Parameterized Hardness of Art Gallery Problems

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The ART GALLERY problems:
Input: a polygon $\mathcal{P}$ with n vertices, a positive integer k.
Point Guard: find a set of at most k **points** guarding $\mathcal{P}$.
Vertex Guard: find a set of at most k **vertices** guarding $\mathcal{P}$.

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Allowing holes make them as hard as Set Cover:

- For parameterized complexity: unlikely to be solvable in $n^{o(k)}$.
- For approximation: very unlikely to be $o(\log n)$-approximable.

Parameterized hardness on simple polygons

Simple polygon: no holes and not self-crossing.
The problems are known to remain NP-hard (even APX-hard).

Theorem (B., Miltzow)

Unless the ETH fails, they cannot be solved in time $n^{o(k/\log k)}$.

ETH: 3-SAT cannot be solved in subexponential time.

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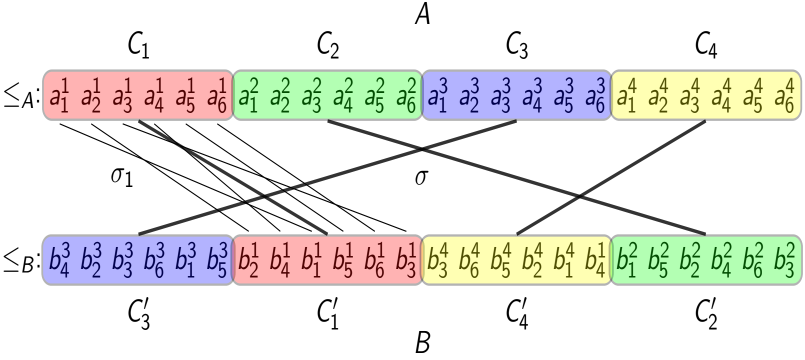
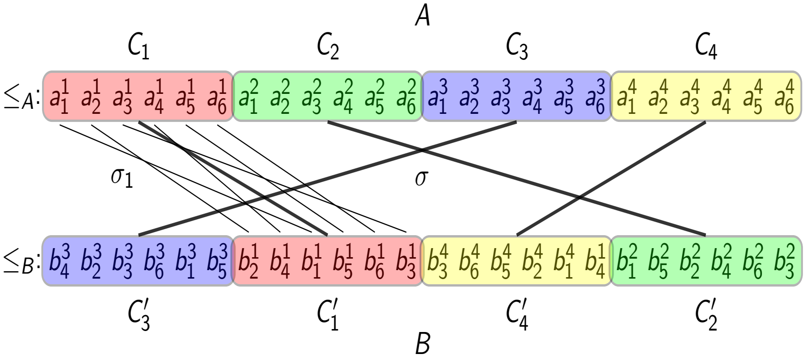
Near tightness: Both are solvable in $n^{O(k)}$
Vertex Guard for an obvious reason
Point Guard for an algebraic reason

Structured 2-Track Hitting Set

2-elements: $\forall i \in [t], \forall j \in [k]$ (a_i^j, b_i^j)
Total orderings of the a-elements and the b-elements
Sets: *A-intervals* and *B-intervals*
Find k 2-elements thats hits all the sets



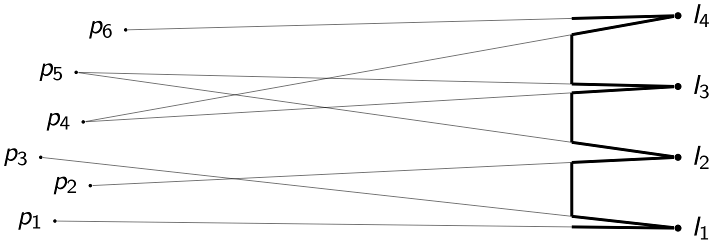
Structured 2-Track Hitting Set



Theorem (B.,Miltzow)

Unless the ETH fails, STRUCTURED 2-TRACK HITTING SET *cannot be solved in time* $n^{o(k/\log k)}$.

Interval gadget



Puzzle[1] for you

Find 2 orderings of $\{1, \overline{1}, 2, \overline{2}, \ldots, n, \overline{n}\}$ and a set-system over those elements such that:

- every set is an *interval* for one of the orders
- the minimum hitting sets are all the pairs $\{i, \overline{i}\}$

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Set-system:
$\forall i,\ \{i, i+1, \ldots, n, \overline{1}, \overline{2}, \ldots, \overline{i-1}\}$
$\forall i,\ \{\overline{i}, \overline{i+1}, \ldots, \overline{n}, 1, 2, \ldots, i-1\}$

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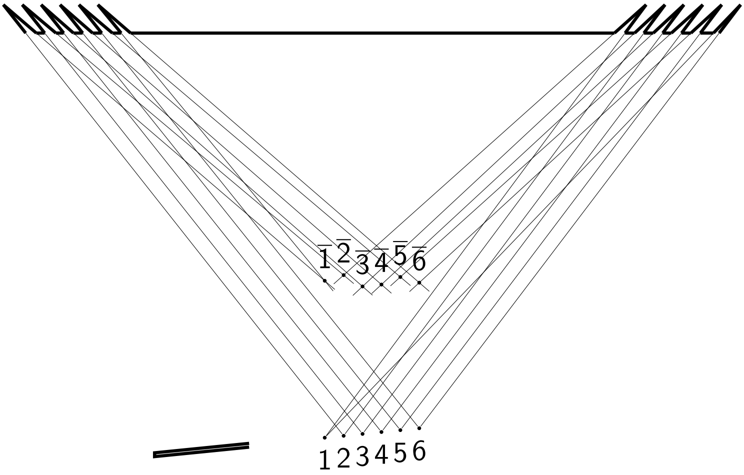
Weak point linker



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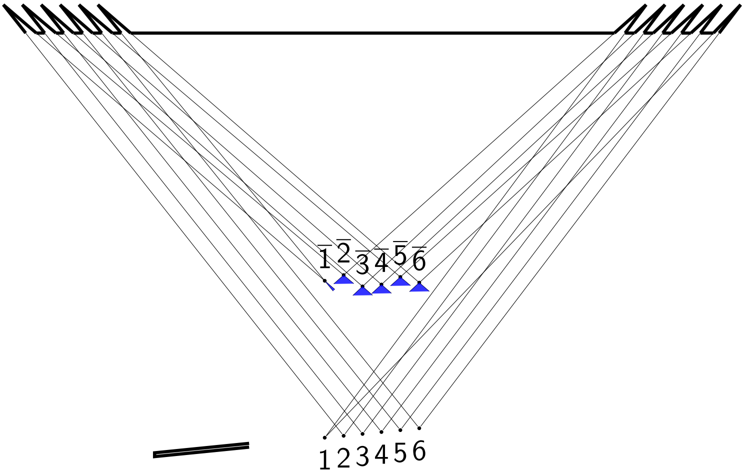
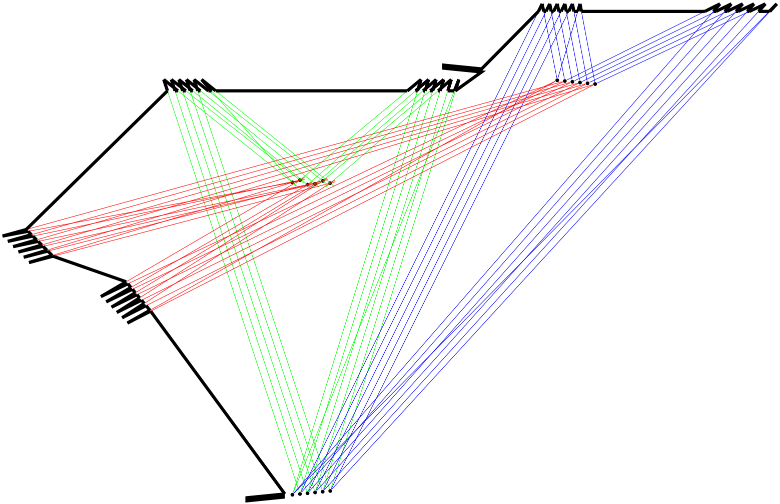
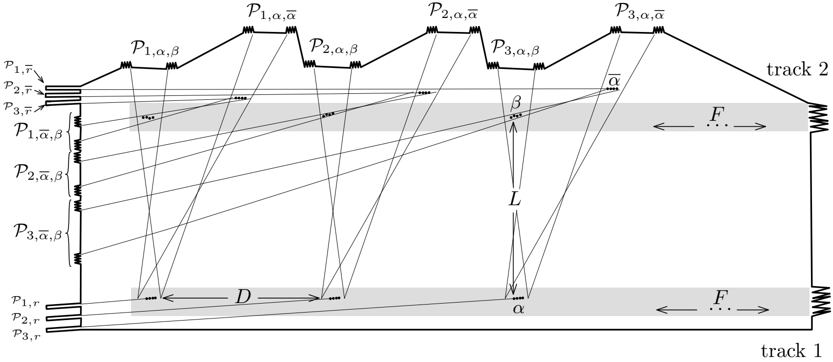


Point linker (triangle of weak linkers)

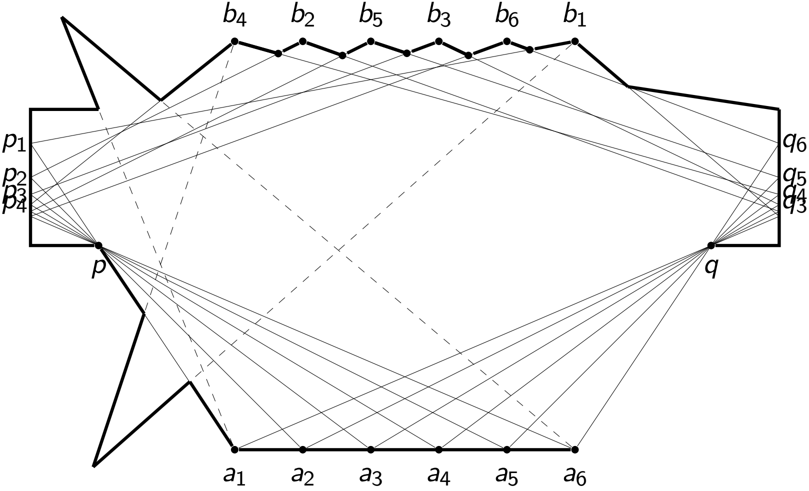


The big picture

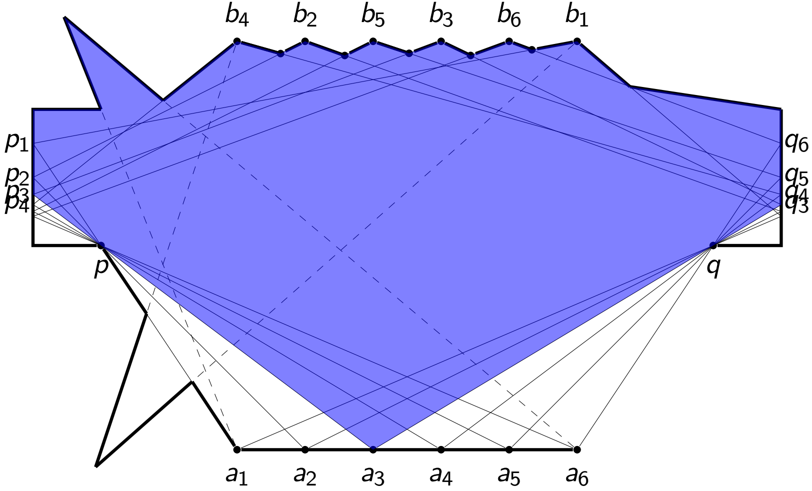


The STRUCTURED 2-TRACK HITTING SET instance is satifiable iff one can guard the polygon with $3k$ points.

Vertex linker



Vertex linker



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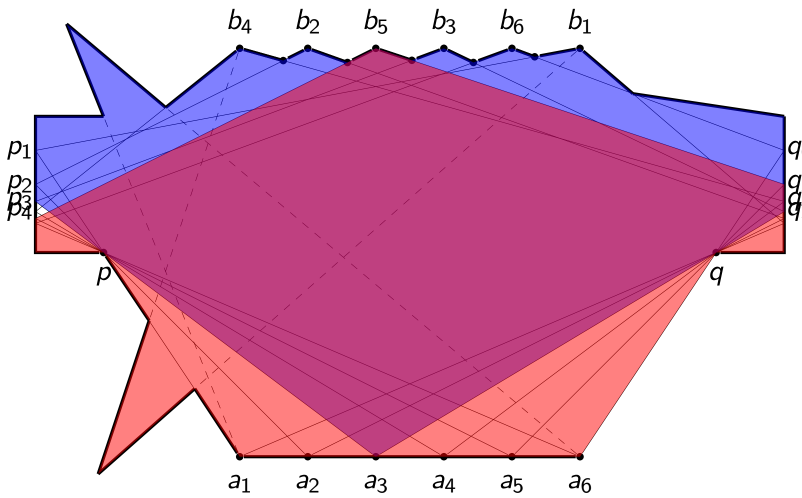

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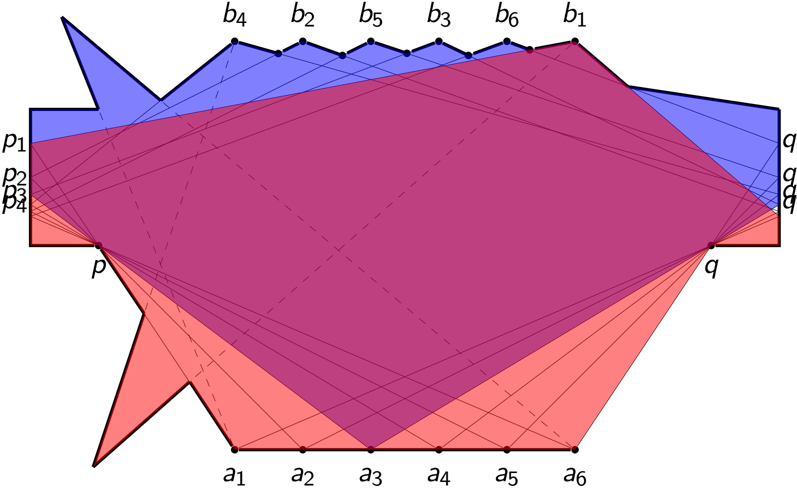

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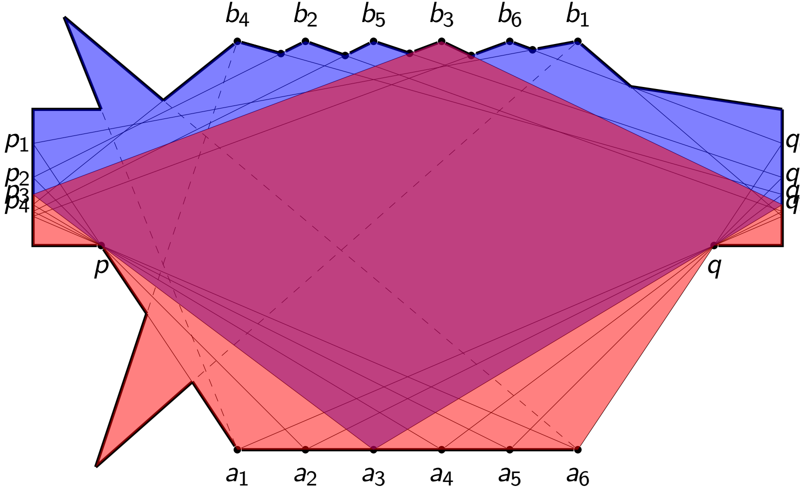
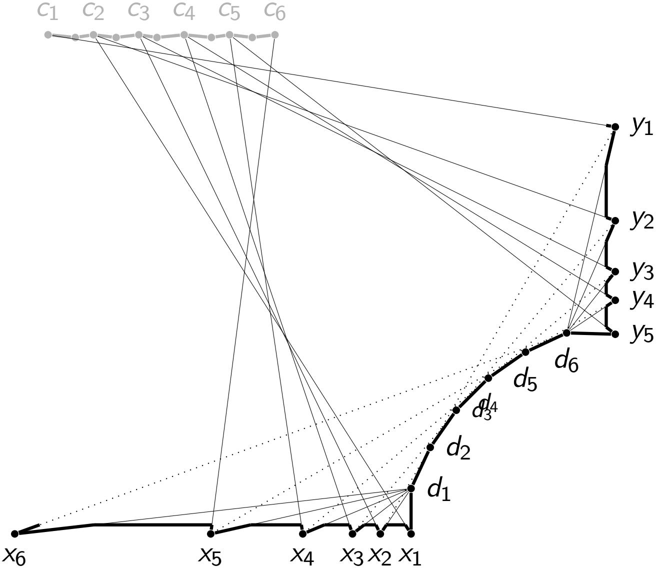


Filter



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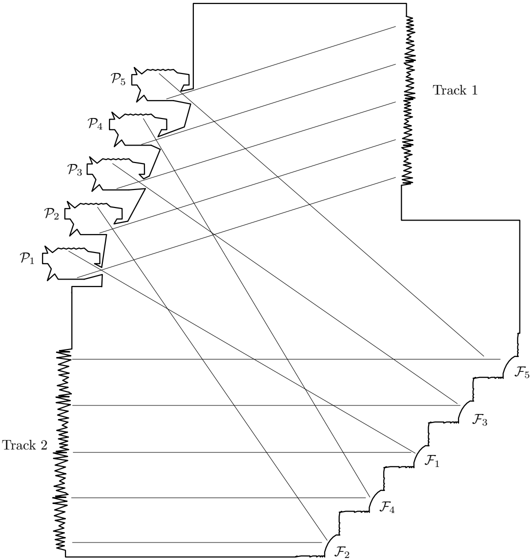

Thank you for attention!