# Twin-width

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ENS Lyon, LIP

Séminaire Algorithmes et Complexité, IRIF, May 26th

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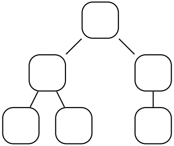

Bounded treewidth

Generalize

trees

tractable $MSO_2$

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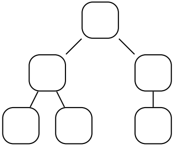

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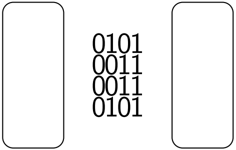


Bounded rank-width

cographs, treewidth

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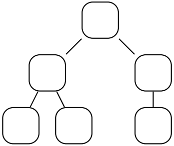

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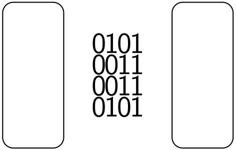


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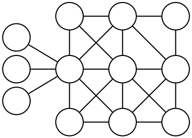

Nowhere dense

bounded degree, $H$-minor free

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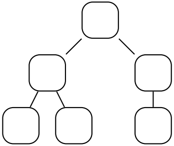

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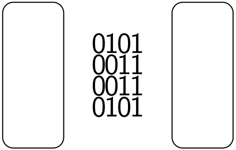


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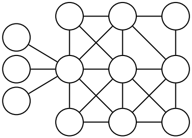

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Perfect graphs
Bounded width posets
Pattern-avoiding permutations
⋮

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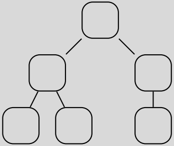

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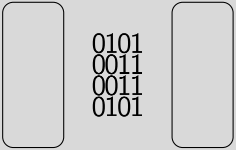


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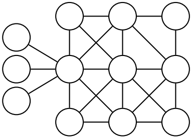

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Iteratively identify **near** twins

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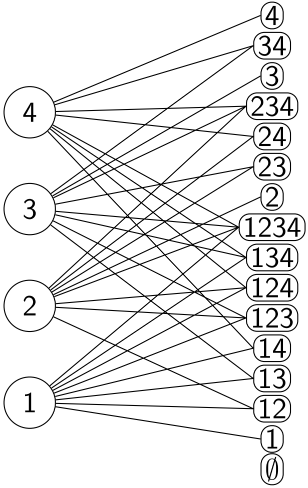


This complicated graph passes the test

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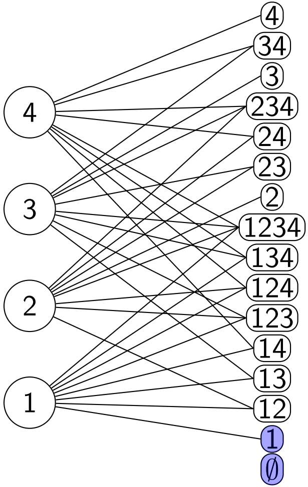


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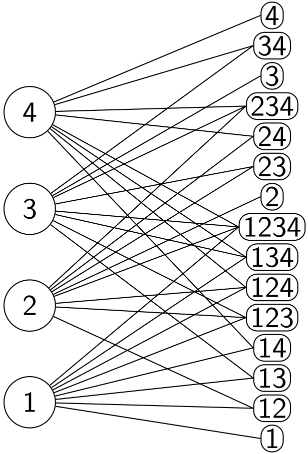


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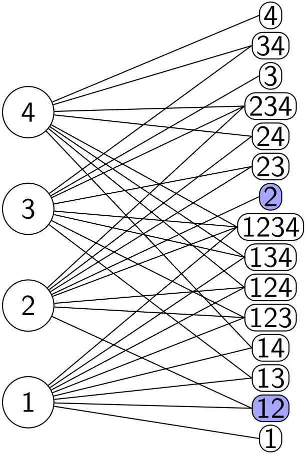


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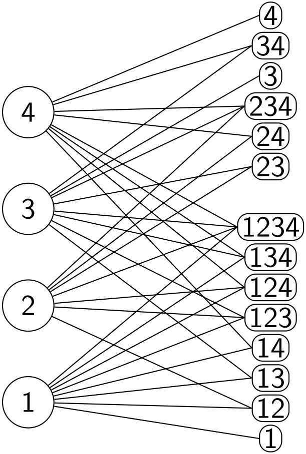


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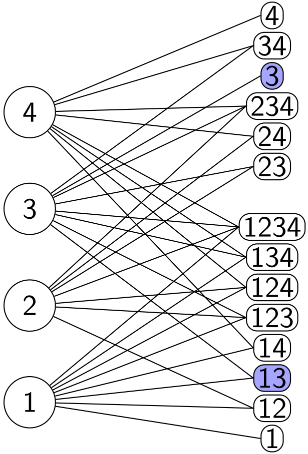


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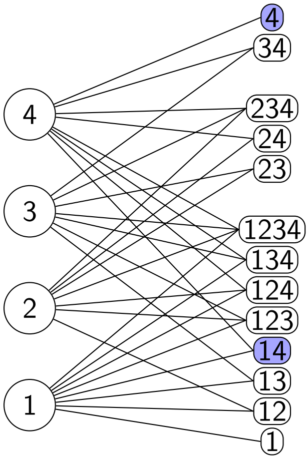


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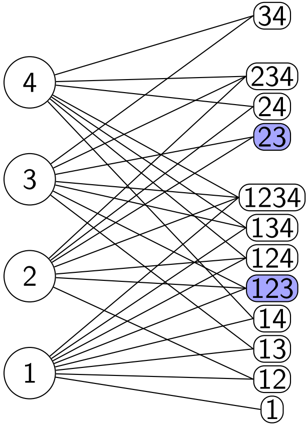


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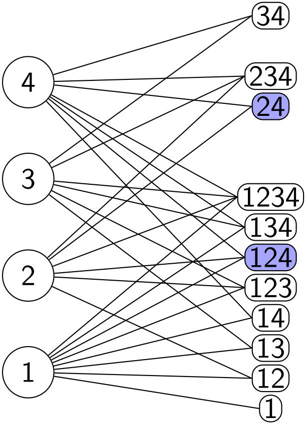


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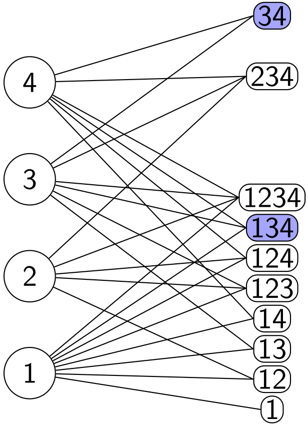


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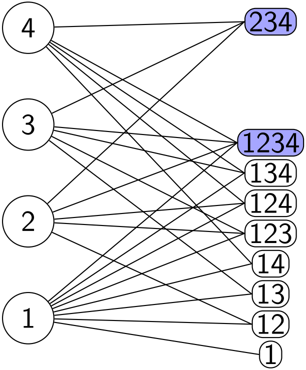


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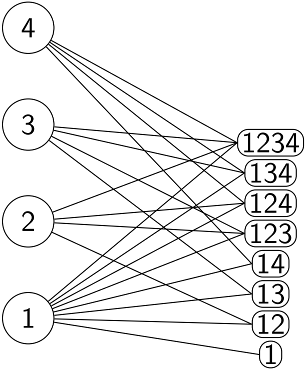


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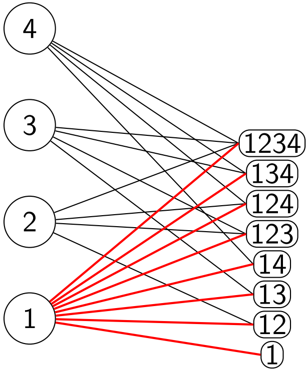
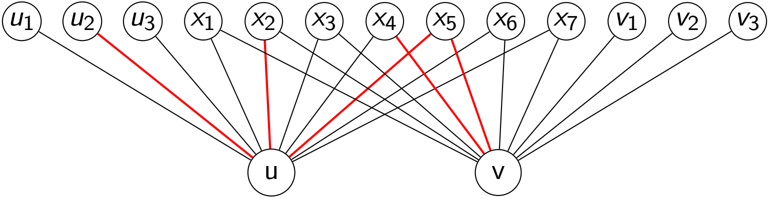
# Cograph generalization

Iteratively identify **near** twins and **keep the error degree small**



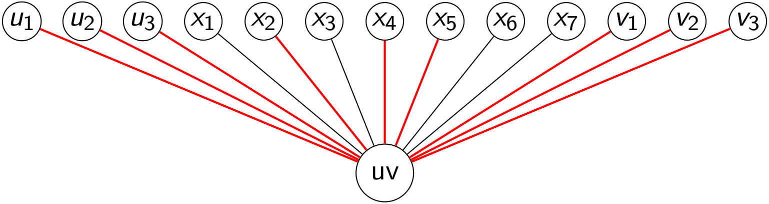

It would not with that further restriction

# Contraction and trigraph



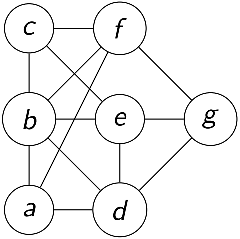

Trigraph: non-edges, edges, and red edges (error)

# Contraction and trigraph




edges to $N(u) \triangle N(v)$ turn red, for $N(u) \cap N(v)$ red is absorbent

# Contraction sequence and twin-width




Maximum red degree $= 0$

**overall maximum red degree = 0**

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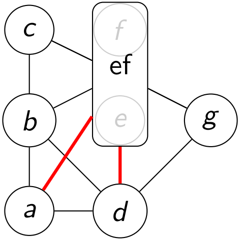


Maximum red degree = 2

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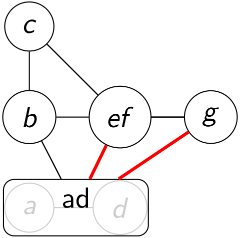


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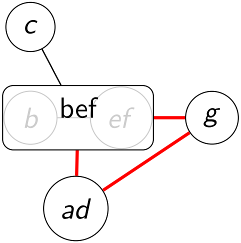


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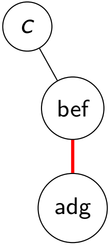


Maximum red degree $= 1$

**overall maximum red degree = 2**

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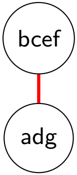



Maximum red degree $= 1$

**overall maximum red degree = 2**

# Contraction sequence and twin-width


Maximum red degree = 0

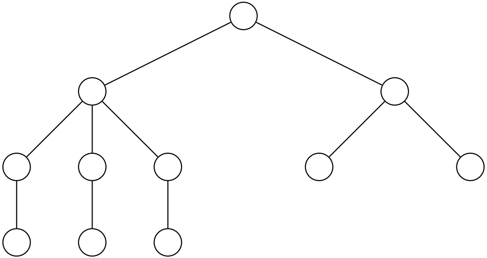
**overall maximum red degree = 2**

# Contraction sequence and twin-width

Sequence of 2-contractions or 2-sequence, twin-width at most 2


Maximum red degree = 0
**overall maximum red degree = 2**

# Graphs with bounded twin-width – trees



If possible, contract two twin leaves

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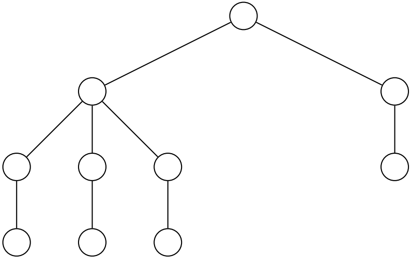

If not, contract a deepest leaf with its parent

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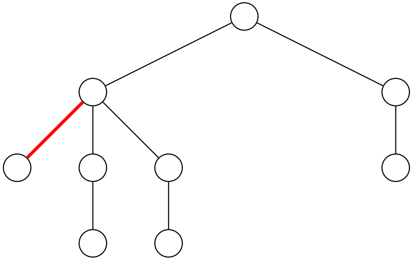

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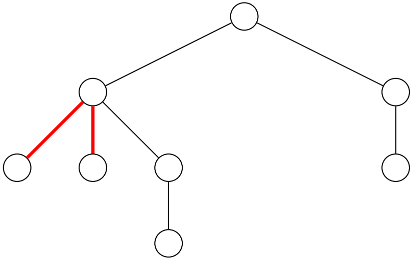

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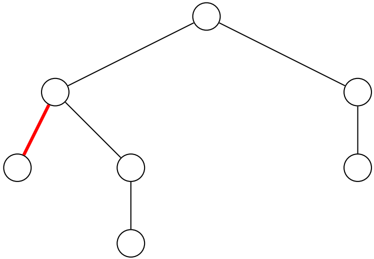

Cannot create a red degree-3 vertex

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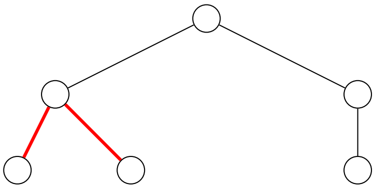

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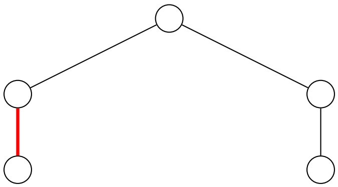

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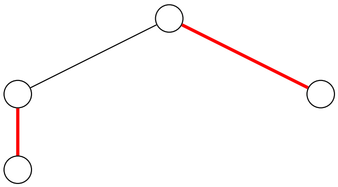

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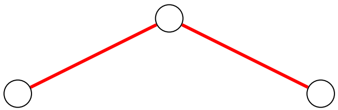

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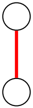

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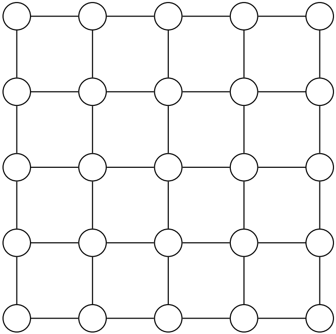
# Graphs with bounded twin-width – trees

Generalization to bounded treewidth and even bounded rank-width

# Graphs with bounded twin-width – grids



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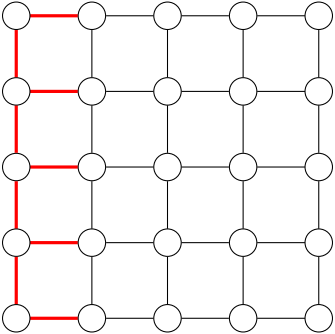

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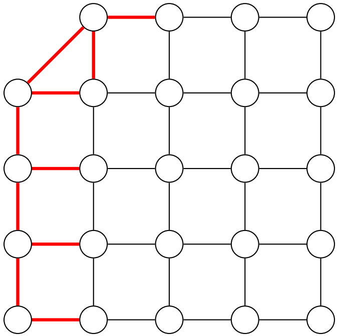

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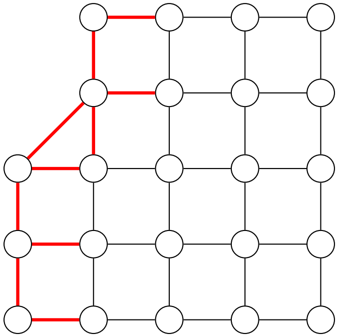

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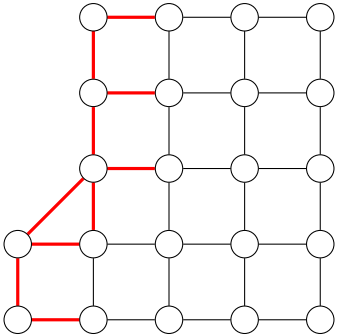

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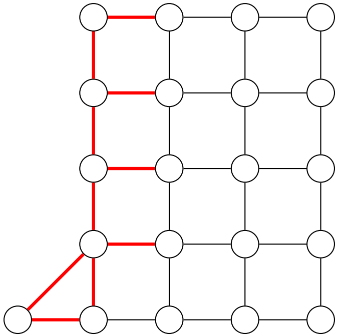

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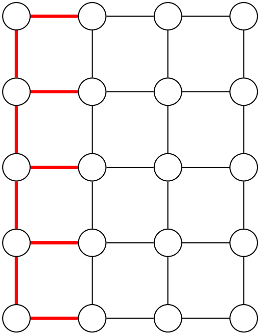

4-sequence for planar grids, $3d$-sequence for $d$-dimensional grids

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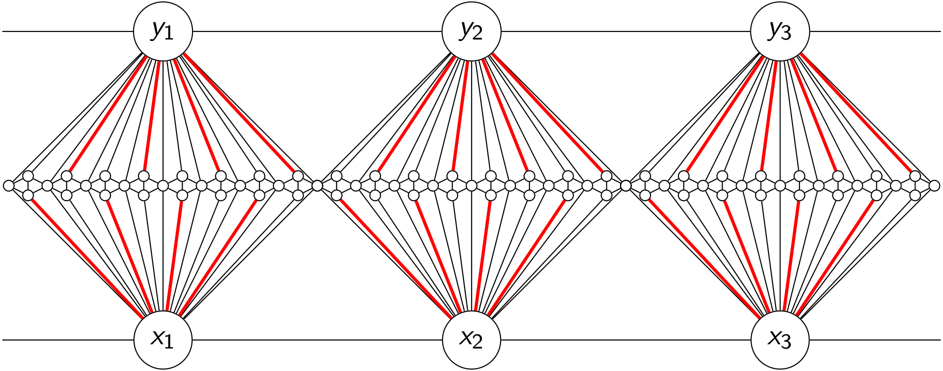


For every $d$, a planar trigraph without planar $d$-contraction

# More powerful tool needed

# First-order model checking on graphs

GRAPH FO MODEL CHECKING **Parameter:** $|\phi|$
**Input:** A graph $G$ and a first-order formula $\varphi \in FO(\{E_2, =_2\})$
**Question:** $G \models \varphi$?

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Example:

$$\varphi = \exists x_1 \exists x_2 \cdots \exists x_k \forall x \bigvee_{1 \leqslant i \leqslant k} x = x_i \vee \bigvee_{1 \leqslant i \leqslant k} E(x, x_i) \vee E(x_i, x)$$

$G \models \varphi$? $\Leftrightarrow$

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$G \models \varphi$? $\Leftrightarrow$ $k$-INDEPENDENT SET

# FO interpretations and transductions

**FO interpretation:** redefine the edges by a first-order formula

$\varphi(x, y) = \neg E(x, y)$ (complement)

$\varphi(x, y) = E(x, y) \vee \exists z E(x, z) \wedge E(z, y)$ (square)

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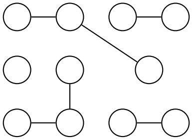

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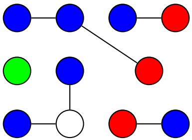

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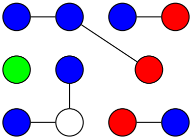
$\varphi(x,y) = E(x,y) \vee \exists z E(x,z) \wedge E(z,y)$ (square)

**FO transduction:** color by $O(1)$ unary relations, interpret, delete



$$\phi(x,y) = E(x,y) \vee (G(x) \wedge B(y) \wedge \neg\exists z R(z) \wedge E(y,z))$$
$$\vee (R(x) \wedge B(y) \wedge \exists z R(z) \wedge E(y,z) \wedge \neg\exists z B(z) \wedge E(y,z))$$

# FO interpretations and transductions

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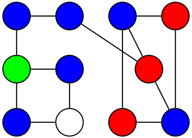

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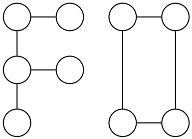

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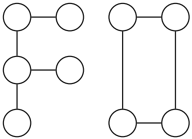
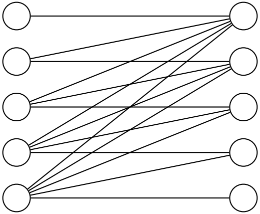


Theorem (B, Kim, Thomassé, Watrigant '20+)

*Bounded twin-width is preserved by transduction.*

# Stable and NIP

**Stable class:** *not* all the ladders can be obtained by transduction
**NIP class:** *not* all the graphs can be obtained by transduction



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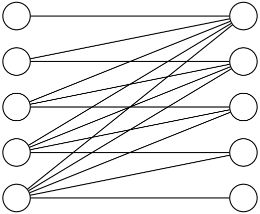

Bounded-degree graphs $\rightarrow$ stable
Unit interval graphs $\rightarrow$ NIP but not stable
Interval graphs $\rightarrow$ not NIP

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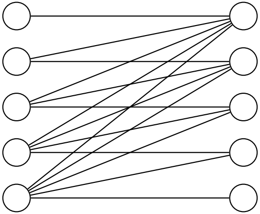

Bounded-degree graphs $\rightarrow$ stable
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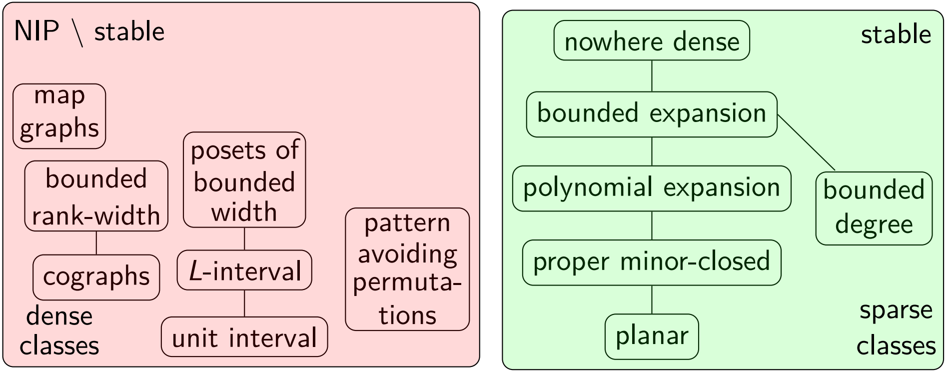
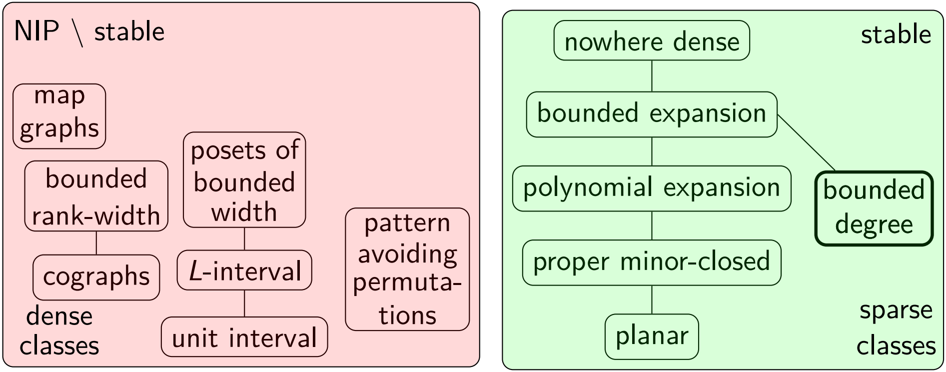
**Bounded twin-width classes $\rightarrow$ NIP but not stable in general**

# Classes with known tractable FO model checking

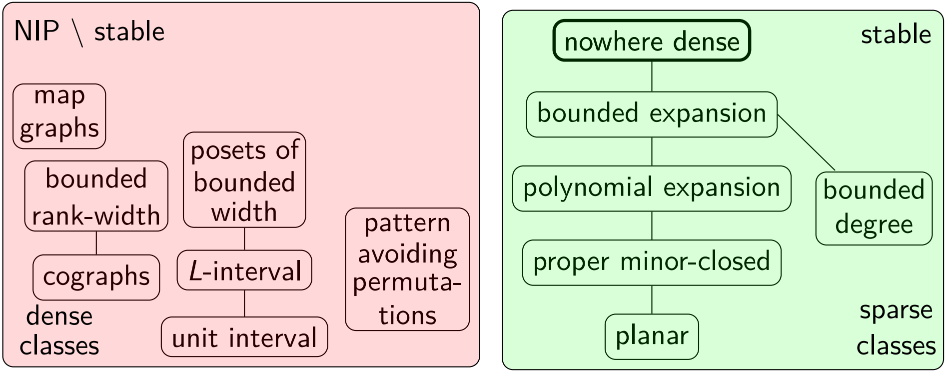


# Classes with known tractable FO model checking



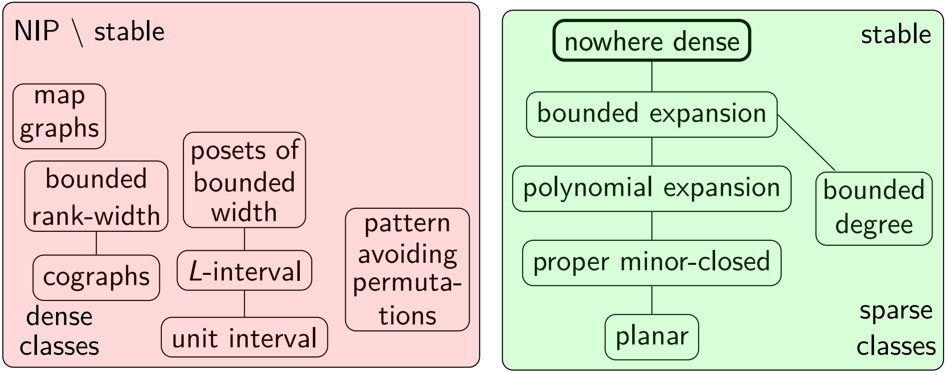

FO Model Checking solvable in $f(|\varphi|)n$ on bounded-degree graphs [Seese '96]

# Classes with known tractable FO model checking




FO Model Checking solvable in $f(|\varphi|)n^{1+\varepsilon}$ on any nowhere dense class [Grohe, Kreutzer, Siebertz '14]

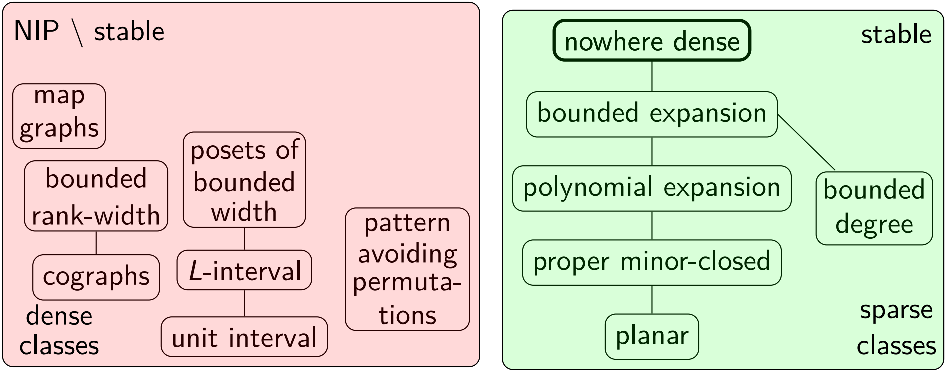
# Classes with known tractable FO model checking




End of the story for the classes closed by taking subgraphs

tractable FO MODEL CHECKING ⇔ nowhere dense ⇔ stable

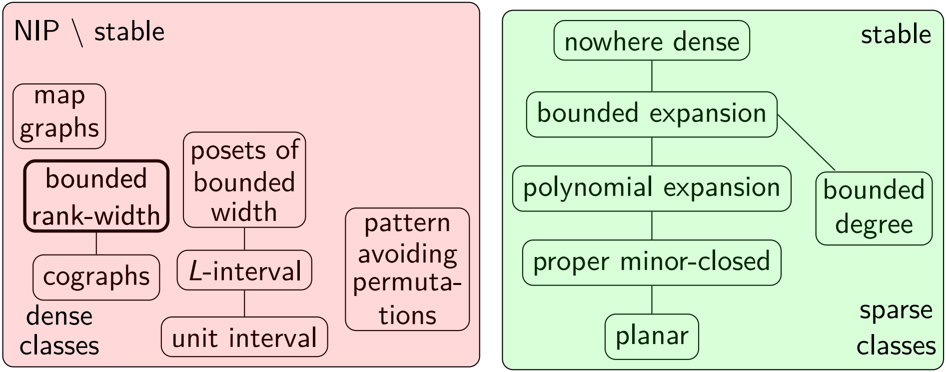
# Classes with known tractable FO model checking




New program: transductions of nowhere dense classes

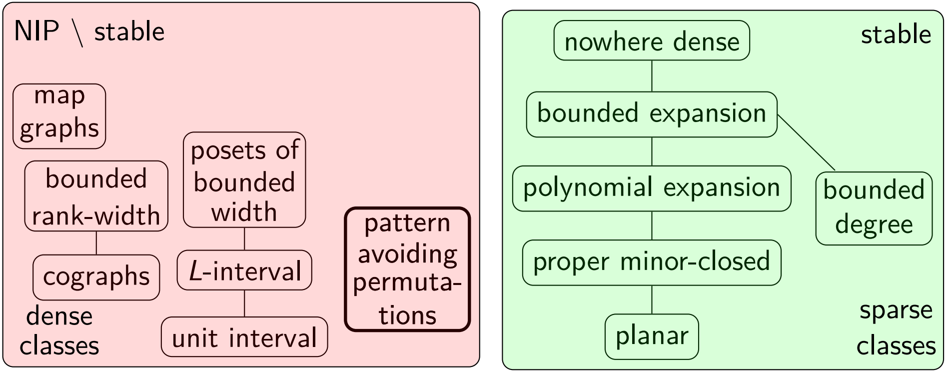
Not sparse anymore but still stable

# Classes with known tractable FO model checking



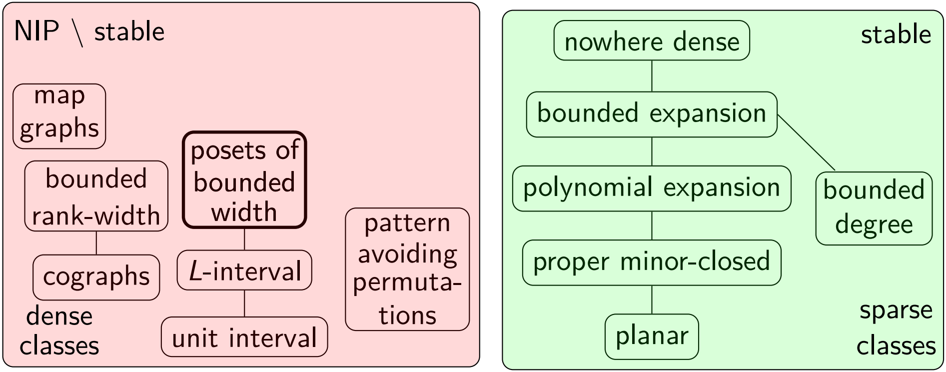

$\mathrm{MSO}_1$ MODEL CHECKING solvable in $f(|\varphi|, w)n$ on graphs of rank-width $w$ [Courcelle, Makowsky, Rotics '00]

# Classes with known tractable FO model checking



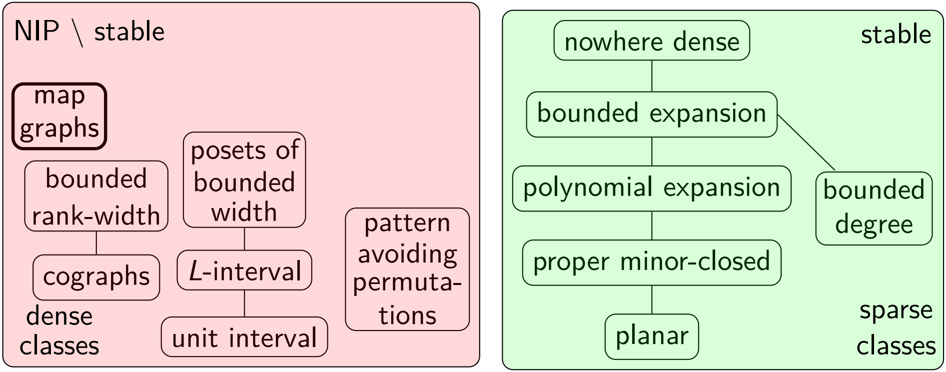

*Is $\sigma$ a subpermutation of $\tau$?* solvable in $f(|\sigma|)|\tau|$
[Guillemot, Marx '14]

# Classes with known tractable FO model checking



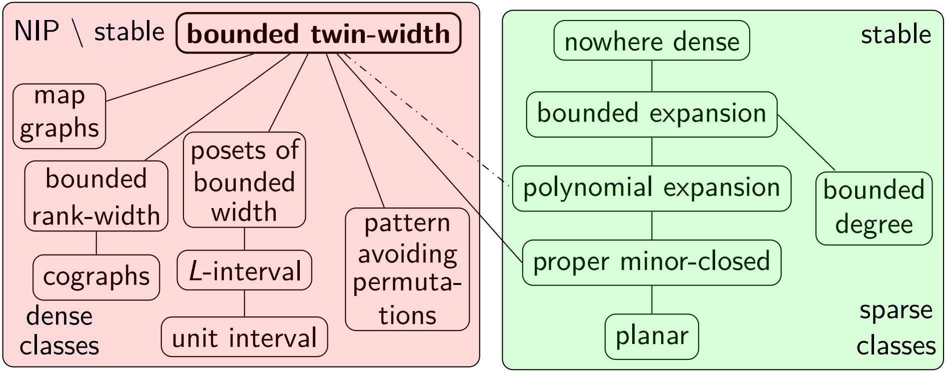

FO MODEL CHECKING solvable in $f(|\varphi|, w)n^2$ on posets of width $w$
[GHLOORS '15]

# Classes with known tractable FO model checking



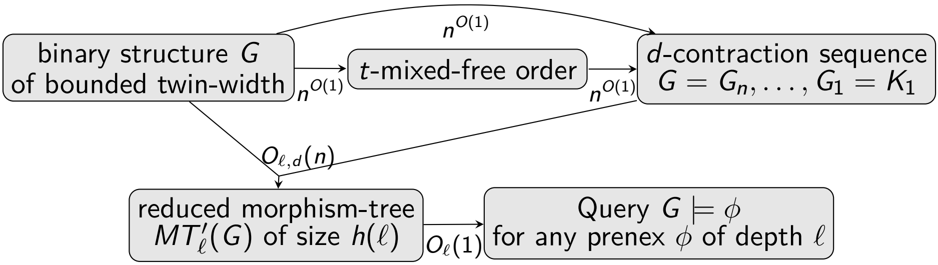

FO MODEL CHECKING solvable in $f(|\varphi|)n^{O(1)}$ on map graphs
[Eickmeyer, Kawarabayashi '17]

# Classes with known tractable FO model checking




FO MODEL CHECKING solvable in $f(|\varphi|, d)n$ on graphs with a $d$-sequence
[B, Kim, Thomassé, Watrigant '20+]

# Workflow of our FO model checking algorithm



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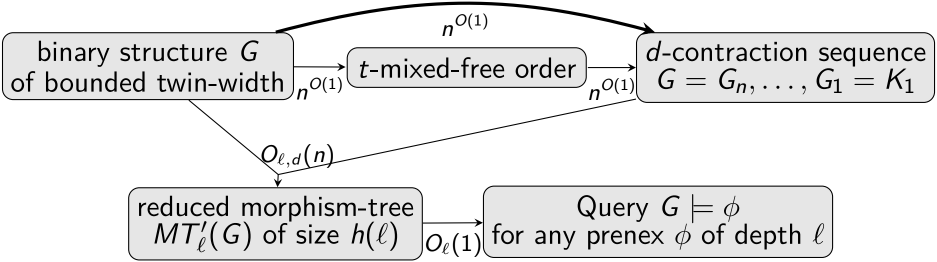


Direct examples: **trees**, bounded rank-width, **grids**, $d$-dimensional grids, unit interval graphs, $K_t$-free unit ball graphs

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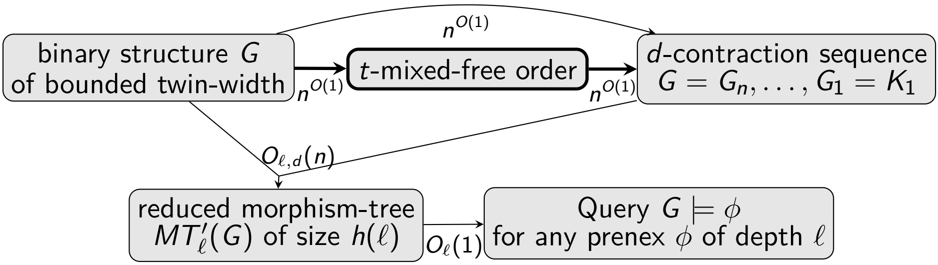
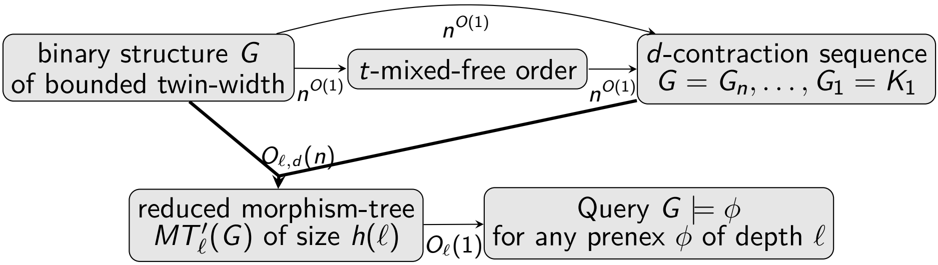


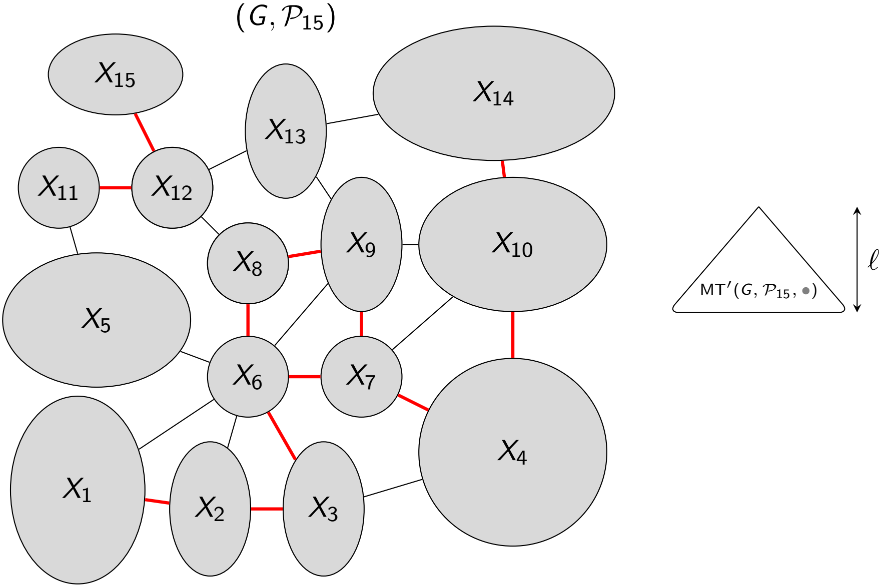

We now explore the detour via mixed minor for:
pattern-avoiding permutations, bounded width posets, $K_t$-minor free graphs

# Workflow of our FO model checking algorithm




But before we give a snapshot of the FO model checking

# DP for FO model checking with $d$-sequence



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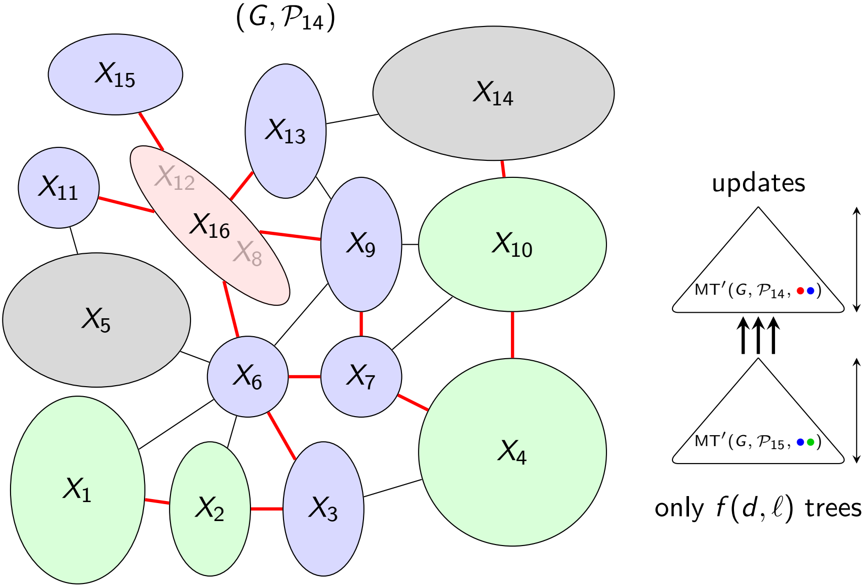
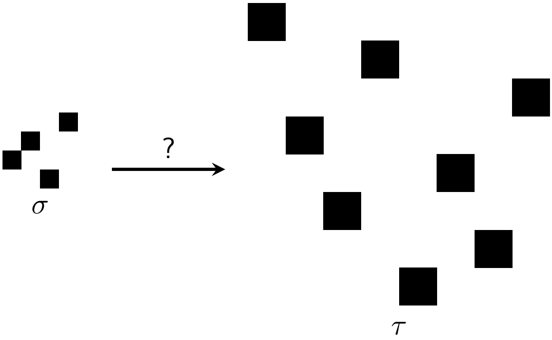


# Permutation Pattern



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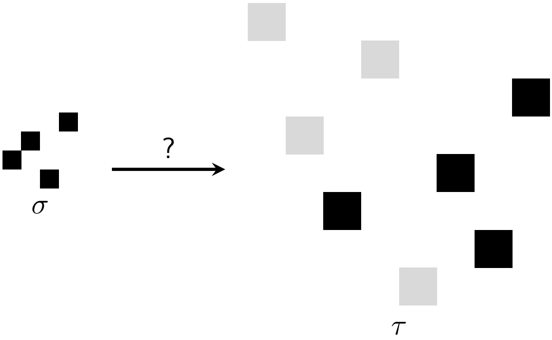

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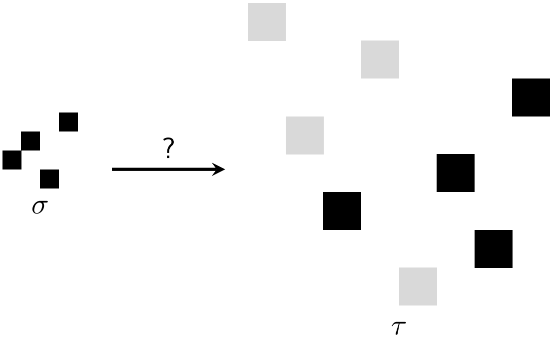


Theorem (Guillemot, Marx '14)

Permutation Pattern *can be solved in time* $2^{|\sigma|^2}|\tau|$.

# Guillemot and Marx's win-win algorithm

**Theorem (Marcus, Tardos '04)**

*$\forall t$, $\exists c_t$ $\forall$ $n \times n$ 0,1-matrix with $\geqslant c_t n$ entries 1 has a t-grid minor.*

$$\text{4-grid minor} \left[\begin{array}{cc|cc|cc|cc}
1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 \\
\hline
0 & 1 & 1 & 0 & 0 & 1 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
\hline
0 & 1 & 0 & 0 & 1 & 0 & 1 & 0 \\
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A) $\geqslant c_{|\sigma|} n$ entries 1 $\rightarrow$ YES from the $|\sigma|$-grid minor.

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If B) always happens $\rightarrow$ DP on this merge sequence

# Our generalization to the dense case – mixed minor

Mixed zone: not horizontal nor vertical

$$\left[\begin{array}{cc|ccc|ccc}
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0 & 1 & 1 & 0 & 0 & 1 & 0 & 1 \\
\hline
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
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\hline
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\end{array}\right]$$

3-mixed minor

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\end{array}\right]$$

3-mixed minor

A matrix is said $t$-**mixed free** if it does not have a $t$-mixed minor

# Grid minor theorem for twin-width

Theorem (B, Kim, Thomassé, Watrigant 20+)

*If $\exists\sigma$ s.t. $Adj_\sigma(G)$ is $t$-mixed free, then $tww(G) = 2^{2^{O(t)}}$.*

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Now to bound the twin-width of a class $\mathcal{C}$:

1) Find a *good* vertex-ordering procedure

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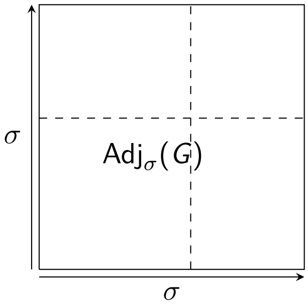


Cutting after the $t/2$-th division of the $t$-grid minor

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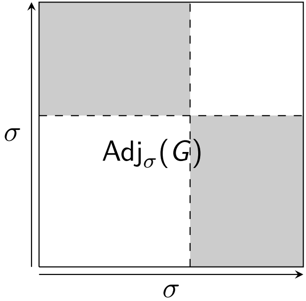
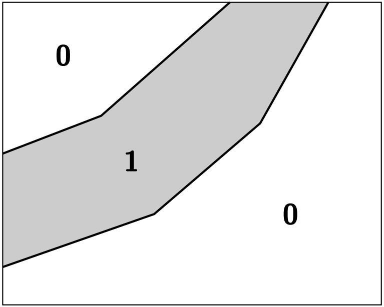


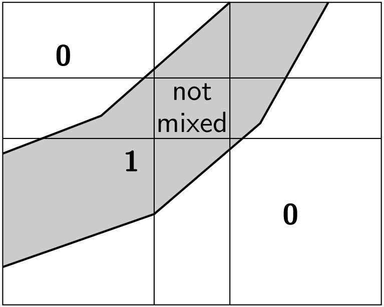

One of the shaded areas contains a $t/2$-grid minor on disjoint sets

# Bounded twin-width – posets of bounded antichain




Warm-up with unit interval graphs: order by left endpoints

# Bounded twin-width – posets of bounded antichain

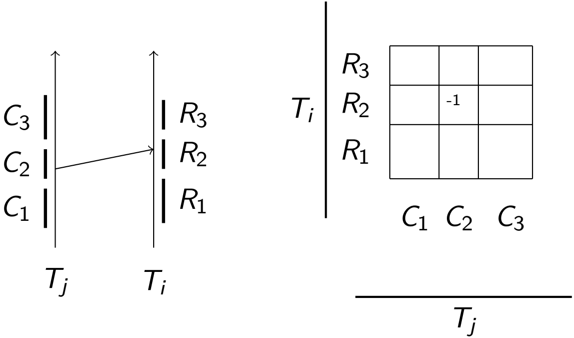



No 3-by-3 grid has all 9 cells crossed by two non-decreasing curves

# Bounded twin-width – posets of bounded antichain

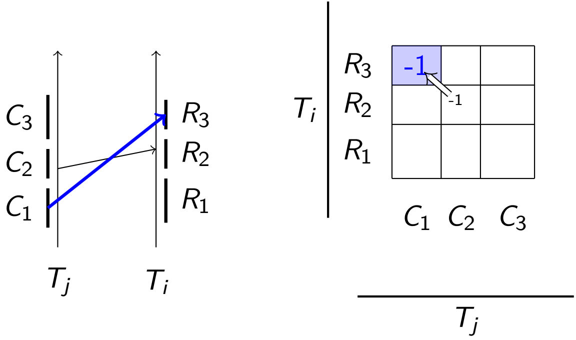

Put the $k$ chains in order one after the other

# Bounded twin-width – posets of bounded antichain



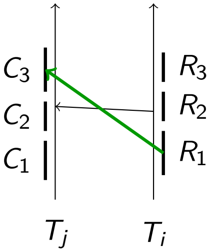

A $3k$-mixed minor implies a 3-mixed minor between two chains

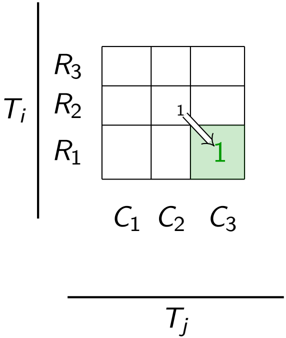
# Bounded twin-width – posets of bounded antichain




Transitivity implies that a zone is constant

# Bounded twin-width – posets of bounded antichain



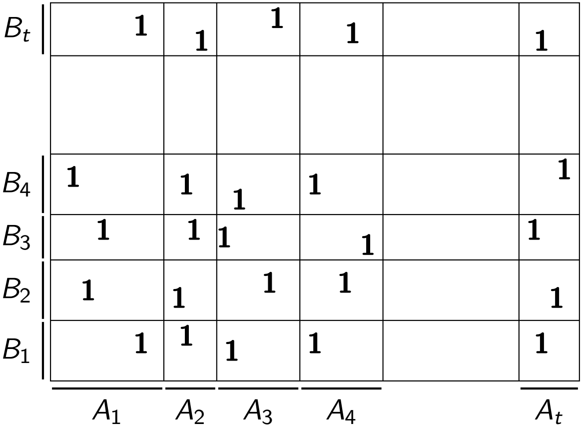




And symmetrically

# Bounded twin-width – $K_t$-minor free graphs

Given a hamiltonian path, we would just use this order

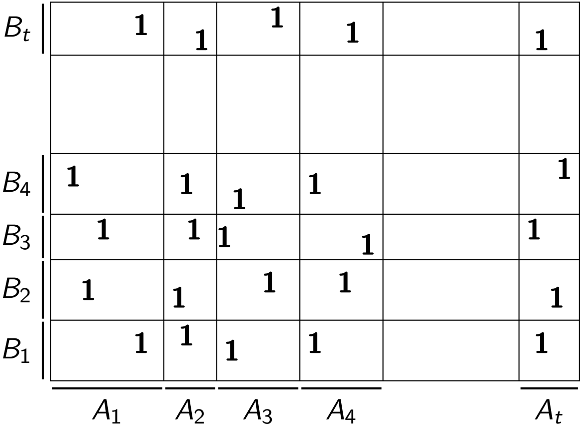
# Bounded twin-width – $K_t$-minor free graphs




Contracting[1] the $2t$ subpaths yields a $K_{t,t}$-minor

[1] Here it is an actual contration, not a mere identification

# Bounded twin-width – $K_t$-minor free graphs




Instead we use a specially crafted lex-DFS discovery order

# Small classes

Classes[1] with at most $n!c^n$ labeled graphs on $[n]$.

Theorem (B, Geniet, Kim, Thomassé, Watrigant 20+)

*Bounded twin-width classes are small.*

Unifies and extends the same result for:
$\sigma$-free permutations [Marcus, Tardos '04]
$K_t$-minor free graphs [Norine, Seymour, Thomas, Wollan '06]

[1] sets closed by taking induced subgraphs

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Theorem (B, Geniet, Kim, Thomassé, Watrigant 20+)

*Bounded twin-width classes are small.*

Subcubic graphs, interval graphs, triangle-free unit segment graphs have **unbounded** twin-width

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Classes[1] with at most $n!c^n$ labeled graphs on $[n]$.

Theorem (B, Geniet, Kim, Thomassé, Watrigant 20+)

*Bounded twin-width classes are small.*

Is the converse true?

Conjecture (small conjecture)

*A class has bounded twin-width if and only if it is small.*

[1] sets closed by taking induced subgraphs

# Future directions

**Obvious questions:**
Algorithm to compute/approximate twin-width in general
Fully classify classes with tractable FO model checking
Small conjecture, polynomial expansion

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⋮

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On arxiv
Twin-width I: tractable FO model checking [BKTW '20]
Twin-width II: small classes [BGKTW '20]
Twin-width III: Max Independent Set and Coloring [BGKTW '20]