Orthogonal Terrain Guarding is NP-complete

Édouard Bonnet and Panos Giannopoulos

LIP, ENS Lyon

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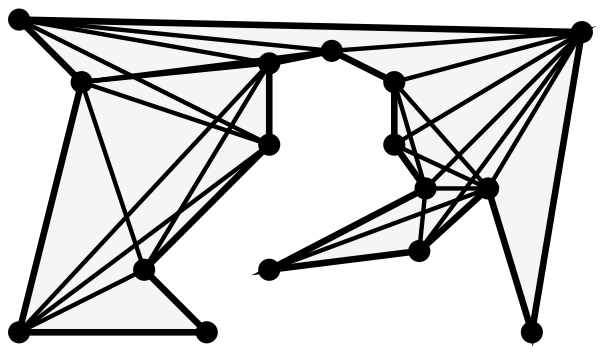
Geometric Set Cover

Covering a universe $\mathcal{U}$ with geometric objects from a class $\mathcal{F}$

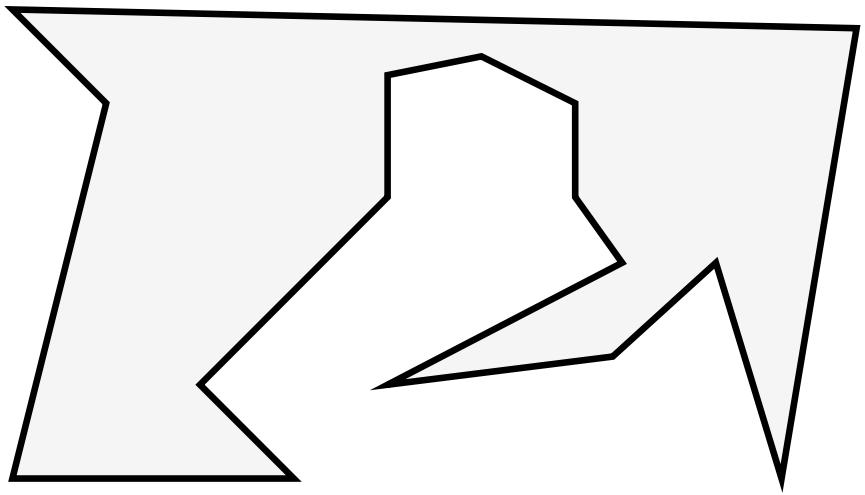
Visibility-based geometric set cover:
$\mathcal{U}$ can be polygons/polygonal chains and $\mathcal{F}$ their points/vertices

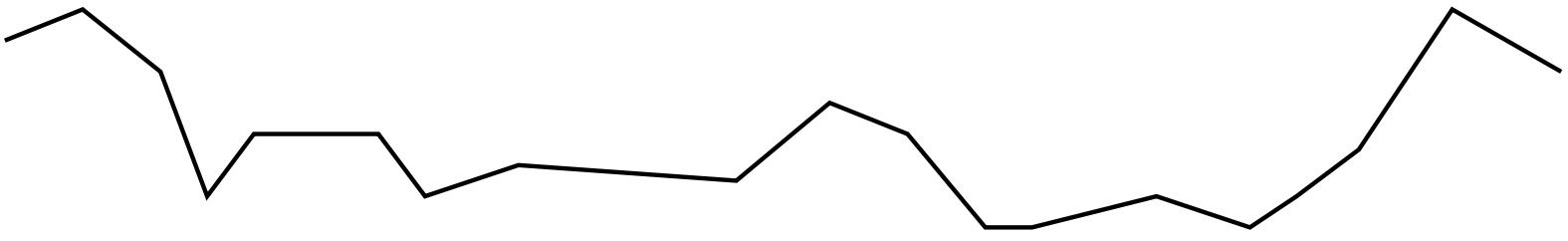


Two equivalent views:

- a point *covers* what it *sees*.
- the objects are in fact the visibility cones (usual cover).

Art Gallery and Terrain Guarding





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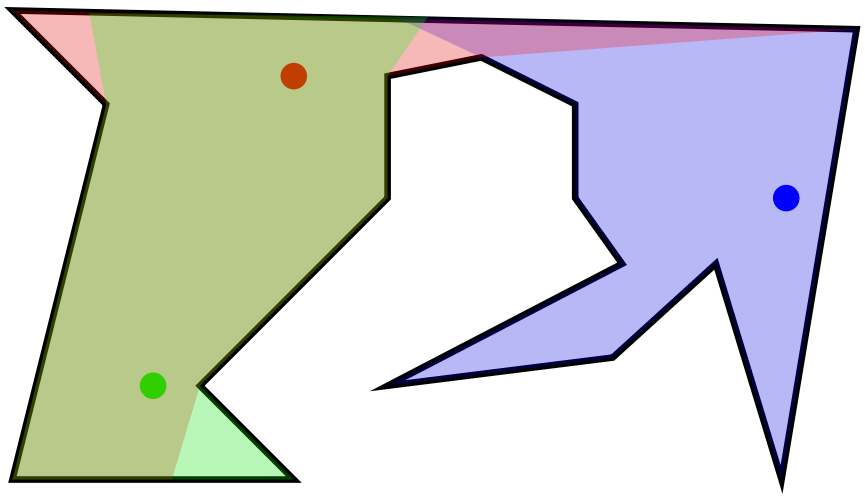

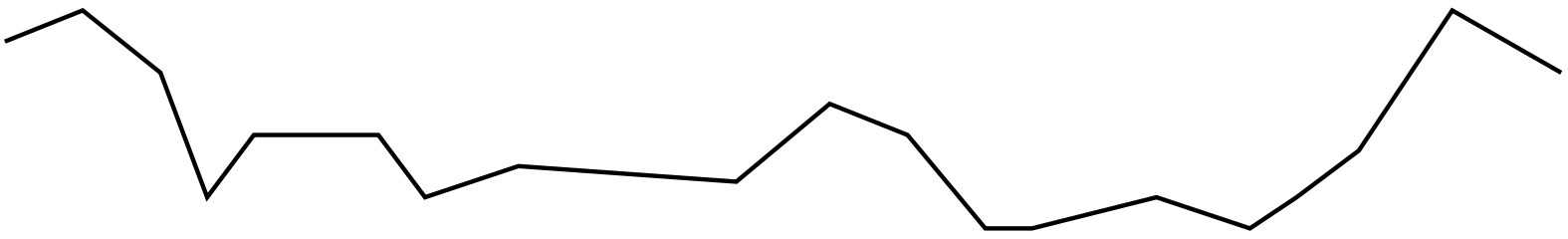

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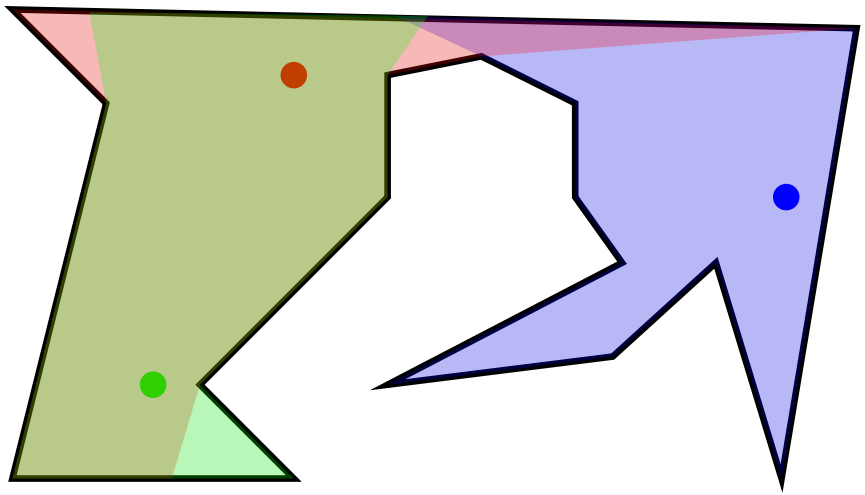
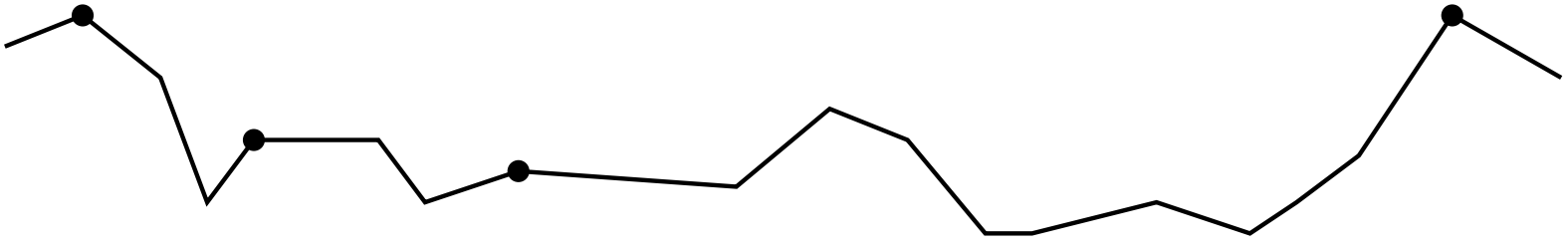
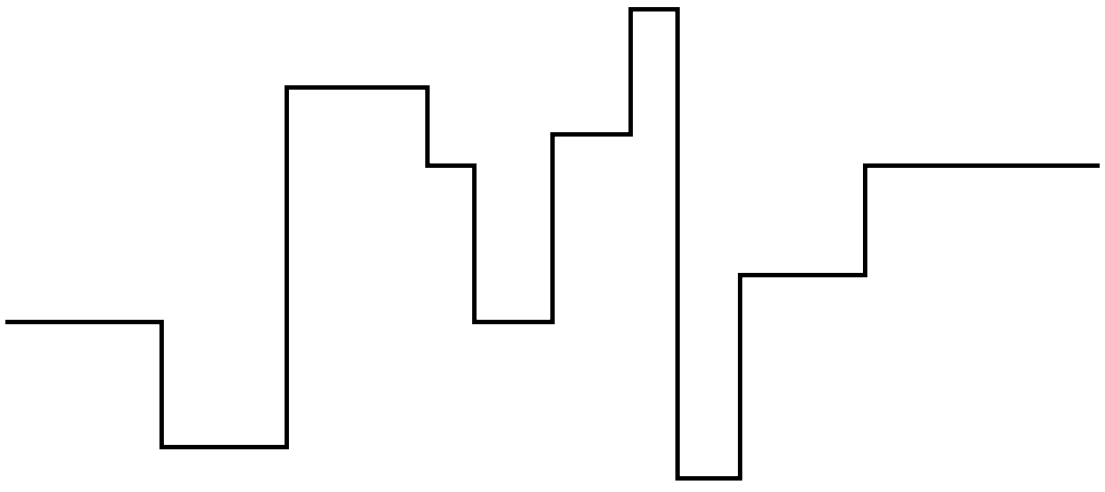




Orthogonal Terrain Guarding



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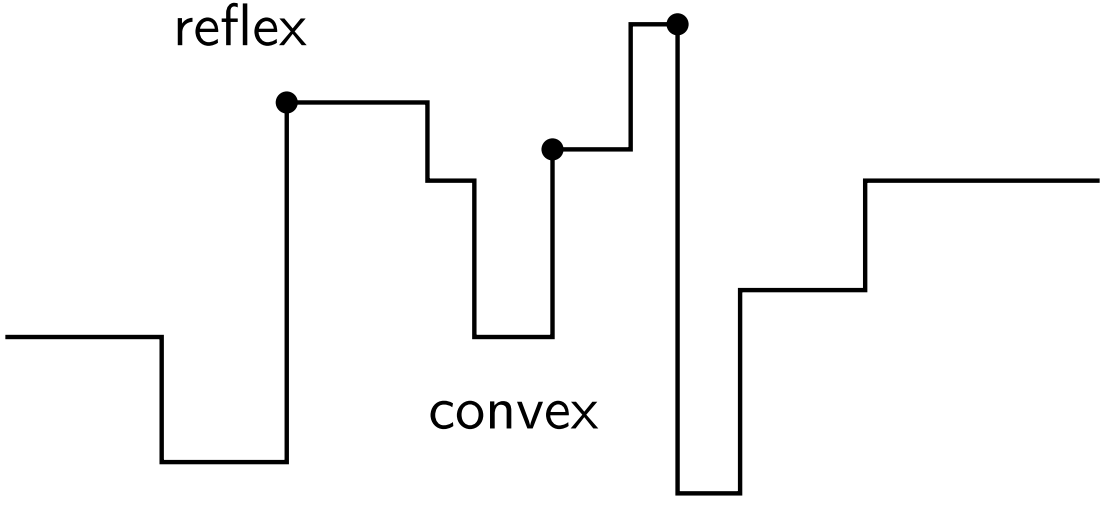
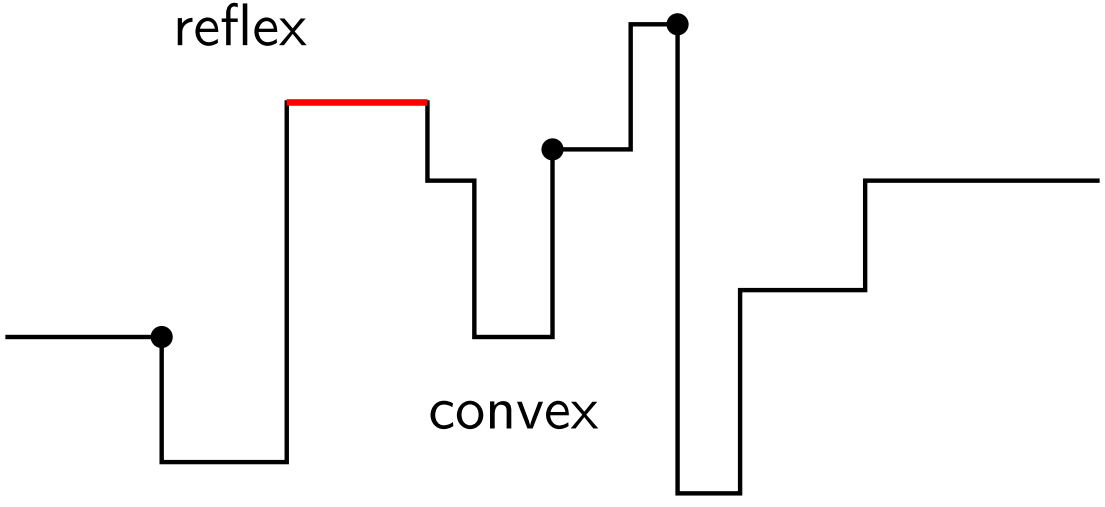


- $\approx$ guarding the convex vertices with reflex vertices

Orthogonal Terrain Guarding



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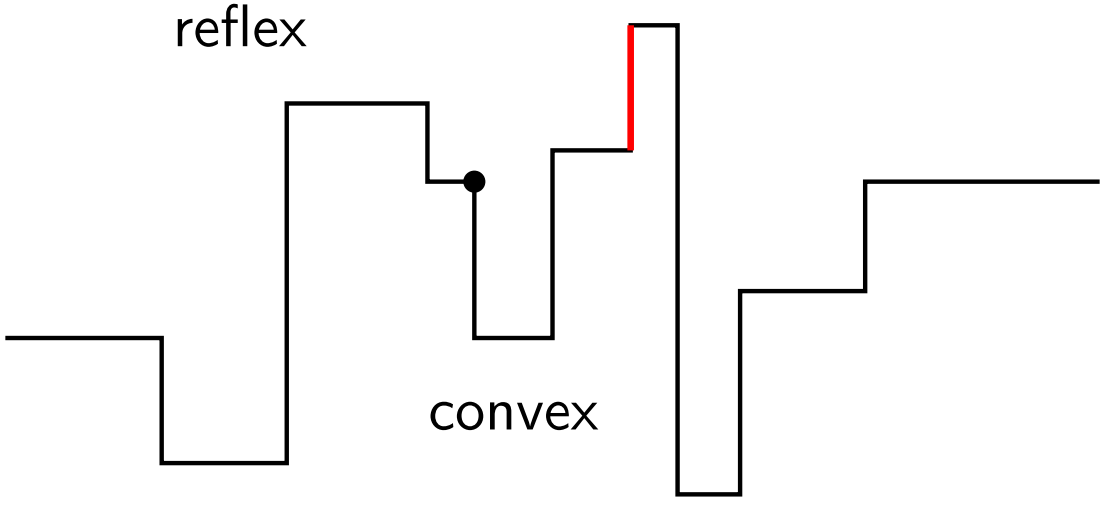


- $\approx$ guarding the convex vertices with reflex vertices
- only what a guard sees at its level and below matters

Simpler or harder than Set Cover?

- Continuum: membership in NP not guaranteed
- $\exists\mathbb{R}$-completeness of Art Gallery, Abrahamsen et al. '18
- If polynomially discretizable, simpler as geometrically realizable

Terrain Guarding can be discretized.

Computational complexity of Art Gallery

- NP-hard, APX-hard, $\exists\mathbb{R}$-complete
- if holes are allowed, as hard as Set Cover
- $O(\log \mathrm{OPT})$-approx in some restricted cases/different setting
- constant-approx for the Vertex Guard, Bhattacharya et al. '18 (come to our workshop!)
- nothing significantly better than $n^{O(k)}$, B. and Miltzow '16

Computational Complexity

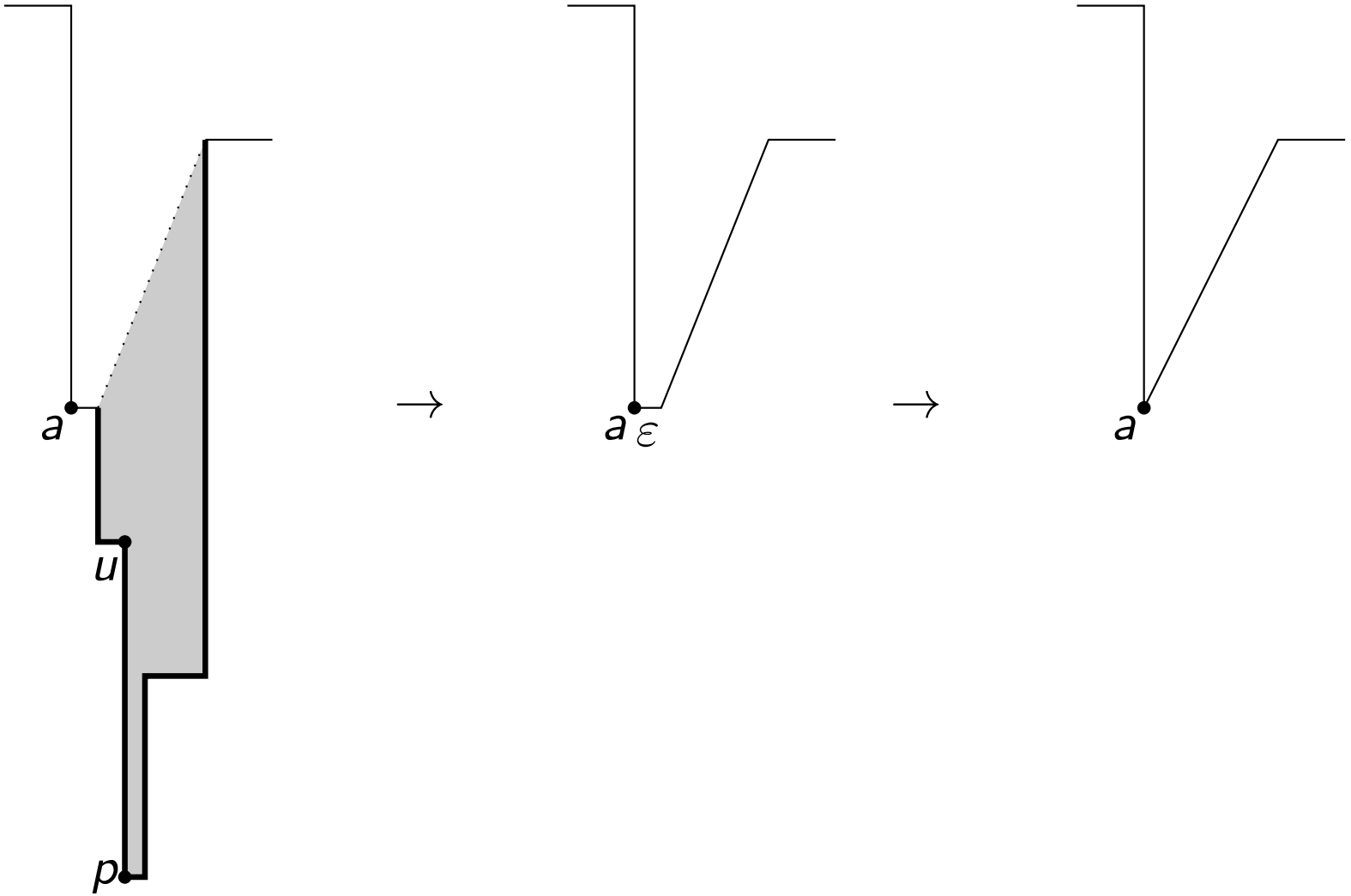
of Terrain Guarding:

- several constant-factor approximations
- **NP-hardness by King and Krohn '11**
- PTAS via local search by Krohn et al. '14
- Subexponential algorithm in $n^{O(\sqrt{k})}$ by Ashok et al. '18

of Orthogonal Terrain Guarding:

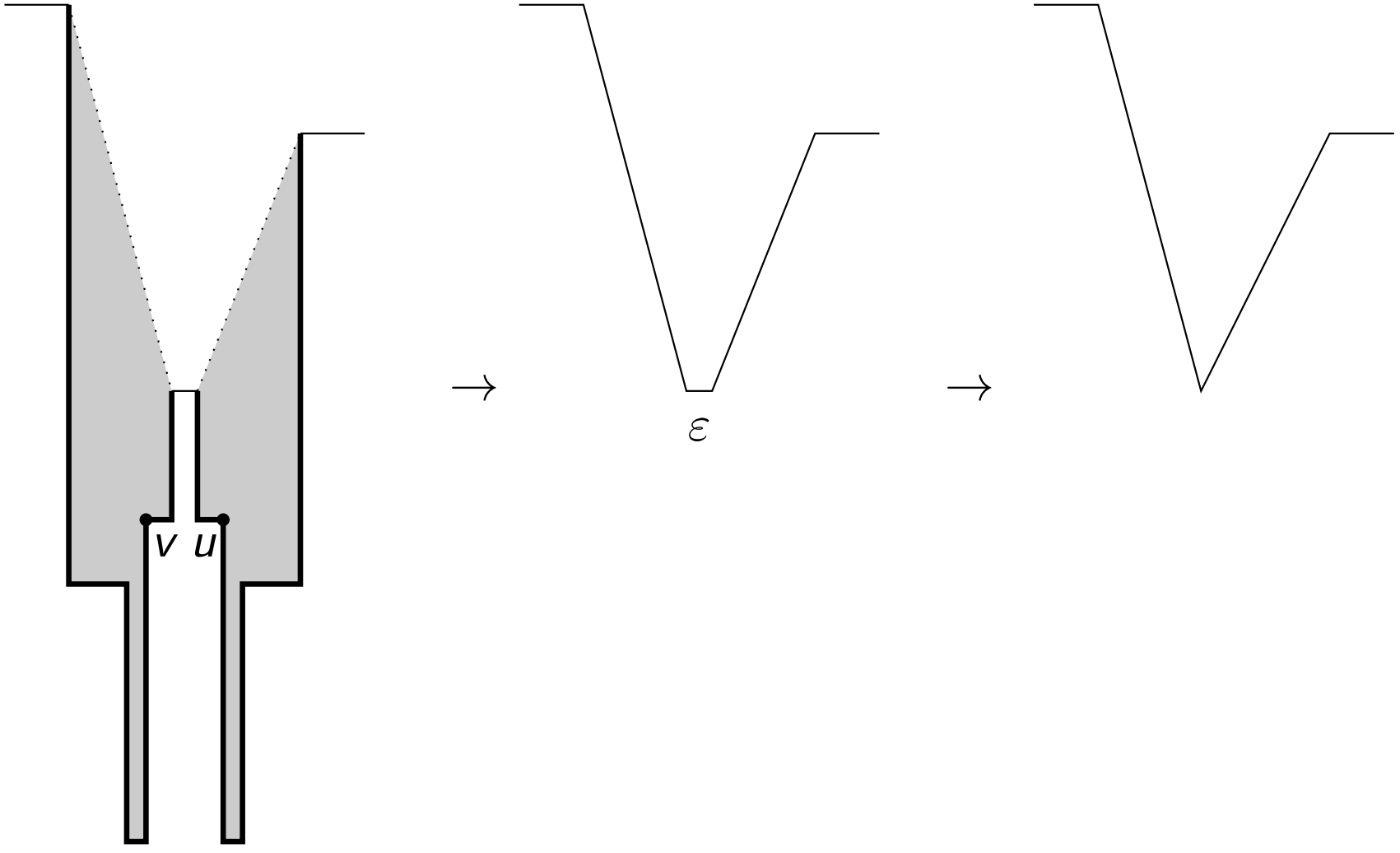
- all positive results of Terrain Guarding
- FPT algorithm $k^{O(k)}$ to guard the vertices, by Ashok et al.

Simulating Triangular Pocket (1)



Vertex u dominates the visibility of every vertex seeing p

Simulating Triangular Pocket (2)



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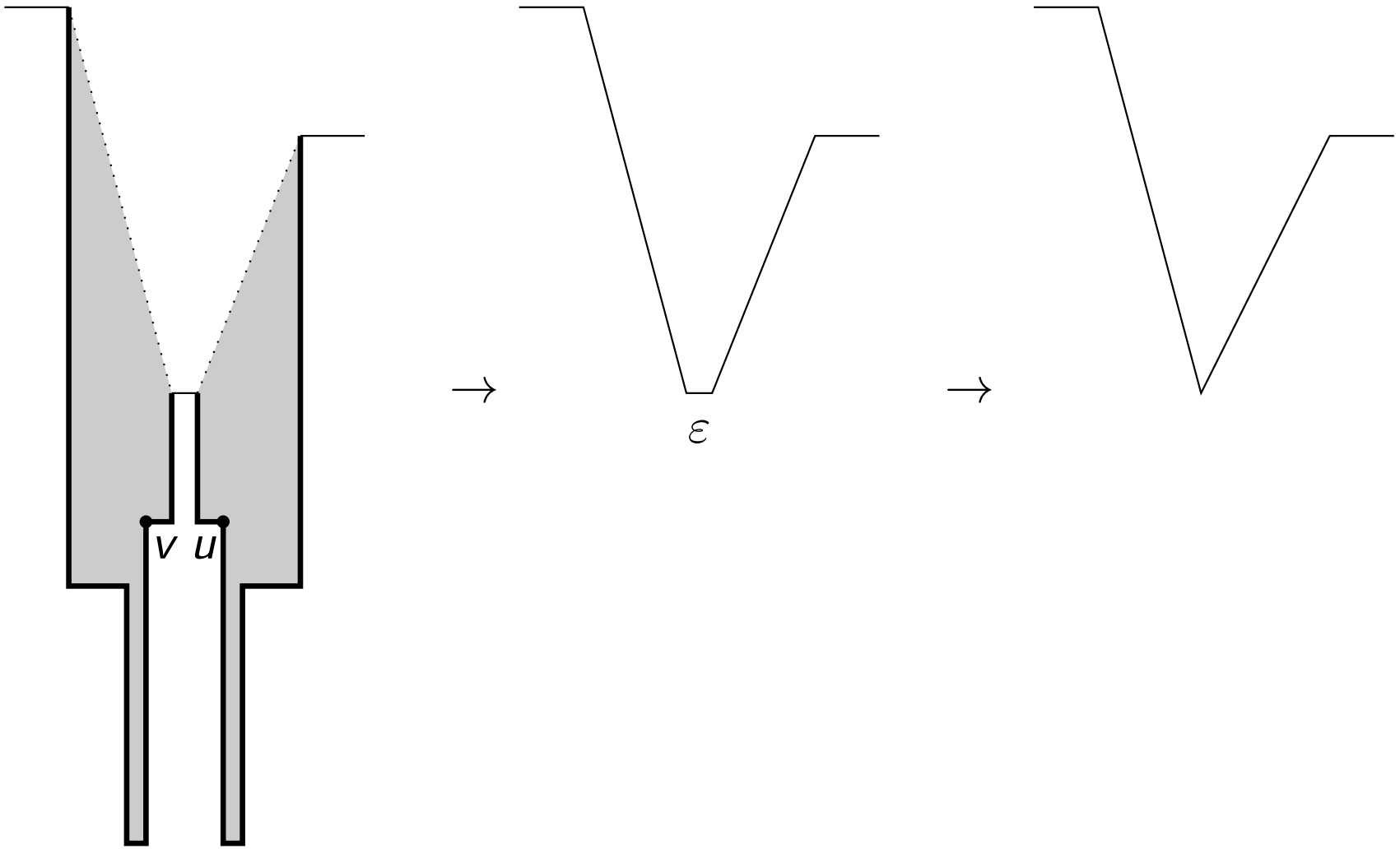
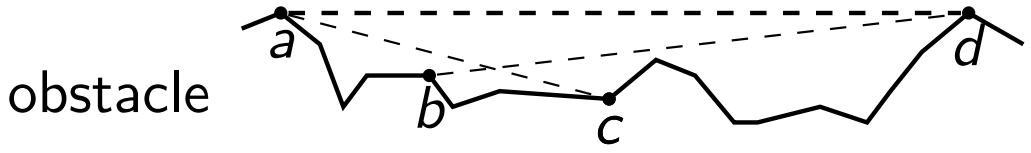


Why is this *not* a direct reduction from Terrain Guarding?

King and Krohn's reduction (1)



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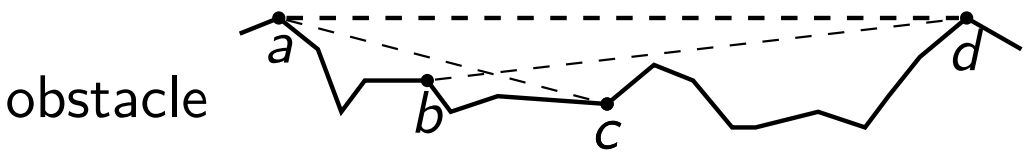
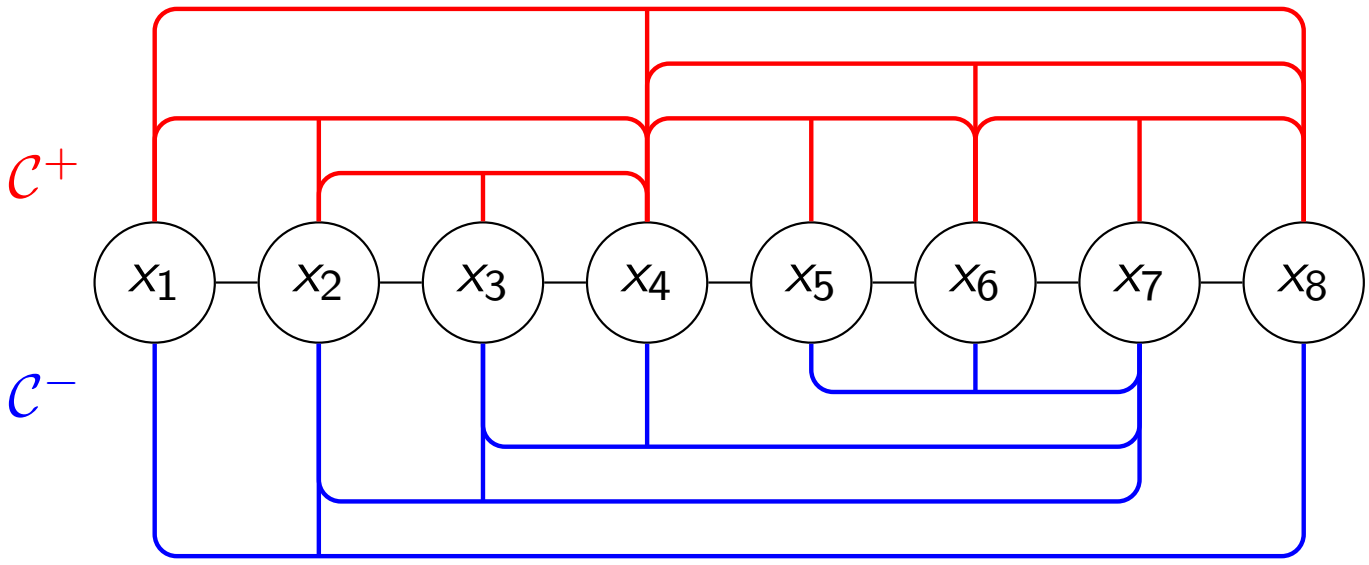


from a rectilinear PLANAR 3-SAT



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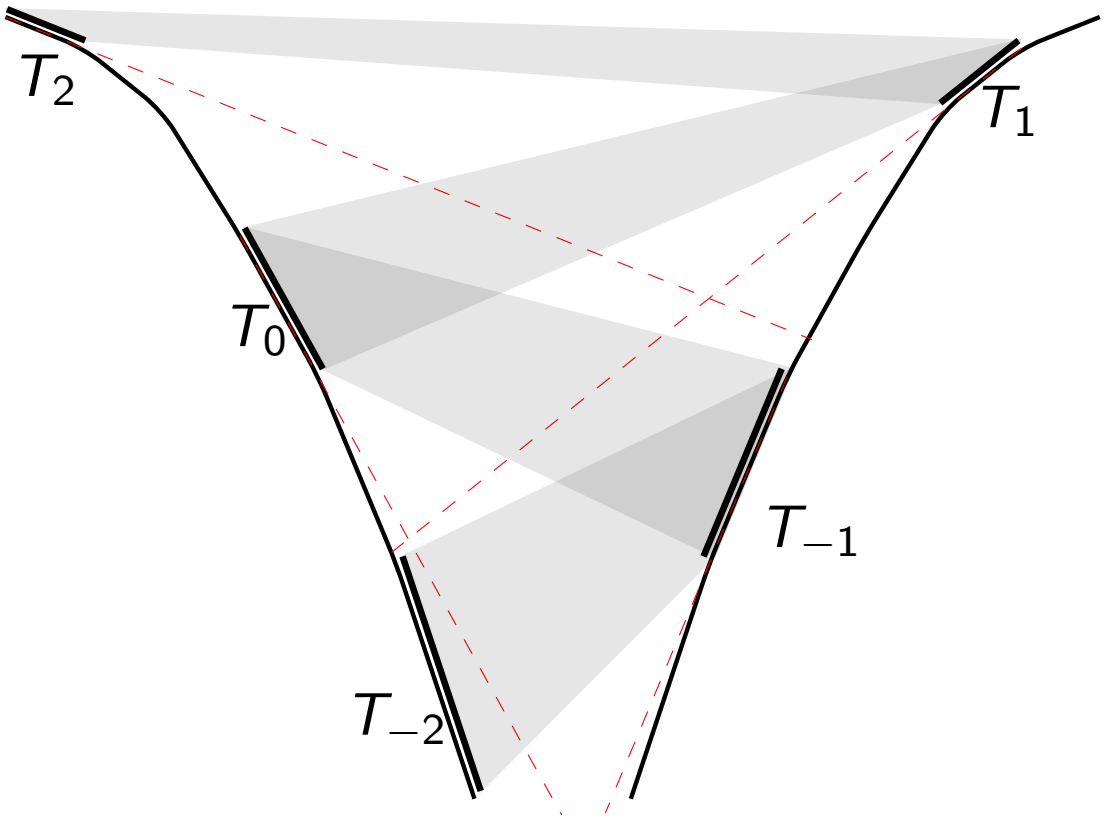

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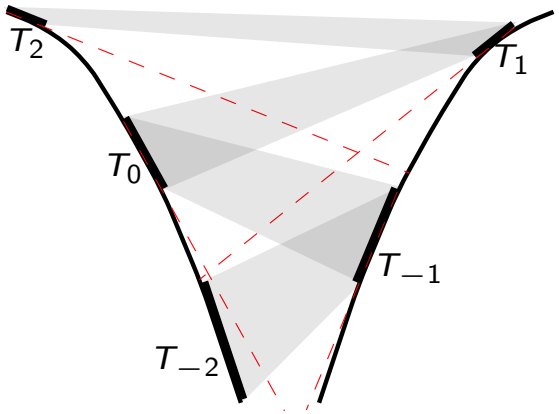

Needed:

- Variable encoding which transmits the value up and down
- Upward/downward clause checker
- Upward/downward variable deletion

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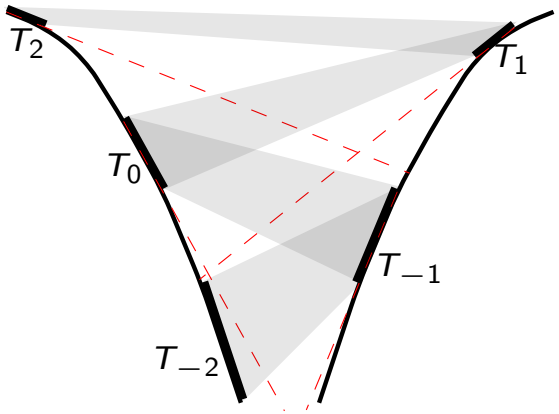
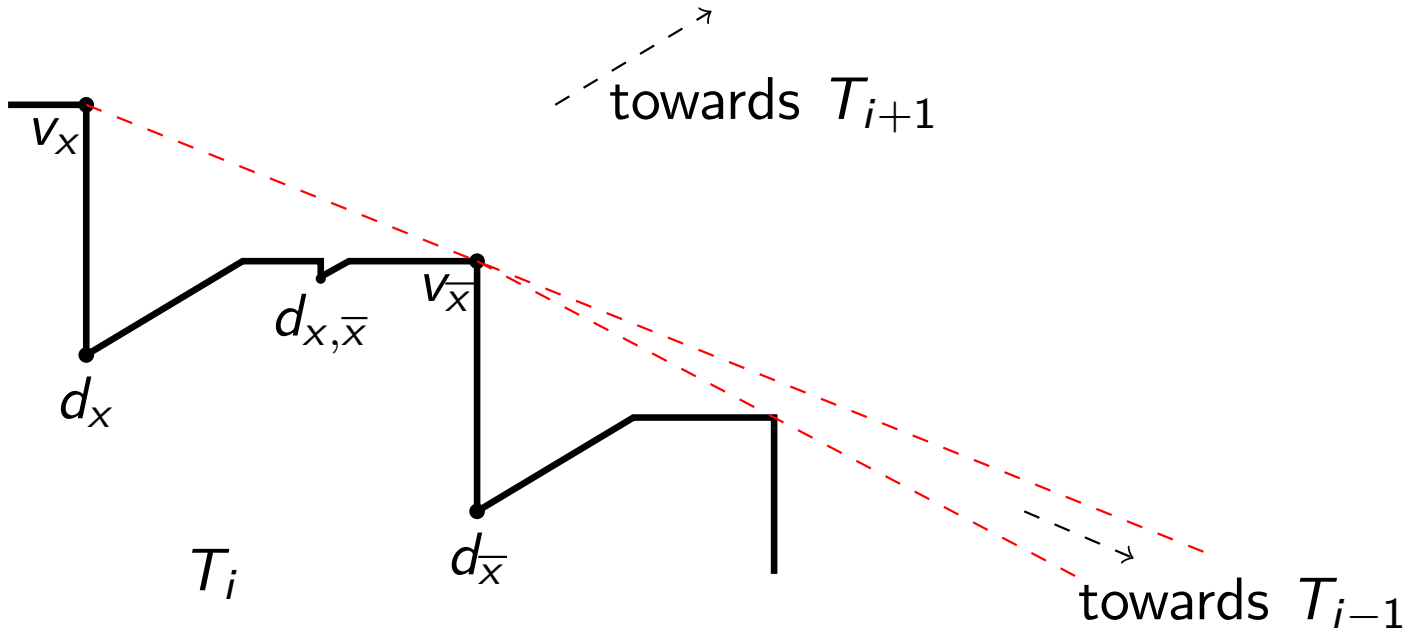


Needed:

- Variable encoding which transmits the value up and down
- Upward/downward clause checker
- Upward/downward variable deletion
- inverter gadget: change the relative position of the literals

Variable encoding



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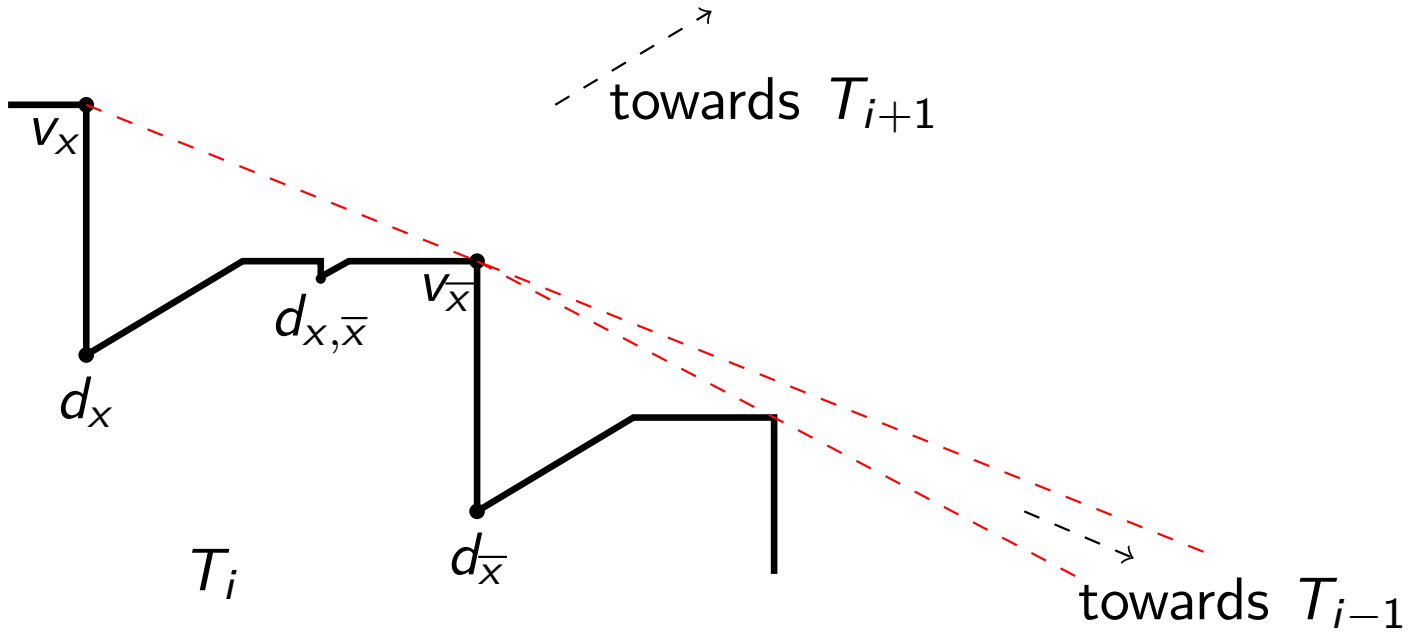
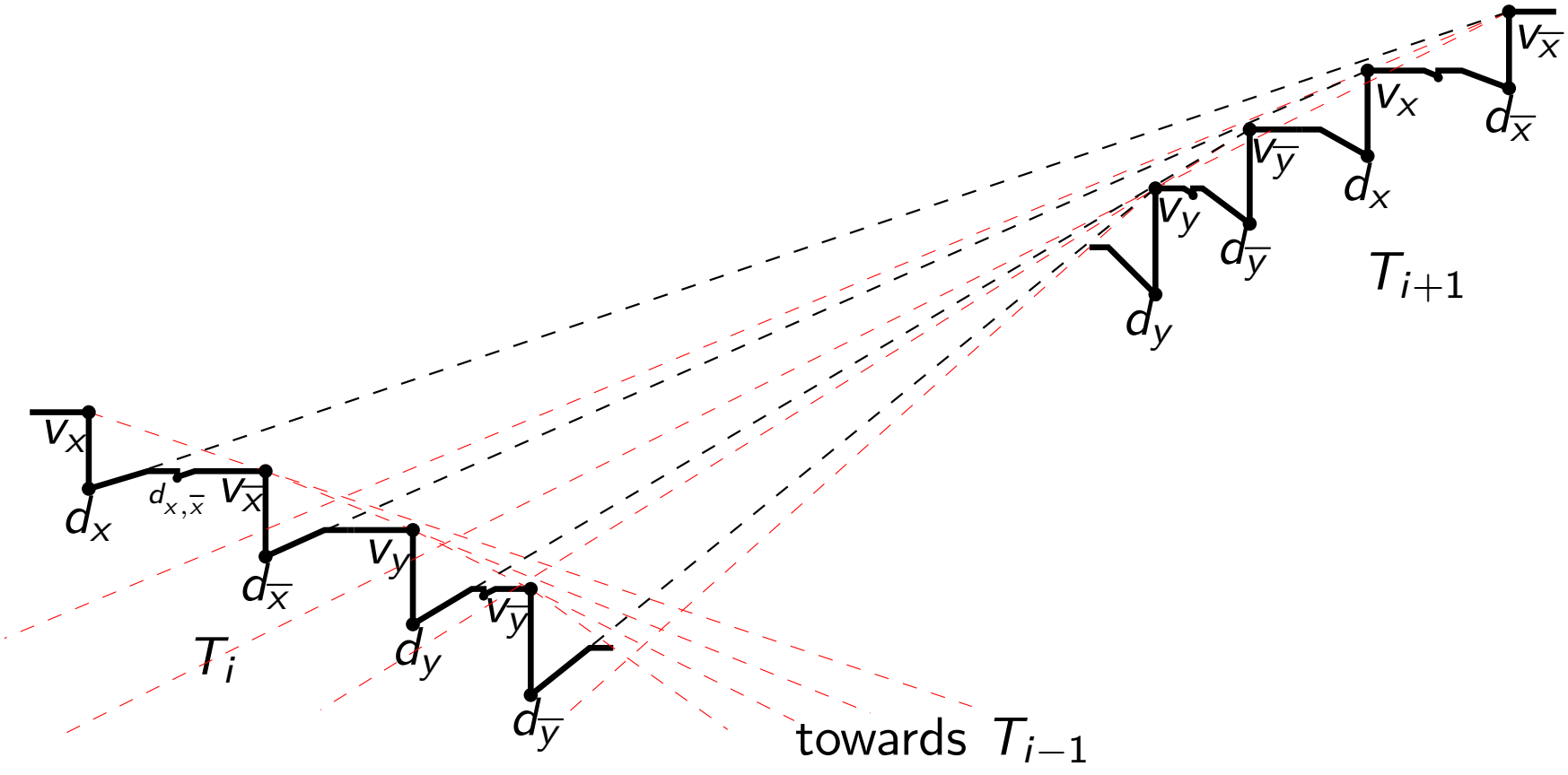


Placing a guard at $v_x \equiv x$ is set to true
Placing a guard at $v_{\overline{x}} \equiv x$ is set to false

Variable propagation



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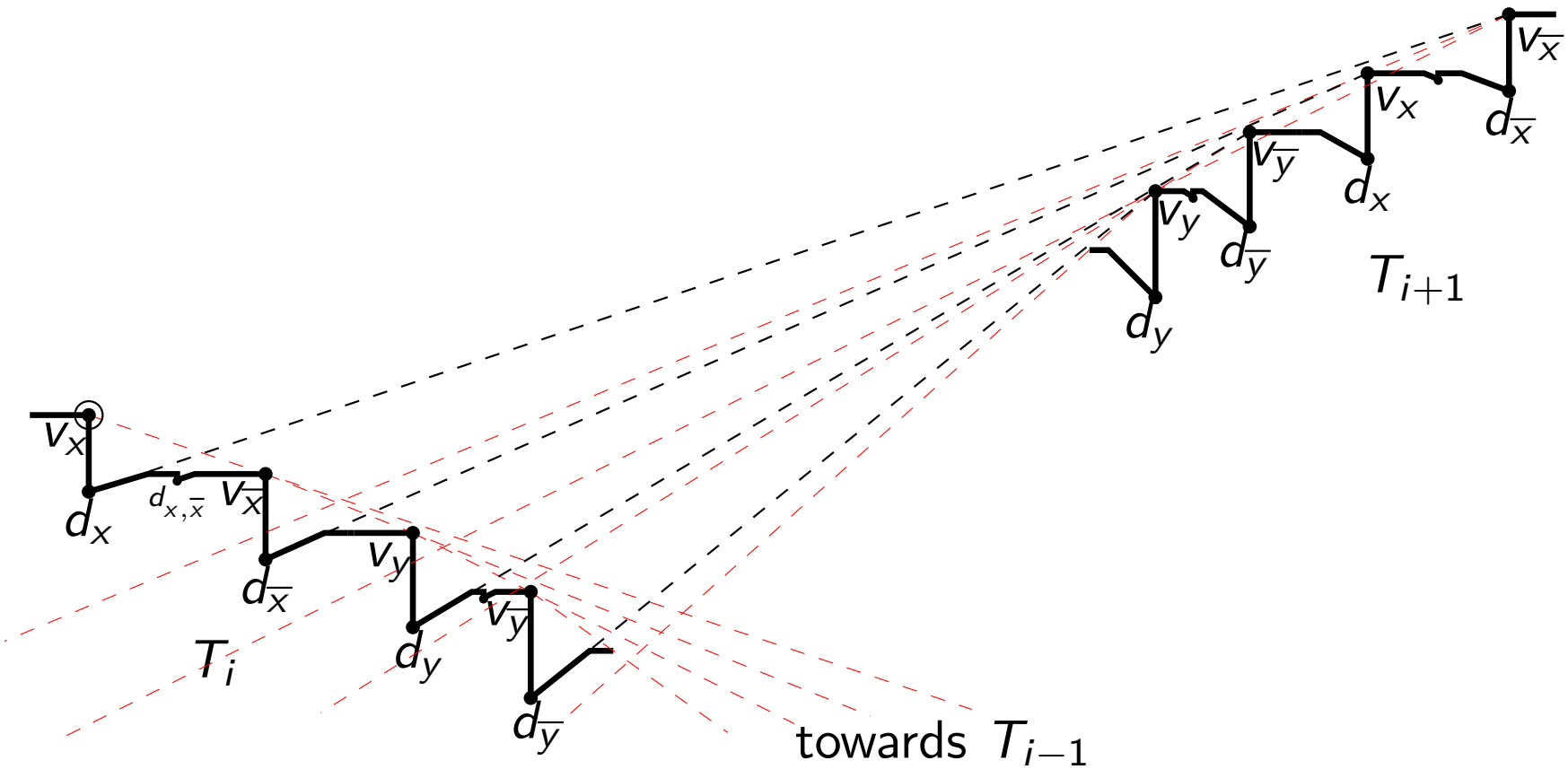

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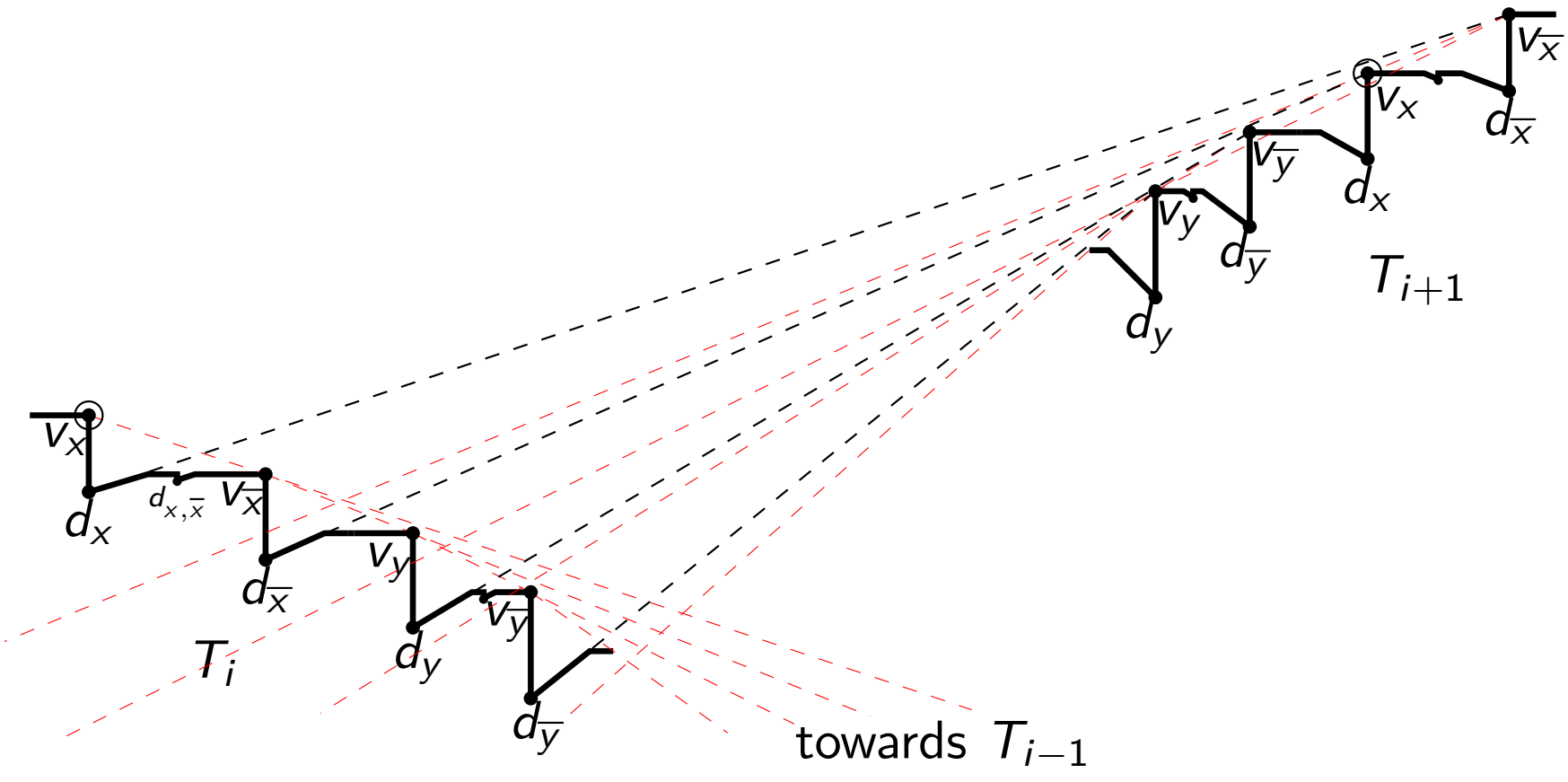

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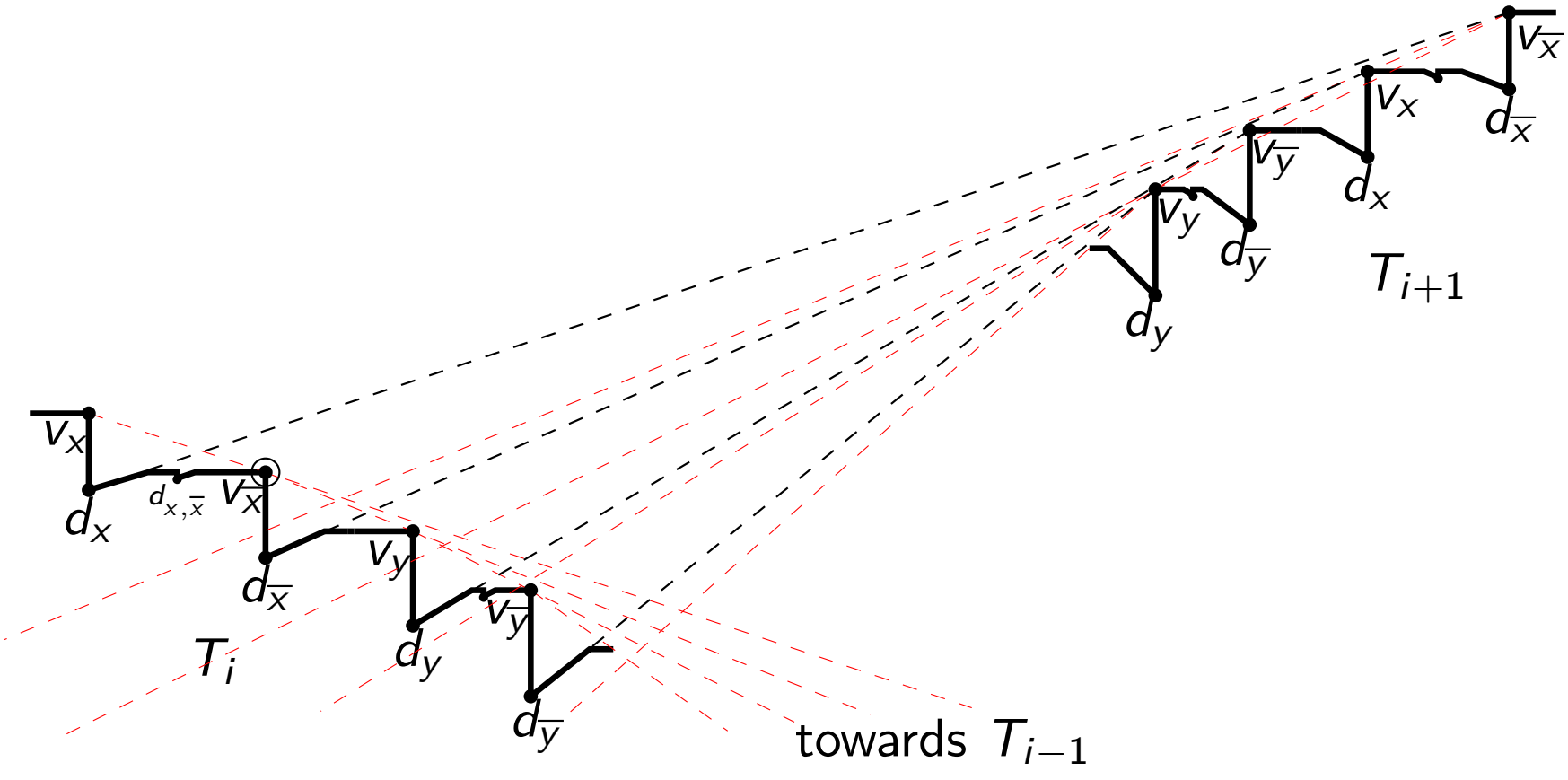

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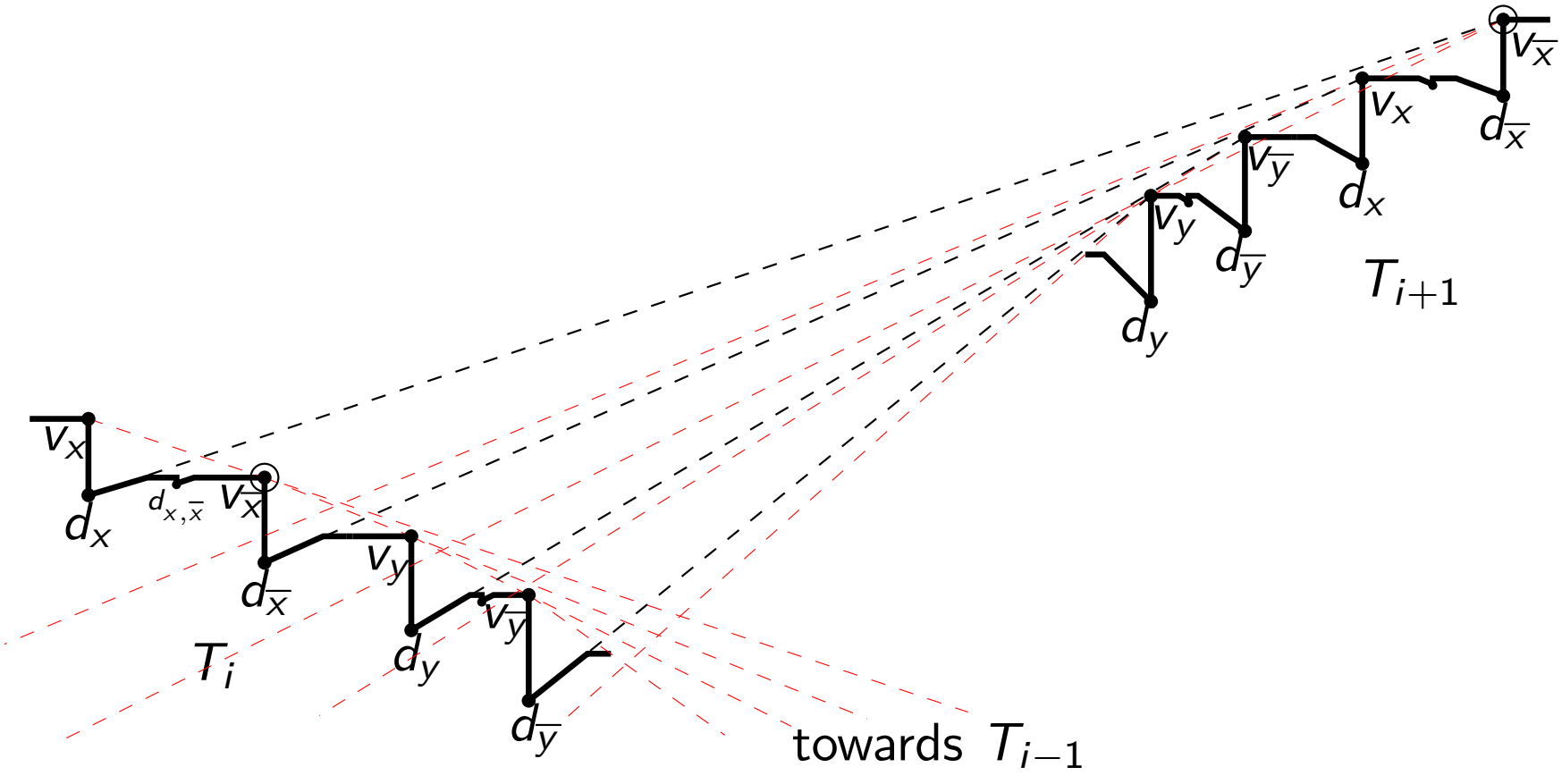
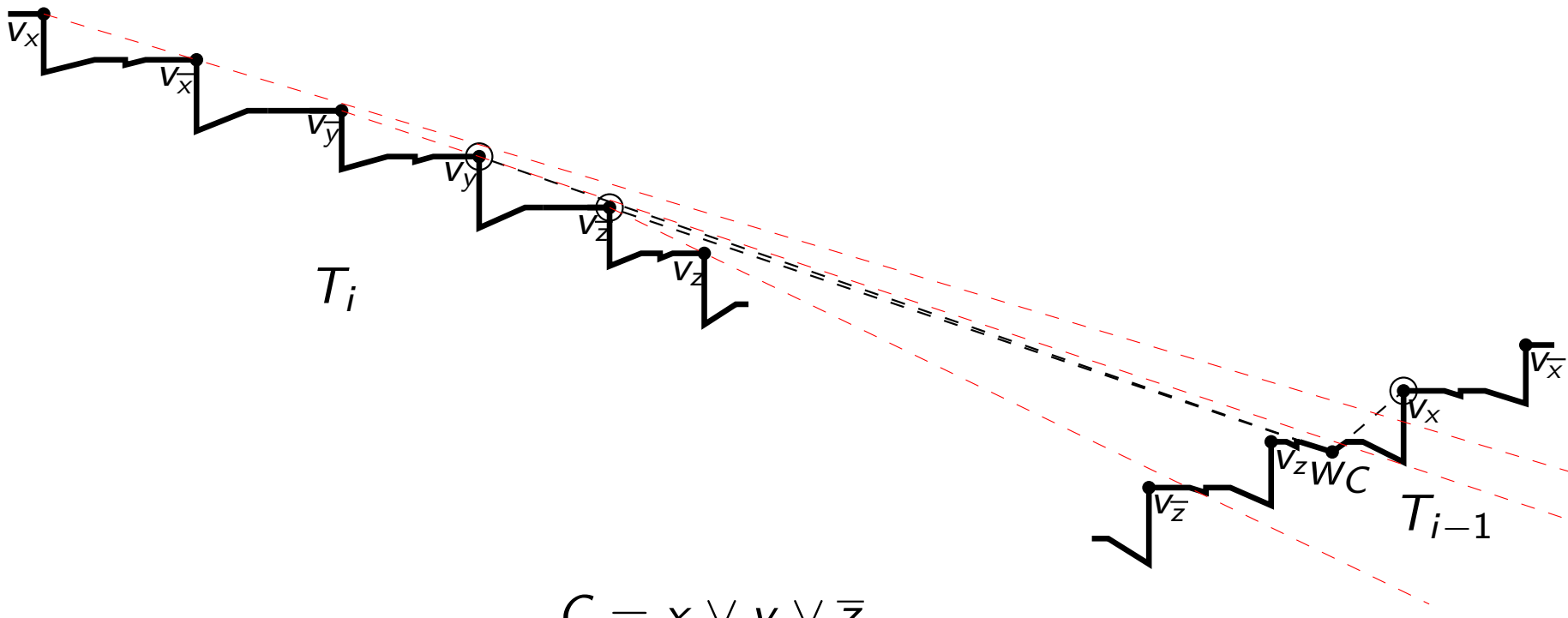
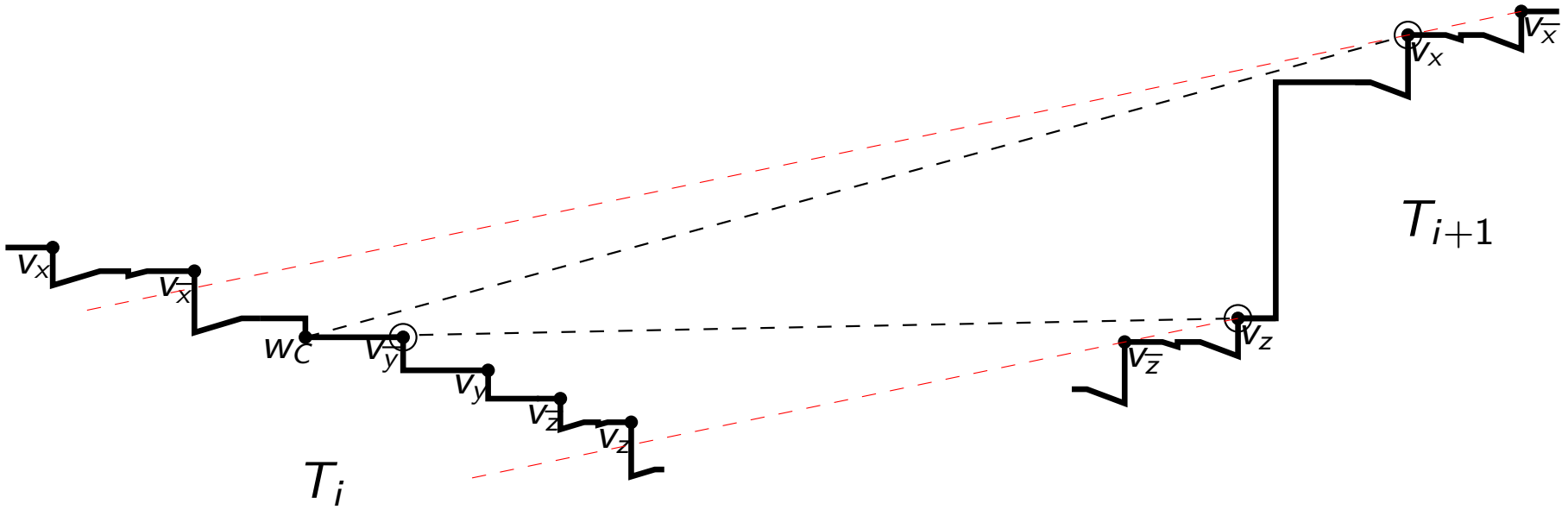


Downward Clause Checker



$$C = x \vee y \vee \bar{z}$$

Upward Clause Checker



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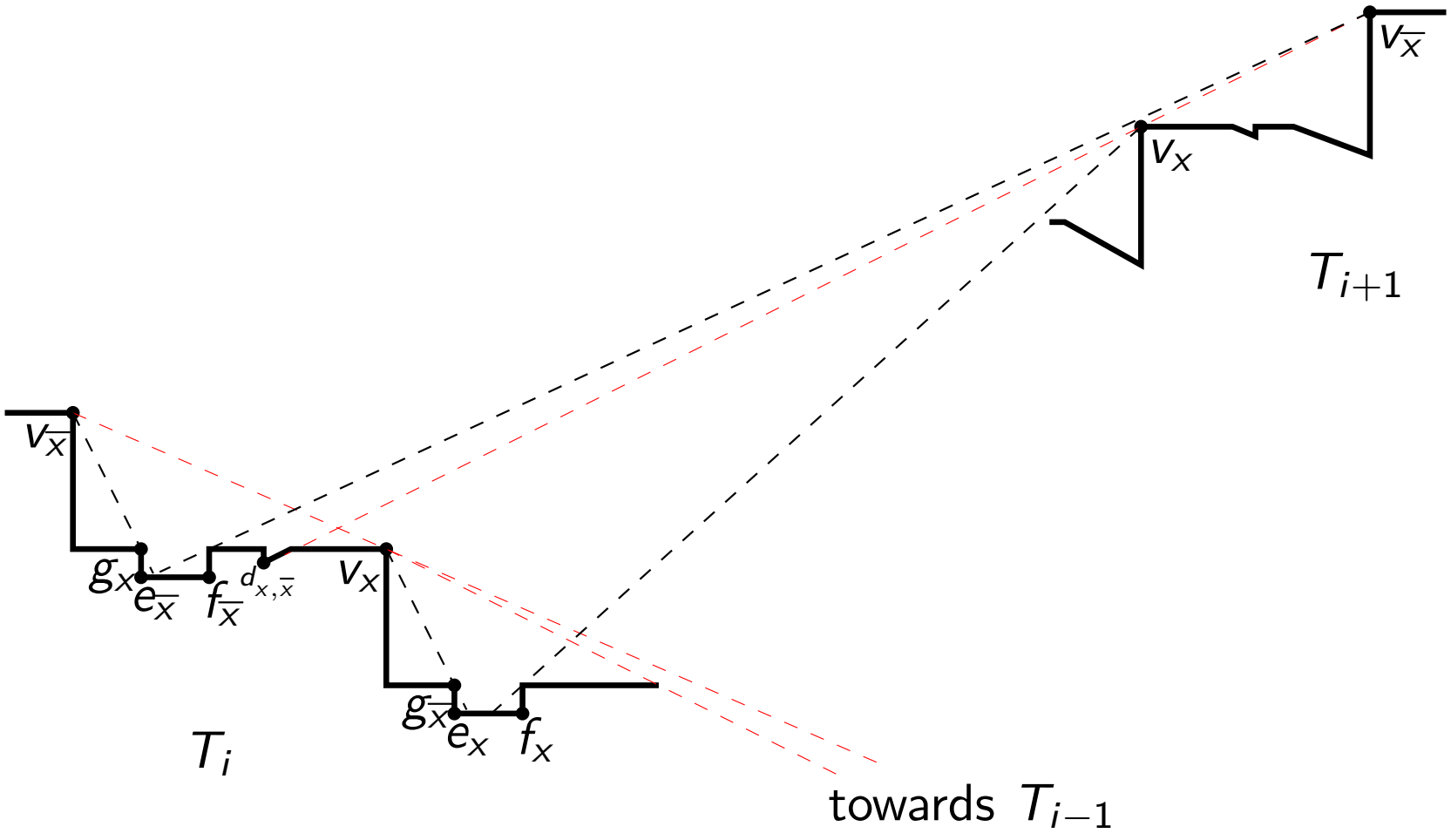
Literal Inverter



The budget allows one extra guard

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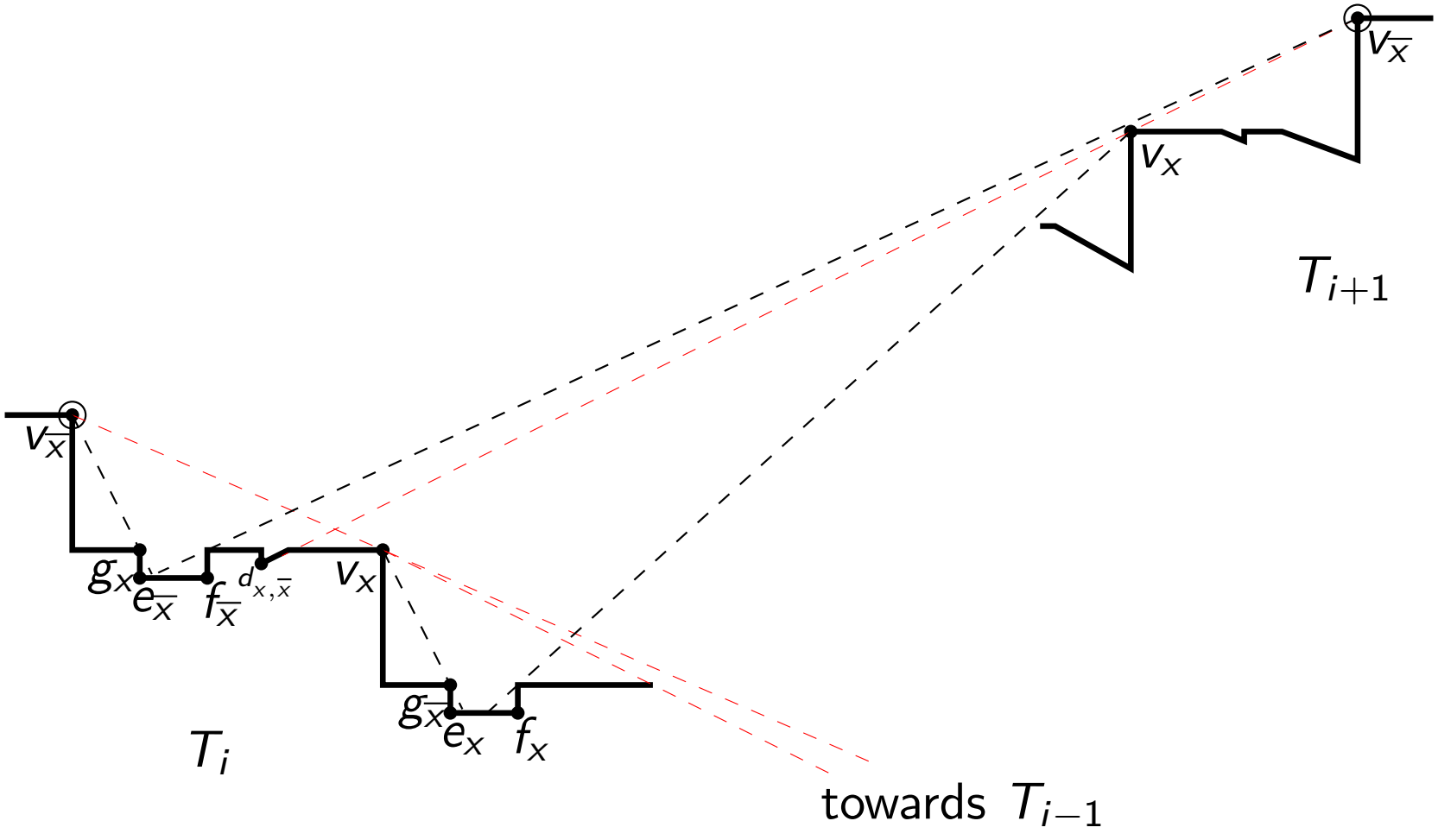

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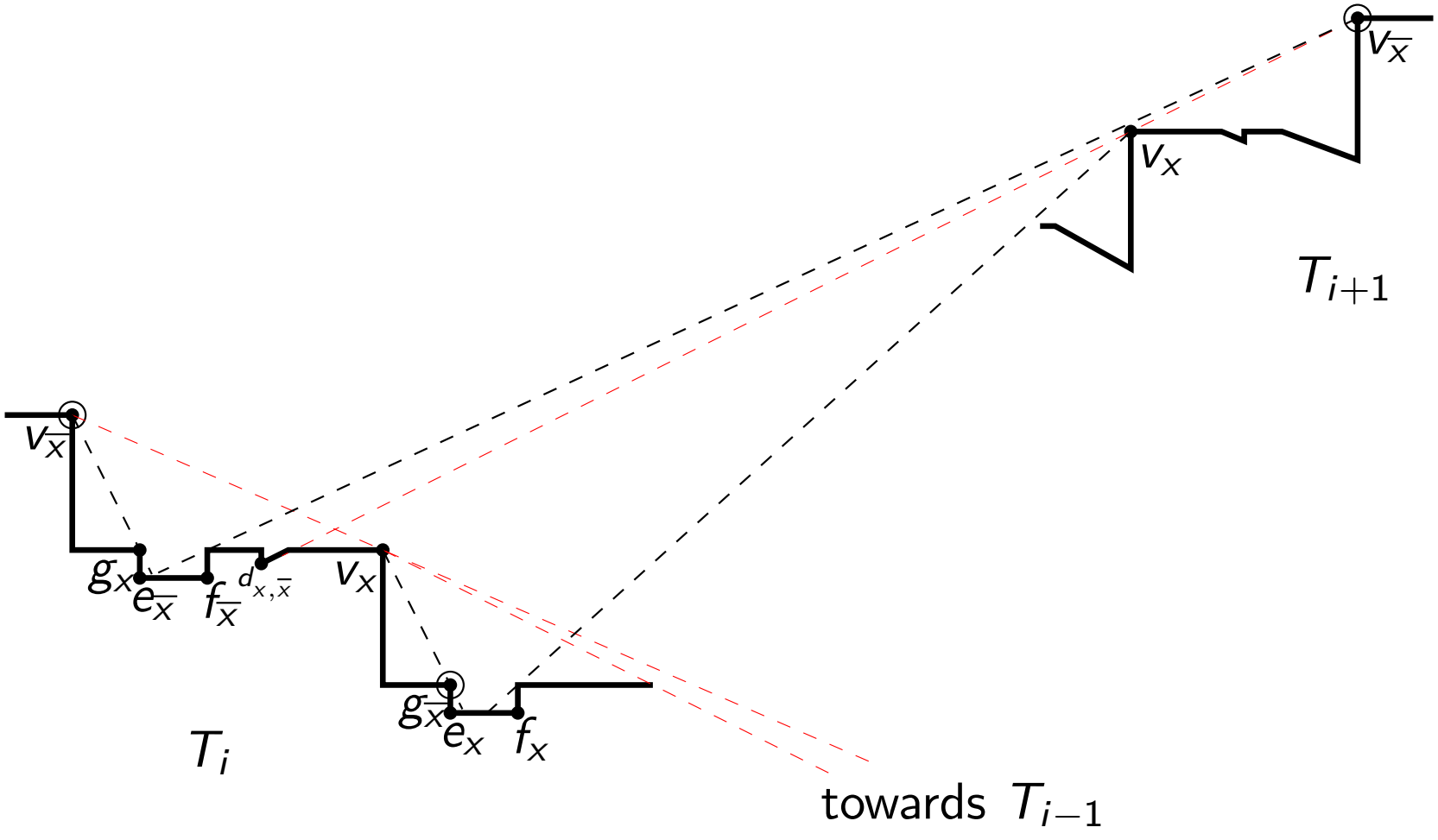

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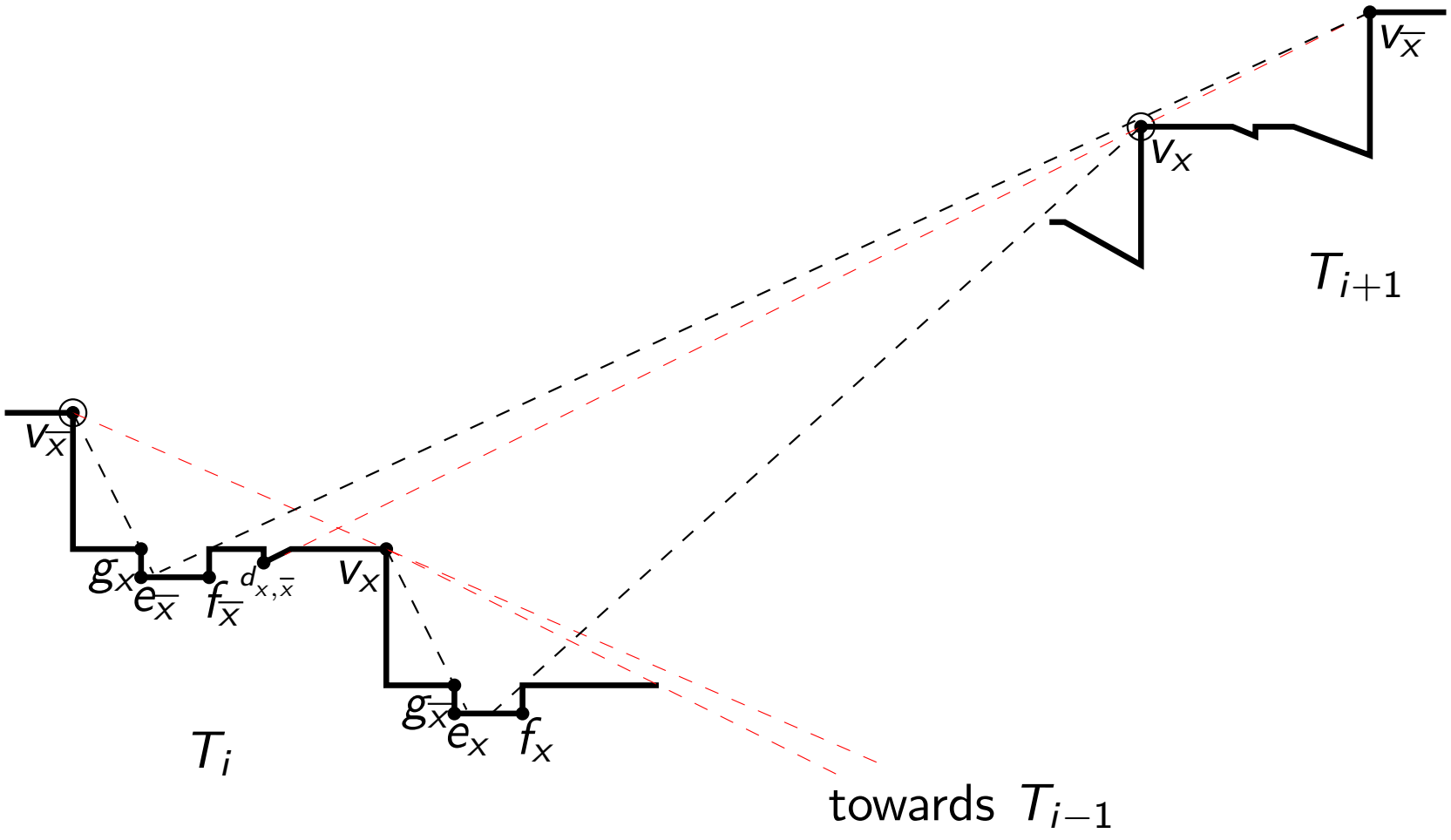

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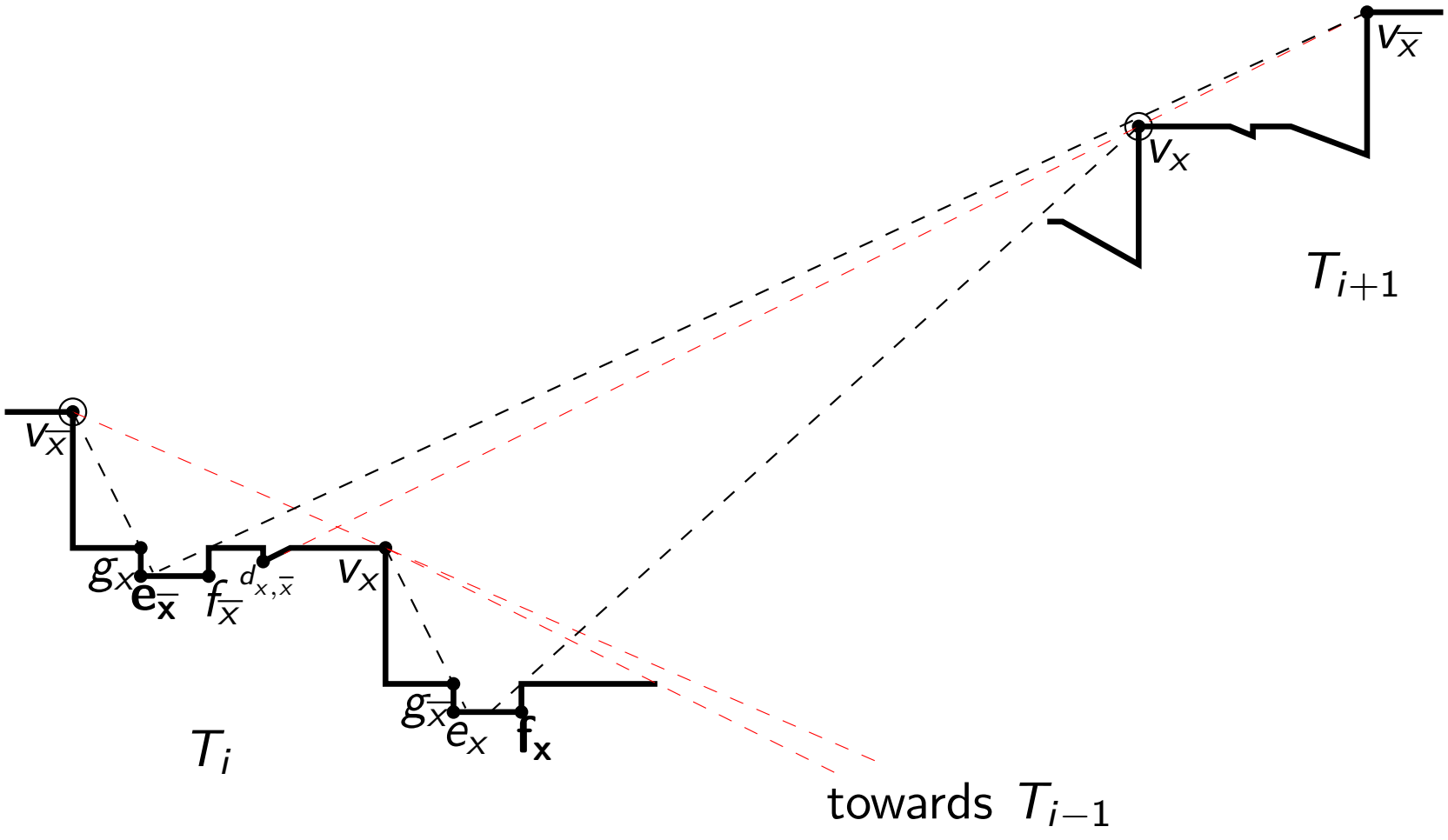

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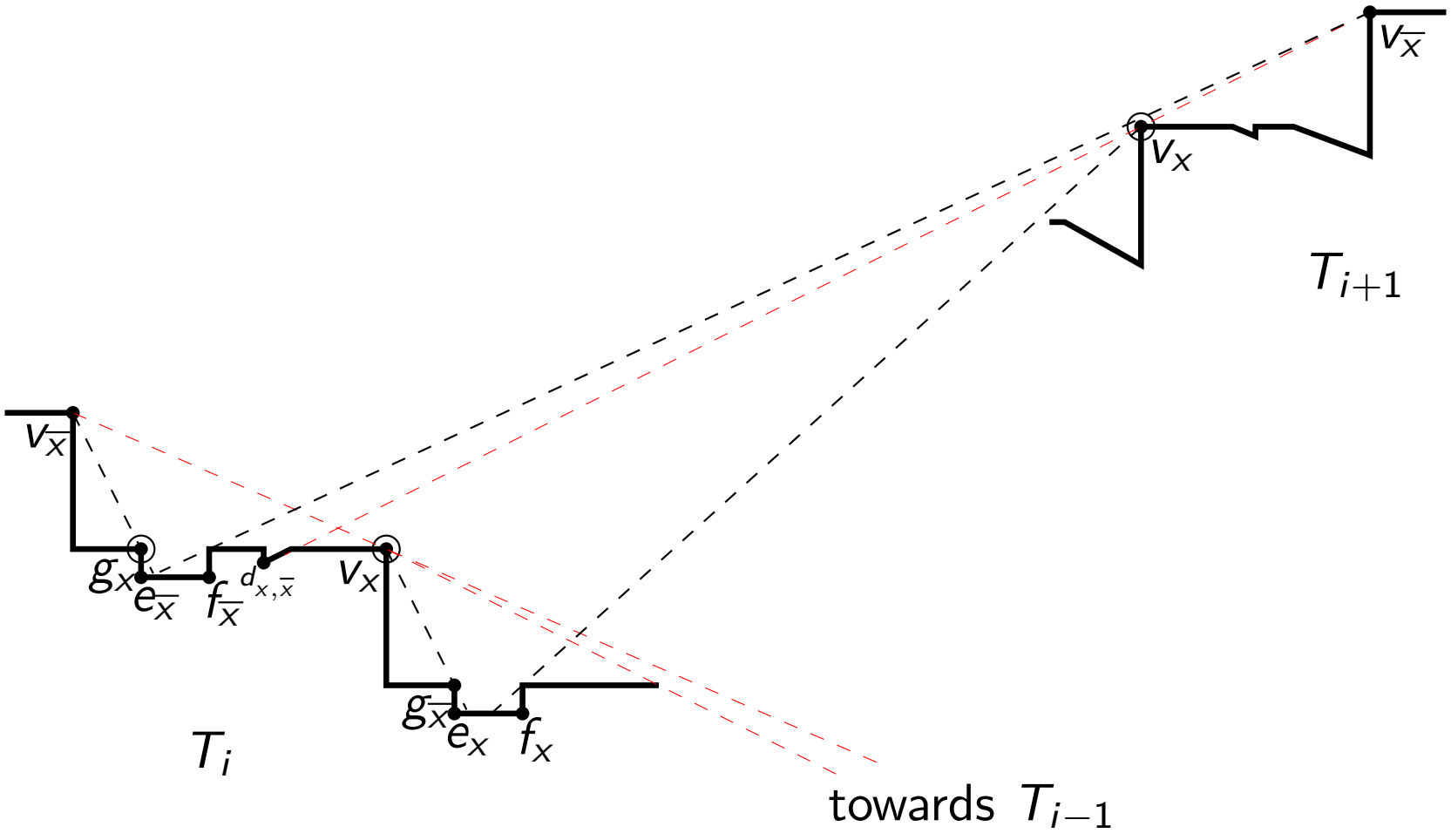

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- Approximation is also well understood with the PTAS.
- FPT algorithms for those problems?

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