pi}\partial_x \phi)$$

$$[\phi(x), \Pi(x')] = i\delta(x-x')$$

Π : stripe displacement

$\partial_x \phi$: stripe width

Action with Coulomb interaction

$$S_0 = \int \frac{dq}{2\pi} \frac{1}{\beta} \sum_n \underbrace{\left[\frac{\omega_n^2}{Q q_x^4 + \frac{\kappa_\perp q_y^2}{|\mathbf{q}|}} + \kappa_\parallel \frac{q_x^2}{|\mathbf{q}|} \right]}_{G_0^{-1}(q,\omega_n)} |\phi(q,\omega_n)|^2$$

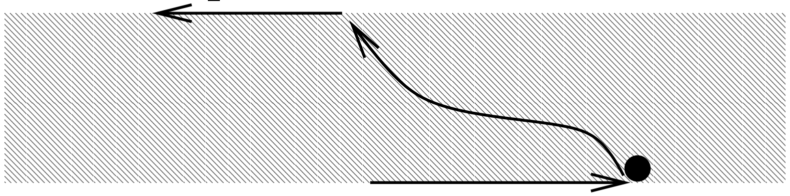
Quasi-long range order

$$\tilde{C}_\phi(x,0,0) \simeq 1/(2\lambda)^2 (\kappa_\perp/\kappa_{||})^{1/2} \ln(|x|/\alpha)$$

- IR divergent correlations in the transverse direction.

Impurity pinning of the quantum smectic ?

Interaction of a quantum smectic with impurities



Backscattering process

$$H_{imp.} = V_0 e^{i2\phi(x_0)} + V_0^* e^{-i2\phi(x_0)}$$

Impurities do not couple directly with stripe displacement

Action of disordered smectic

$$S = S_0 + \int d^2\mathbf{r}d\tau \left[\xi(\mathbf{r})e^{2i\phi(\mathbf{r},\tau)} + \mathrm{H.c.}\right]$$

$$\overline{\xi(\mathbf{r})\xi^*(\mathbf{r}')} = \frac{\Delta}{4}\delta(\mathbf{r} - \mathbf{r}')$$

Replica trick

$$\overline{\ln Z} = \lim_{n\to 0} \frac{\partial}{\partial n} \overline{Z^n}$$

n fields ϕ_a, $a = 1, \ldots, n$.

$$\begin{aligned} S_{\text{replicated}} &= \sum_{a=1}^{n} S_0[\phi_a] \\ &- \frac{\Delta}{4} \sum_{ab} \int dx dy d\tau d\tau' \cos 2[\phi_a(x,y,\tau) - \phi_b(x,y,\tau')] \end{aligned}$$

limit $n \to 0$?

Variational approach

(M. Mézard and G. Parisi J. Phys. I **1** 809 (1991))

$$S_{\mathrm{trial}} = \frac{1}{2\beta} \sum_n \int \frac{d^2\mathbf{q}}{(2\pi)^2} \sum_{a,b} G^{-1}_{ab}(\mathbf{q}, i\omega_n)\phi_a(\mathbf{q}, i\omega_n)\phi_b(\mathbf{q}, i\omega_n)$$

$$F_{\mathrm{var}} = F_{\mathrm{trial}} + \frac{1}{\beta}\langle S_{\mathrm{r}} - S_{\mathrm{trial}}\rangle_{S_{\mathrm{trial}}}$$

$$G^{-1}_{ab}(\mathbf{q}, i\omega_n) = G^{-1}_0(\mathbf{q}, i\omega_n)\delta_{ab} - \Sigma_{ab}(\mathbf{q}, i\omega_n)$$

$$\frac{\partial F}{\partial G^{-1}_{ab}} = 0$$

Saddle point equations

$$\Sigma_{ab}(q,\omega_n) = \Delta \int_0^\beta e^{-4B_{ab}(\tau)} e^{i\omega_n \tau} d\tau$$

$$G_c^{-1}(q,\omega_n) = \sum_b G_{ab}^{-1}(q,\omega_n)$$

$$G_c^{-1}(q,\omega_n) = G_0^{-1}(q,\omega_n) + \Delta \sum_b \int_0^\beta e^{-4B_{ab}(\tau)} (1 - e^{i\omega_n \tau}) d\tau$$

$$B_{ab}(\tau) = G_{aa}(x=0,\tau=0) - G_{ab}(x=0,\tau)$$

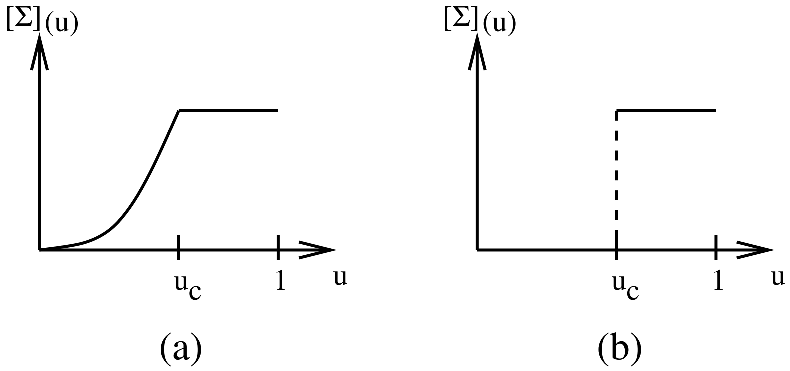
$$\frac{d}{d\tau} G_{a\neq b}(x,\tau) = 0$$

Limit $n \to 0$

Parisi parametrization

$$\begin{aligned}
\Sigma_{aa} &\to \tilde{\Sigma} \\
\Sigma_{a\neq b}(q,\omega_n) &\to \Sigma(q,\omega_n,u), u \in [0,1] \\
\sum_{a\neq b} &\to -\int_0^1 du \\
[\Sigma](u) &= u\Sigma(u) - \int_0^u \Sigma(v)dv
\end{aligned}$$

$$
\begin{aligned}
\Sigma(q,\omega_n,u) &= \Delta\beta\delta_{\omega_n,0}e^{-4B(u)} \\
G_c^{-1}(q,\omega_n) &= G_0^{-1}(q,\omega_n) + \Delta\int d\tau\left[e^{-4\tilde{B}(\tau)} - \int_0^1 du e^{-4B(u)}\right] \\
&\times \quad (1-\cos\omega_n\tau)d\tau
\end{aligned}
$$



(a) Full RSB (b) One-step RSB

One-step replica symmetry breaking solution

$$G_c^{-1}(q, i\omega_n) = G_0^{-1}(q, i\omega_n) + I(i\omega_n) + \Sigma(1 - \delta_{\omega_n,0})$$

$$\Sigma = \frac{\kappa_\parallel}{R_a}$$

$$R_a = \text{Larkin length}$$

$$R_a \simeq \frac{4\pi^2\kappa_\parallel^2\lambda}{\Delta}$$

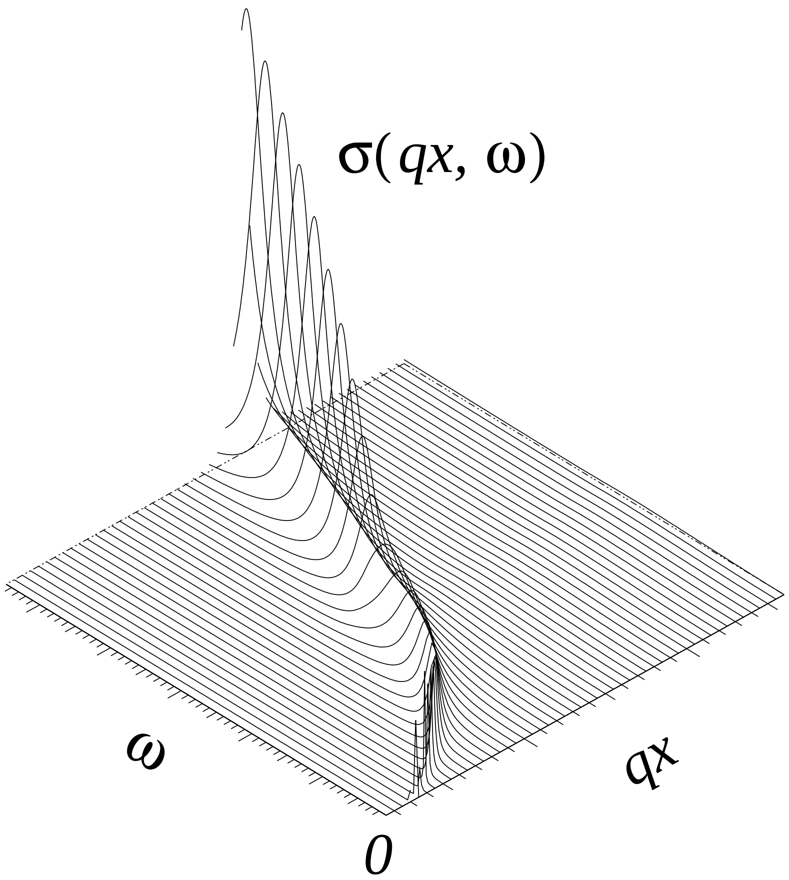
Threshold field: $E_{Tx} = \kappa_\parallel R_a^{-1} l$

$$I(\omega_n) = \text{dissipation}$$

$$I(i\omega_n) = \frac{\lambda\kappa_{||}\Sigma}{4\pi}\int_{-\infty}^{\infty} dq_x \int_{-\frac{\pi}{\lambda}}^{\frac{\pi}{\lambda}} dq_y \left[\frac{1}{\kappa_{||}|q_x| + \Sigma} - \frac{Qq_x^4 + \frac{\kappa_\perp}{|q_x|}q_y^2}{\omega_n^2 + (\kappa_{||}|q_x| + \Sigma + I(i\omega_n))(Qq_x^4 + \frac{\kappa_\perp}{|q_x|}q_y^2)}\right]$$

$$\omega_n \to 0 \Rightarrow I(\omega_n) \sim |\omega_n|^{7/8}$$

Conductivity



$$\sigma_{xx}(q_x, q_y = 0, \omega) = Ci\omega \frac{Qq_x^4}{\omega^2 - (\kappa_{\|}|q_x| + \Sigma + I)Qq_x^4}$$

Quasi-long range order persists

$$\tilde{C}_\phi(x,0,0) \simeq \frac{1}{2\pi\lambda\kappa_{||}} \ln\left(\frac{x}{R_a}\right)$$

- IR divergent correlations in the transverse direction.

Conclusions

- Strong wavelength dependence of conductivity in the pinned quantum smectic
- persistence of quasi-long range order in the presence of impurities

Questions

- Effect of temperature ?
- Conductivity in the transverse direction ?
- Hall conductivity ?