Answering Related Questions

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Sidestepping a question

Given instance I of problem Π, answer Π on a nearby instance

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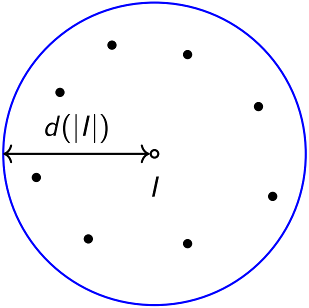

$$\textsc{Sidestep}(\Pi, \mathrm{dist}, d): \quad \text{input } I \longmapsto \text{ output } (J, \Pi(J)),$$

with $\mathrm{dist}(I, J) \leqslant d(|I|)$.

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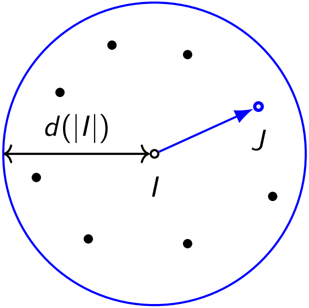

$\Pi(J)$

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Related notions

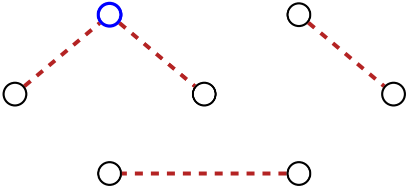
Closest to *certified algorithms*

	who chooses?	guarantee
Sidestep	the algorithm	exact answer on J
Smoothed analysis	Nature	running time on random J
Edge modification	the algorithm	positive $J \in \mathcal{C}$

also linked to hardness of approximation and planted problems

Our two metrics

$$D := G \triangle H.$$



D

$$\mathrm{dist}_e(G,H) = |E(D)| = 4, \qquad \mathrm{dist}_\Delta(G,H) = \Delta(D) = 2.$$

Results

Problem	NP-hard up to	in P at
	dist_e	
Hamiltonian Cycle	$n^{1/2-o(1)}$	$n/3$
Dominating Set	$n^{1-o(1)}$	n/e
Independent Set, Clique, Vertex Cover	$n^{4/3-o(1)}$	$n^{4/3}$
Coloring	$n^{4/3-o(1)}$	$n^{4/3}$
Clique Cover	$n^{4/3-o(1)}$	$n^{4/3}$
	dist_Δ	
Hamiltonian Cycle	0	1
Independent Set, Clique, Vertex Cover	$n^{1/2-o(1)}$	$n^{1/2}$
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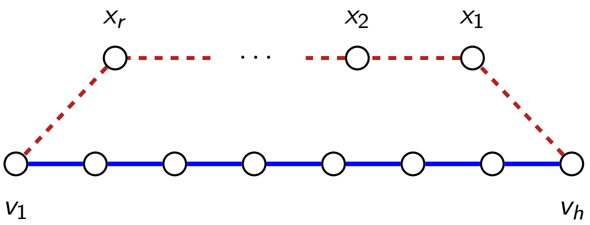
presented

Hamiltonian Cycle: tractability at $n/3$

Theorem

$\textsc{Sidestep}(\textsc{Hamiltonian Cycle}, \mathsf{dist}_e, n/3)$ *is in* P*.*

$P = v_1 \cdots v_h$ edge-maximal, $\quad r = n - h$.



If $r < n/3$: add the (absent) dashed edges $\rightarrow$ positive instance

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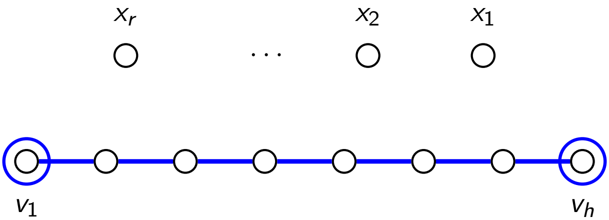
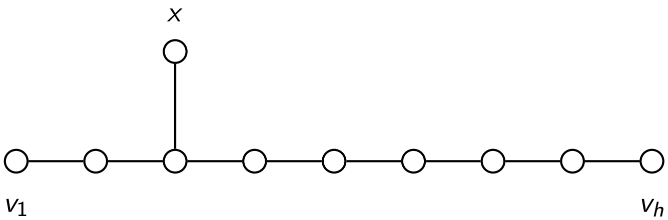


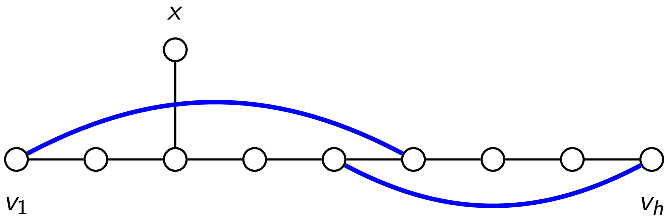
If $r \geqslant n/3$: $h \leqslant 2n/3$, and $N(v_1) \cup N(v_h) \subseteq V(P)$ by maximality.

Path extension



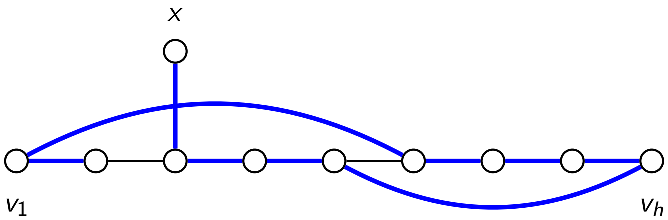
$x \notin V(P)$ and $xv_j \in E(G)$

Path extension



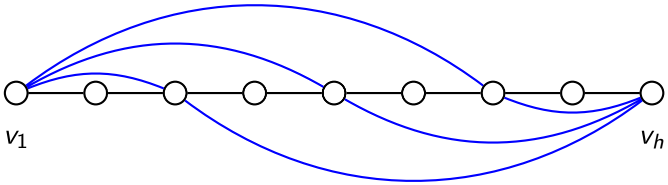
when $v_1 v_{i+1} \in E(G)$ and $v_i v_h \in E(G)$

Path extension



$$P' = xv_j \cdots v_i v_h v_{h-1} \cdots v_{i+1} v_1 \cdots v_{j-1}, \qquad |P'| = h + 1.$$

When no such configuration



$$A := \{i : v_1 v_{i+1} \in E\}, \qquad B := \{i : v_i v_h \in E\}, \qquad A \cap B = \varnothing.$$

$$\deg(v_1) + \deg(v_h) \leqslant h \implies \min\{\deg(v_1), \deg(v_h)\} \leqslant h/2 \leqslant n/3.$$

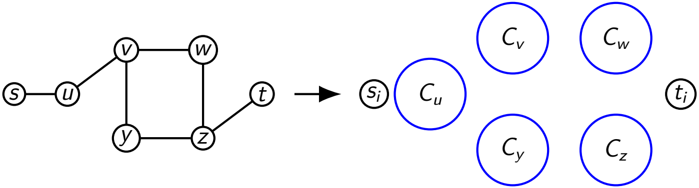
Hamiltonian Cycle: a robust reduction

Theorem

$\textsc{Sidestep}(\textsc{Hamiltonian Cycle}, \mathsf{dist}_e, n^{1/2-o(1)})$ *is* NP*-hard.*

subcubic s–t $\textsc{Hamiltonian Path}(H)$ and $d(s) = d(t) = 1$ is NP-hard,

Garey–Johnson–Stockmeyer '76; Arkin et al. '09.



$$u \longmapsto C_u \simeq K_{q+2},$$

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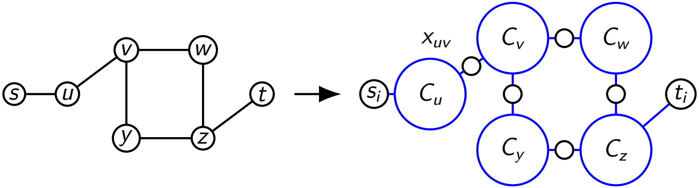
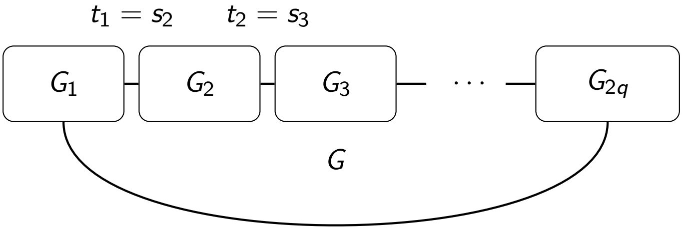


$$u \longmapsto C_u \simeq K_{q+2},$$

$$uv \longmapsto x_{uv} \text{ complete to } C_u \cup C_v.$$

Amplification

Make $2q$ copies and identify $t_i = s_{i+1}$ cyclically.

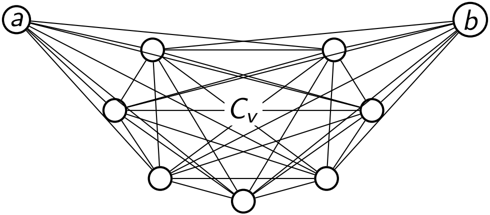


$$\nu := |V(H)|, \qquad n := |V(G)| = \Theta(q^2 \nu), \qquad q = n^{1/2-o(1)}.$$

Yes-instance: robustness against deletions

$$G^- := G - F, \qquad |F| \leqslant q - 1.$$

Ore's theorem with prescribed endpoints:
For every a, b initially complete to C_v, the graph $G^-[C_v \cup \{a, b\}]$ has a spanning a–b path.



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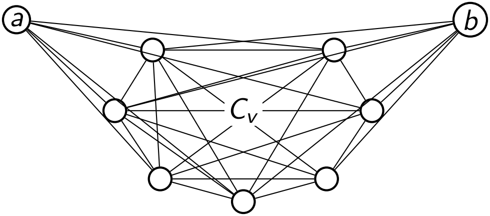

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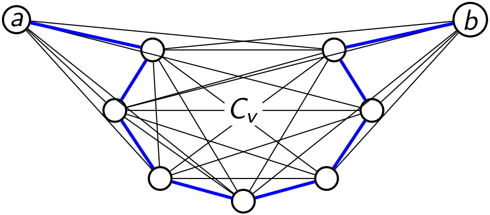

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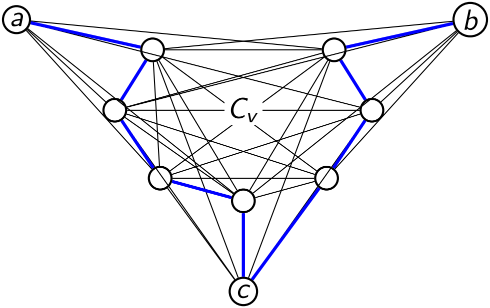
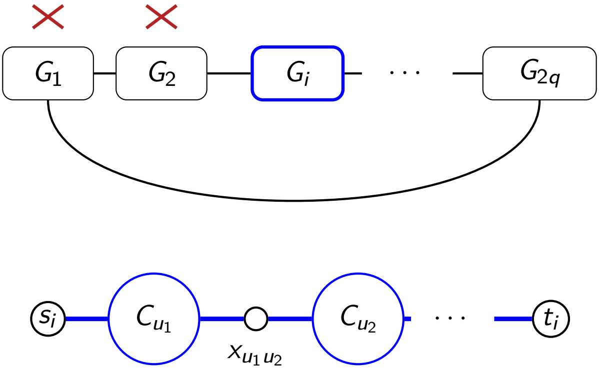


No-instance: robustness against additions

$$G^+ := G + F, \qquad |F| \leqslant q - 1$$



An s_i–t_i spanning path in G_i contradicts the negativity of H

Open questions and directions

Open questions

- Close the gap for HAMILTONIAN PATH, dist_e.
- MAX CLIQUE: optimization radii under both distances.
- TRIANGLE PARTITION: is radius 1 tractable under dist_Δ?

Directions

- Optimization variants.
- One-sided variants.
- Other input spaces: formulas, strings, hypergraphs, and geometry.

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Thank you for your attention!

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