Flip Distance to a Non-crossing Perfect Matching

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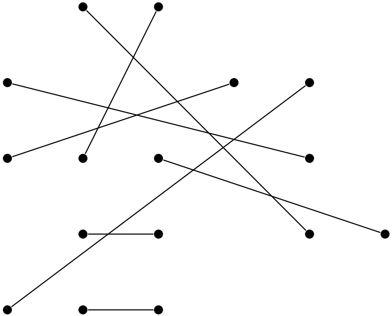
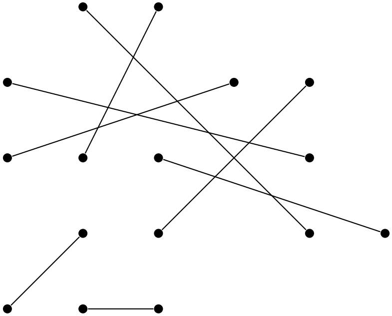
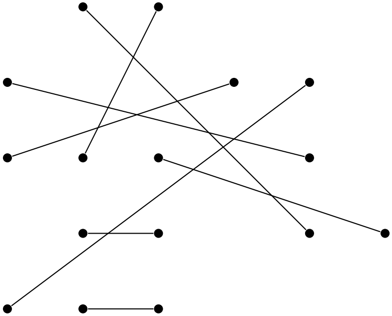


Matching of n edges in the plane



Matching of n edges in the plane

Uncross with a flip; goal: reach a non-crossing configuration



Matching of n edges in the plane

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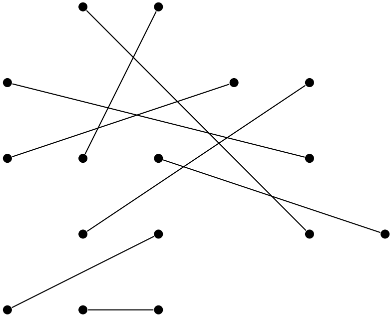

Matching of n edges in the plane

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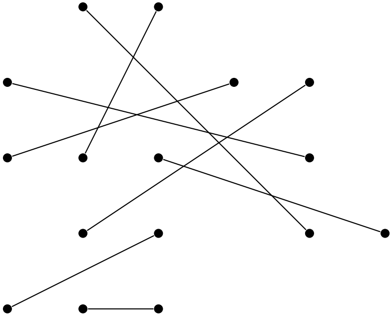
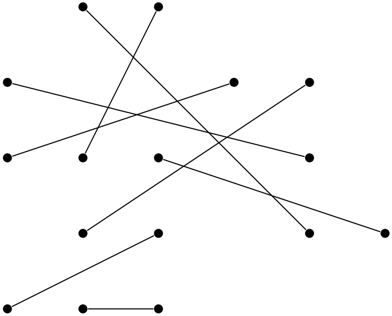


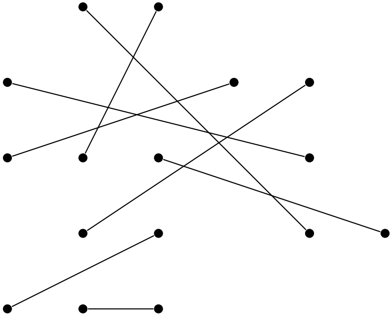
Matching of n edges in the plane

Total length decreases $\Rightarrow$ terminates in exponentially many flips



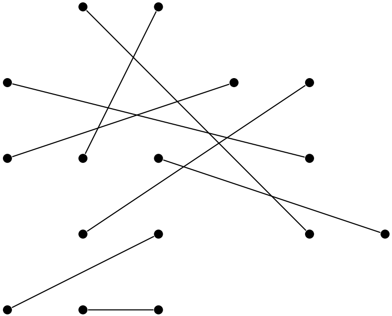
Matching of n edges in the plane

How many flips do I need to do from the worst initial configuration



Matching of n edges in the plane

How many flips do I need to do from the worst initial configuration for a fastest sequence?



Matching of n edges in the plane

How many flips do I need to do from the worst initial configuration

for a fastest sequence?
for a slowest sequence?

Overview

Min version: $\max_{M \in n-\text{matching}} \min_{s \in \text{flip-seq}(M)} |s|$
Max version: $\max_{M \in n-\text{matching}} \max_{s \in \text{flip-seq}(M)} |s|$

Version	Lower bound	Upper bound
Min	$n-1$	$n^2/2$
Max	$\binom{n}{2}$	n^3

Overview

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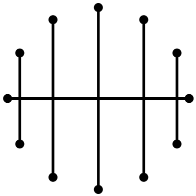
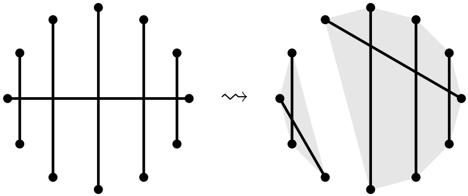
Max	$\binom{n}{2}$	n^3 [van Leeuwen and Schoone '80]

Lower bound for the min version

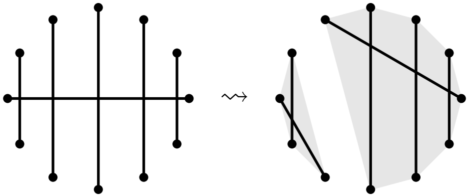


Lower bound for the min version



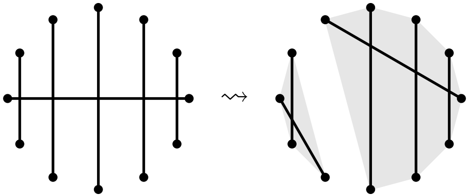
$s(n) = \min_{a\in[1,n]} s(n-a) + s(a)$, $s(1) = 0$

Lower bound for the min version



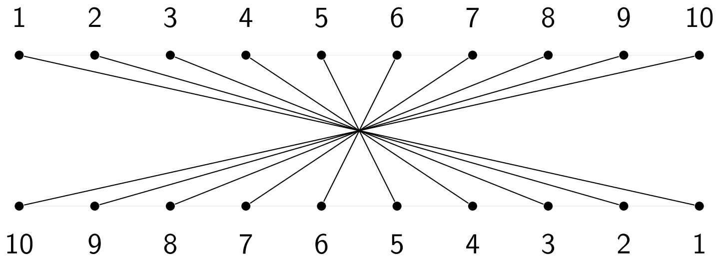
$s(n) = \min_{a \in [1,n]} s(n-a) + s(a)$, $s(1) = 0$
$s(n) = n - 1$

Lower bound for the min version



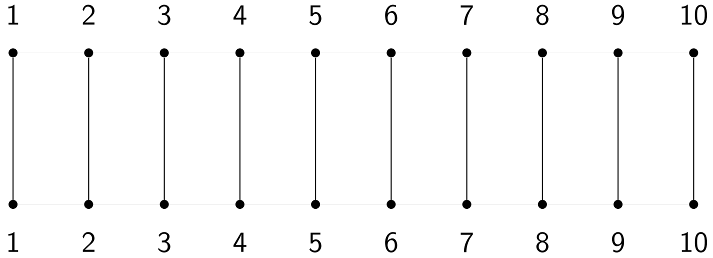
$s(n) = \min_{a \in [1,n]} s(n-a) + s(a)$, $s(1) = 0$
$s(n) = n - 1$
Also: all the solutions have the same length.

Lower bound for the max version



Think bubble sort

Lower bound for the max version



Think bubble sort

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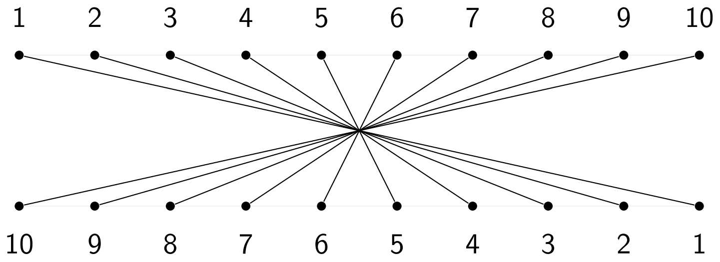

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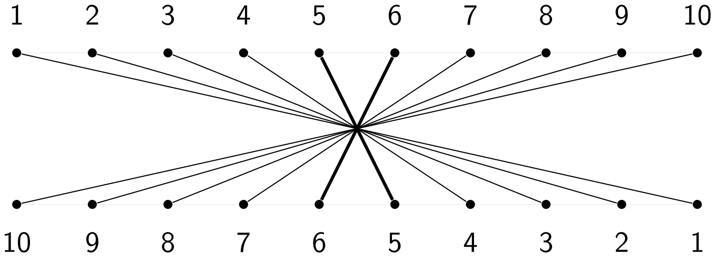
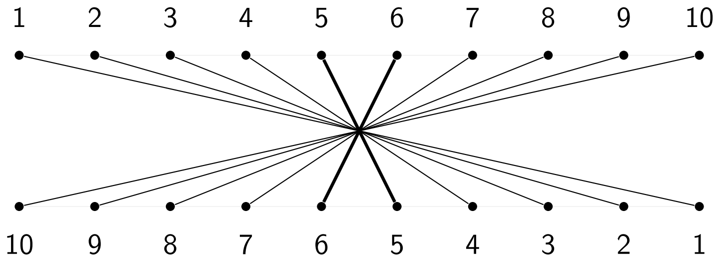


Think bubble sort
Only do *consecutive* flip

Lower bound for the max version



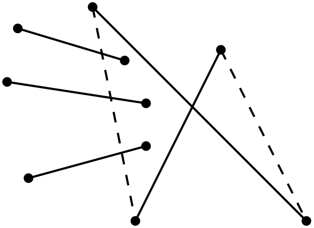
Think bubble sort
Only do *consecutive* flip
Number of flips = number of inversions = $\binom{n}{2}$

Ideas that cannot work

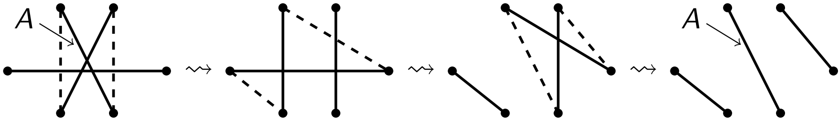
- Tracking the number of crossings
- Argument based on the fact that an edge cannot reappear[1]

[1] Because it's not true

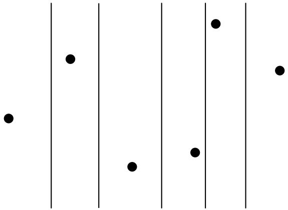
The number of crossings increase after a flip



An edge can easily disappear and reappear

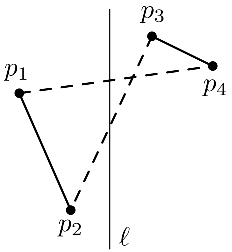


Upper bound for the min version



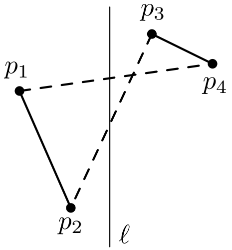
Sort your point by increasing x-coordinates and add vertical separators between two consecutive points.

Upper bound for the min version (2)



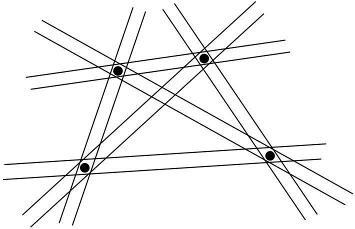
The measure that decreases efficiently is the number of intersections segment-separator.

Upper bound for the min version (2)



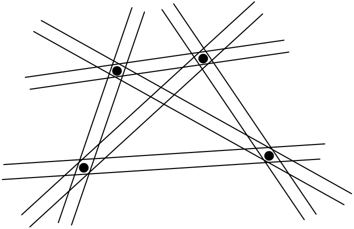
The measure that decreases efficiently is the number of intersections segment-separator.
It goes down by 2 at each flip and is initially bounded by n^2.

Upper bound for the max version



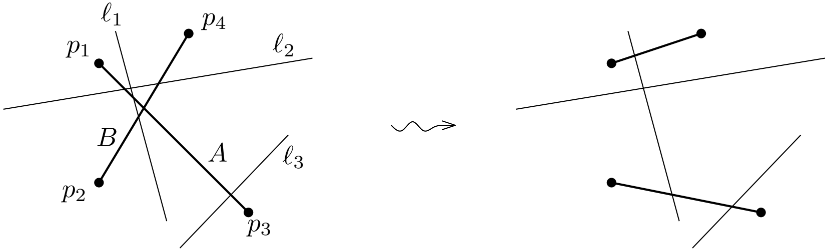
Those are the $2\binom{2n}{2}$ separators that we consider.

Upper bound for the max version



Those are the $2\binom{2n}{2}$ separators that we consider.
Again, the measure is the number of intersections segment-separator and is bounded by $4n^3$ initially.

Upper bound for the max version (2)



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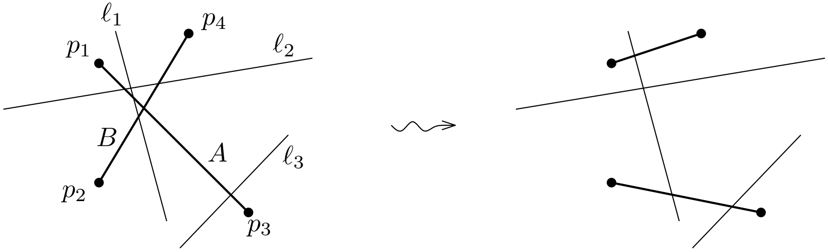

ℓ_1 and ℓ_3 intersect the same number of edges.
The number of intersections segment-ℓ_2 drops by 2.

Thank you for your attention!

Version	Lower bound	Upper bound
Min	$\Omega(n)$	$O(n^2)$
Max	$\Omega(n^2)$	$O(n^3)$

Open question: close the gap for both versions