Faster Parameterized Broadcasting

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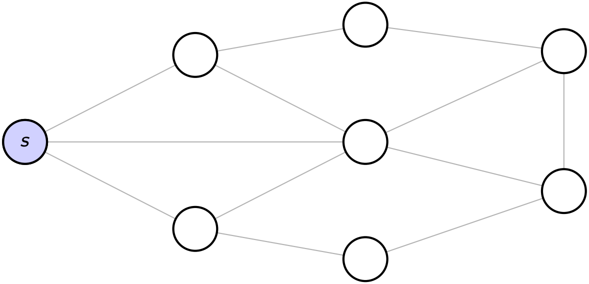
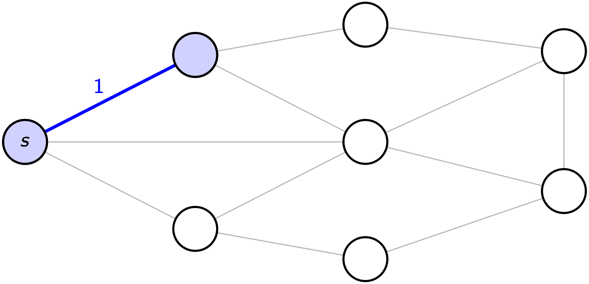
Carl Feghali (1986–2026)

Telephone Broadcast



A designated vertex s is initially the only informed vertex

Telephone Broadcast



Each round: each informed vertex informs (at most) one neighbor

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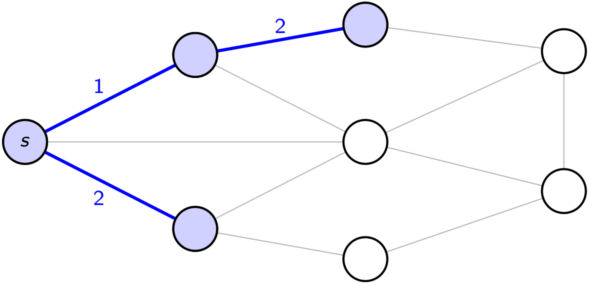

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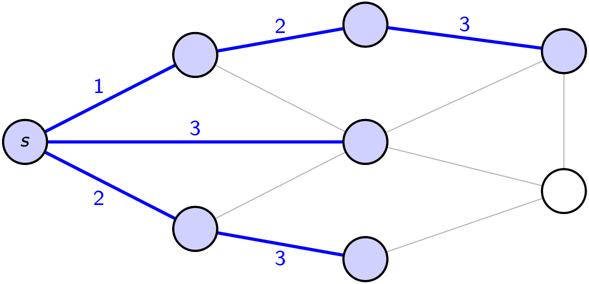

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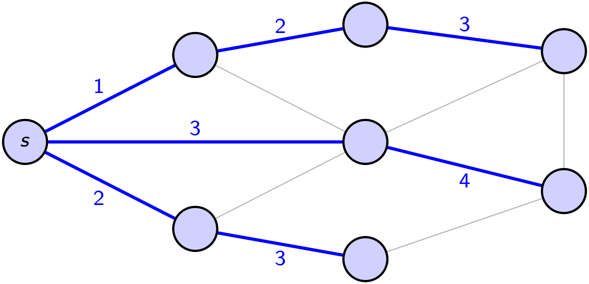


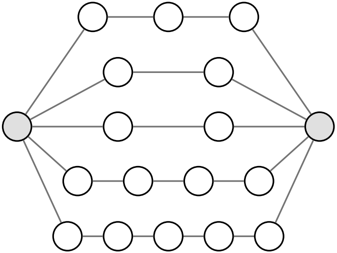
$b(G, s) :=$ minimum number of rounds to inform all vertices

Given $G,\ s \in V(G),\ t \in \mathbb{N}$, does $b(G, s) \leqslant t$ hold?

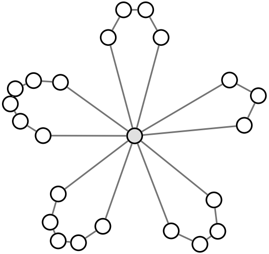
Formally introduced by Slater, Cockayne, and Hedetniemi in 1981

Hardness on very restricted graphs

TELEPHONE BROADCAST is NP-complete even on:



subdivisions of a multiedge
[Tale '25]



cycles identified at one vertex
[Aminian et al. '25]

and on graphs of treedepth at most 6 [Egami et al. '25]

Faster parameterized algorithms

parameter	previous		this work
vertex cover vc	$2^{\mathcal{O}(\mathsf{vc}^3)}$	[Fomin et al. '24]	$2^{\mathcal{O}(\mathsf{vc}\log \mathsf{vc})}$
vertex integrity vi	$2^{2^{\mathcal{O}(\mathsf{vi}^2)}}$	[Egami et al. '25]	$2^{\mathcal{O}(\mathsf{vi}^2\log \mathsf{vi})}$
distance to clique k	$2^{\mathcal{O}(k^2)}$	[Egami et al. '25]	$2^{\mathcal{O}(k\log k)}$

running times above $\times\, n^{\mathcal{O}(1)}$

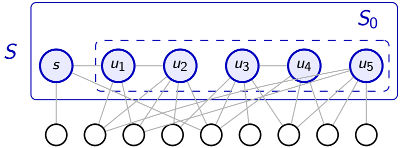
presented

Vertex-cover prefix

We compute

$$S_0: \text{ vertex cover of } G - s$$

$S := S_0 \cup \{s\}$ of size $k \leqslant \mathrm{vc} + 1$



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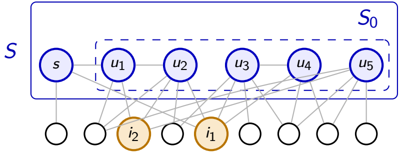
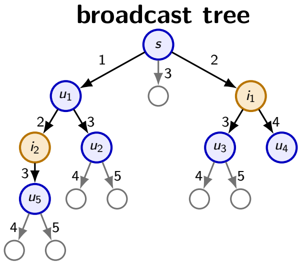
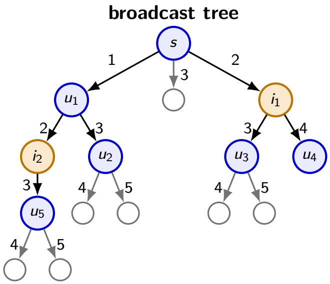




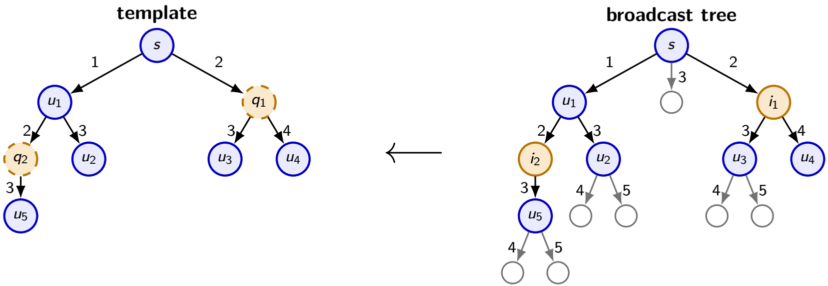
Lemma (Fomin, Fraigniaud, Golovach '24)

Some optimal protocol has S fully informed by round $2k - 1$.

Vertex-cover template



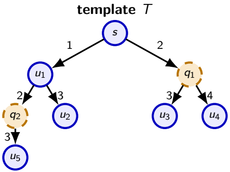
Vertex-cover template



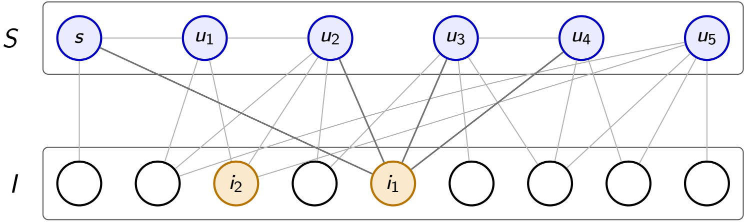
$$\#\text{templates} \leqslant k \cdot (2k)^{2k} \cdot (2k)^{2k} = k^{\mathcal{O}(k)}$$

We try all possible templates

I_h: the candidates i_h for q_h



$$I_h := \{v \in I : N_G(v) \supseteq N_T(q_h)\}$$



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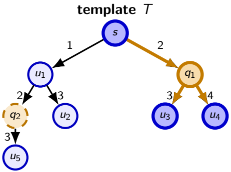

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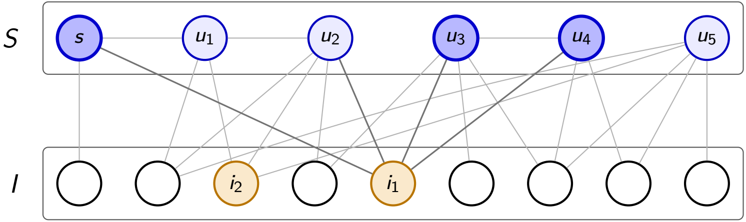

$$N_T(q_1) = \{s, u_3, u_4\}$$

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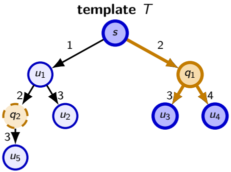

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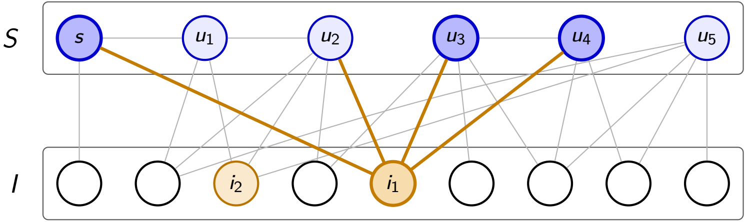
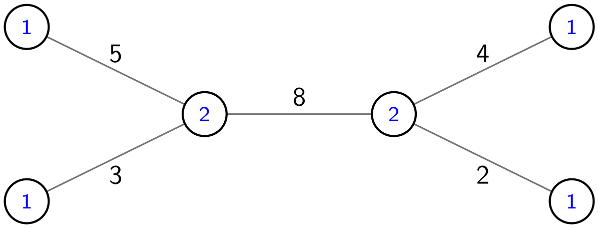


$$N_G(i_1) \supseteq N_T(q_1) \qquad \Longrightarrow \qquad i_1 \in I_1$$

Edge-weighted b-matching

$\beta : V \to \mathbb{N}$ (vertex capacities), $w : E \to \mathbb{N}$ (edge weights)



$M \subseteq E$ is a b-matching: $\forall v \in V, \deg_M(v) \leqslant \beta(v)$

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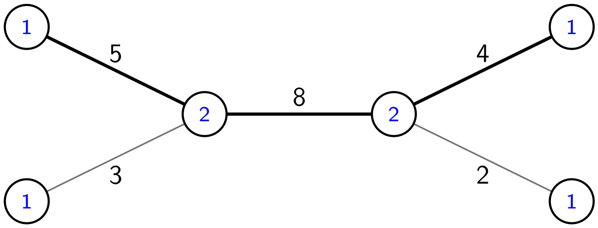


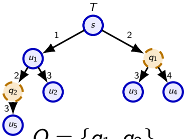
$M \subseteq E$ is a b-matching: $\forall v \in V, \deg_M(v) \leqslant \beta(v)$

A b-matching of maximum weight can be found in polynomial time

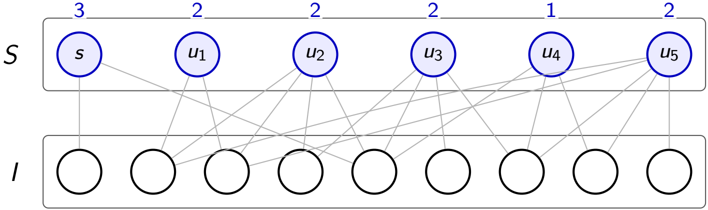
The weighted b-matching of template T

Capacity of $v \in S$: its number of *free* rounds

$$\beta(v) = t - (\text{time } v \text{ is informed}) - |\text{children}_T(v)|$$



$Q = \{q_1, q_2\}$



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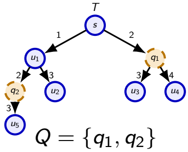

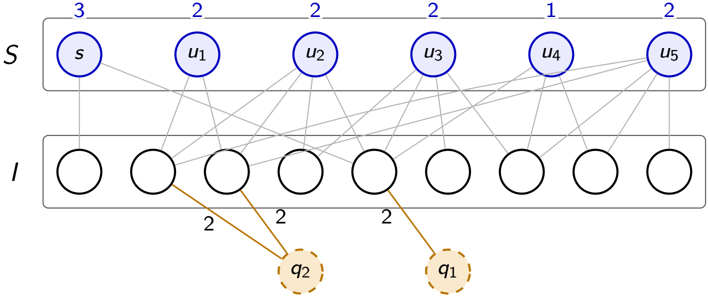

Add q_h linked to every vertex of I_h with an edge of weight 2

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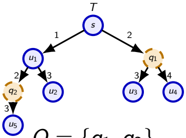

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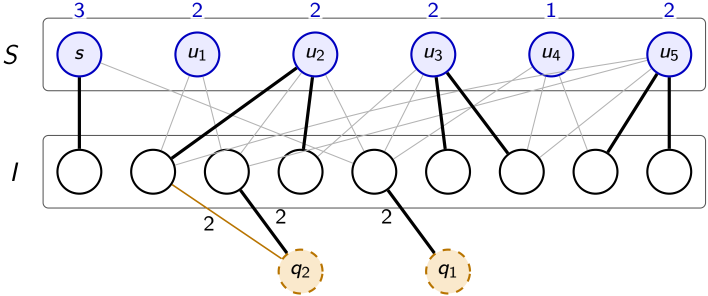
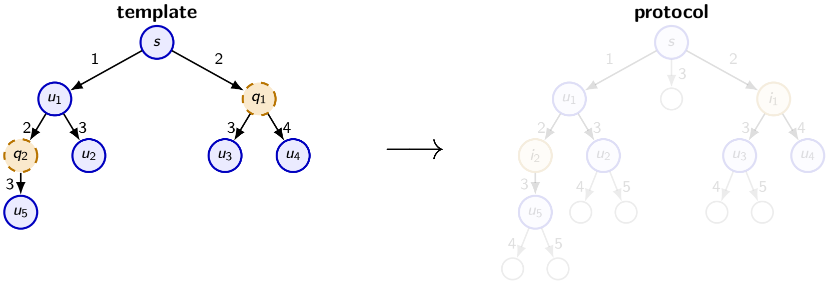


Is there a b-matching M with $w(M) = |I| + |Q|$?

From a matching to a protocol

$$w(M) = |I| + |Q| \quad \Longleftrightarrow \quad I \text{ and all } q_h \text{ are saturated}$$



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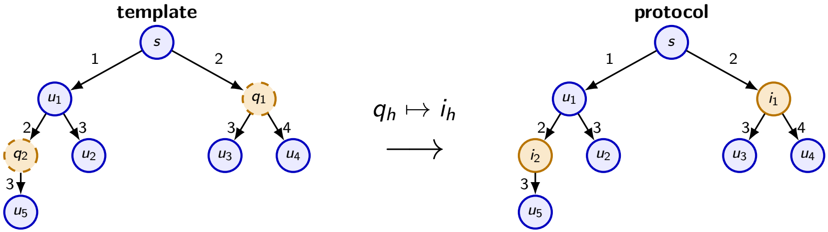

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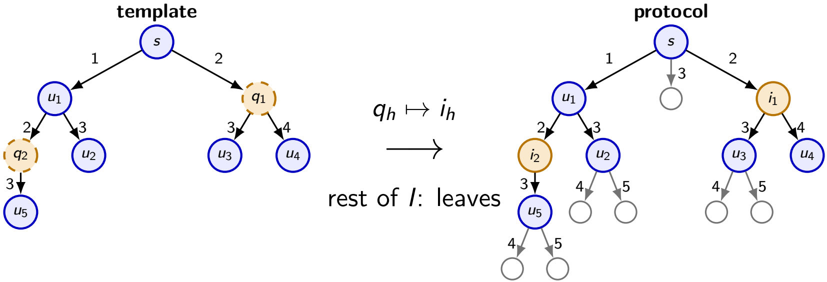

$$b(G,s) \leqslant t \iff \text{some template has } \max_M w(M) = |I| + |Q|$$

Faster parameterized broadcasting

$$k^{\mathcal{O}(k)} \text{ templates} \quad \times \quad n^{\mathcal{O}(1)}\text{-time weighted } b\text{-matching}$$

Theorem

TELEPHONE BROADCAST *can be solved in time* $\mathsf{vc}^{\mathcal{O}(\mathsf{vc})} n^{\mathcal{O}(1)}$.

Faster parameterized broadcasting

$k^{\mathcal{O}(k)}$ templates $\quad\times\quad$ $n^{\mathcal{O}(1)}$-time weighted b-matching

Theorem

Telephone Broadcast *can be solved in time* $\mathsf{vc}^{\mathcal{O}(\mathsf{vc})} n^{\mathcal{O}(1)}$.

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Telephone Broadcast *can be solved in time* $\mathsf{vi}^{\mathcal{O}(\mathsf{vi}^2)} n^{\mathcal{O}(1)}$.

Theorem

Telephone Broadcast *can be solved in time* $k^{\mathcal{O}(k)} n^{\mathcal{O}(1)}$ *for distance to clique* k.

require more elaborate templates

Faster parameterized broadcasting

$k^{\mathcal{O}(k)}$ templates $\times$ $n^{\mathcal{O}(1)}$-time weighted b-matching

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Open questions: improve or match with an ETH lower bound

Thank you for your attention!