Twin-Width: Algorithmic Applications and Open Questions

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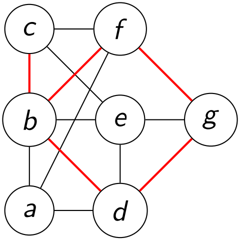
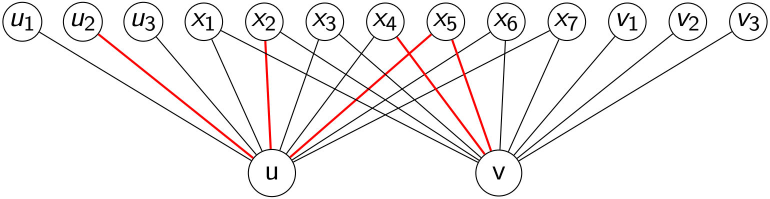
20th January 2025, Solving Problems on Graphs: From Structure to Algorithms

Trigraphs



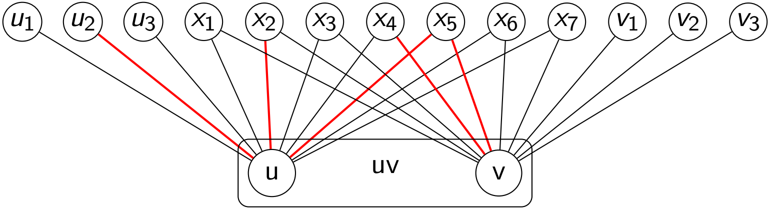
Three outcomes between a pair of vertices:
edge, or non-edge, or red edge (error edge)

Contractions in trigraphs



Identification of two non-necessarily adjacent vertices

Contractions in trigraphs



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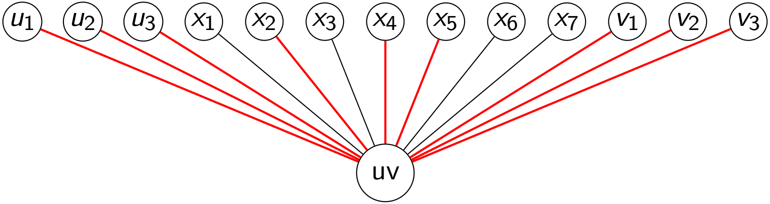
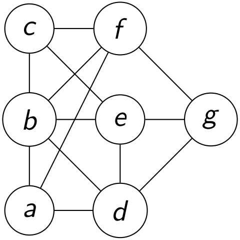


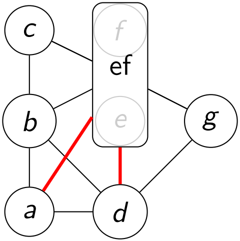
edges to $N(u) \triangle N(v)$ turn red, for $N(u) \cap N(v)$ red is absorbing

Contraction sequence



A contraction sequence of G:
Sequence of trigraphs $G = G_n, G_{n-1}, \ldots, G_2, G_1$ such that
G_i is obtained by performing one contraction in G_{i+1}.

Contraction sequence



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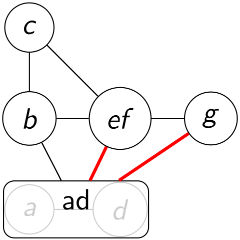

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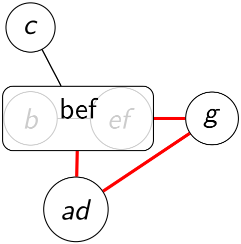

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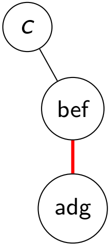

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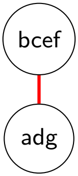

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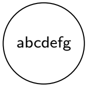

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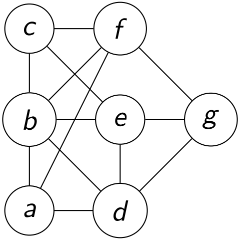
Twin-width

tww(G): Least integer d such that G admits a contraction sequence where all trigraphs have *maximum red degree* at most d.



Maximum red degree = 0

overall maximum red degree = 0

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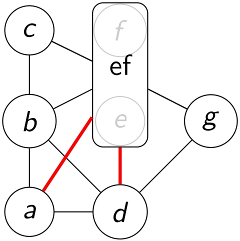

Maximum red degree $= 2$

overall maximum red degree = 2

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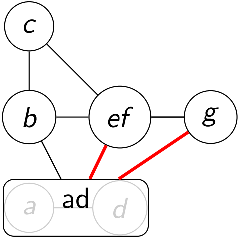

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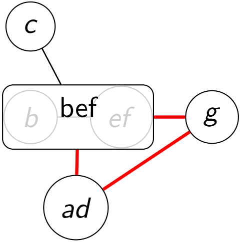

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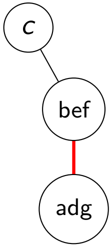

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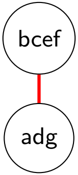

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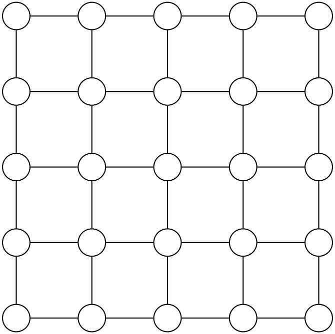
Twin-width

tww(G): Least integer d such that G admits a contraction sequence where all trigraphs have *maximum red degree* at most d.

Maximum red degree $= 0$

overall maximum red degree = 2

Grids have twin-width at most 4



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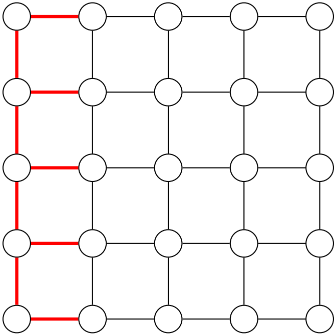

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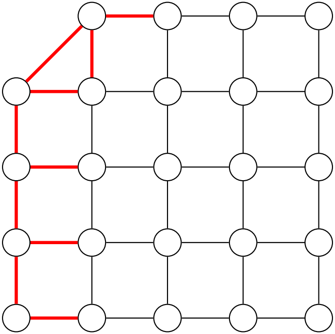

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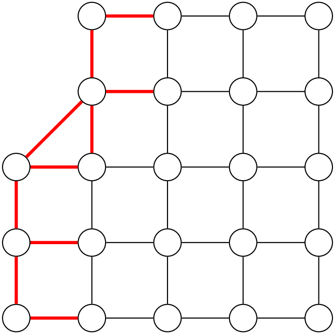

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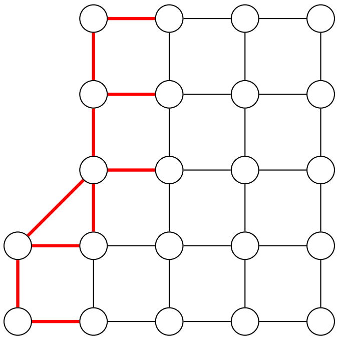

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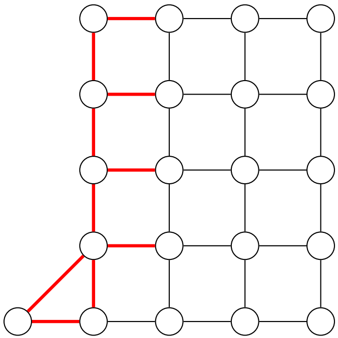

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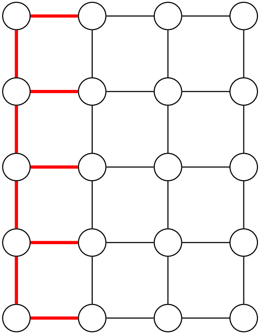

Theorem (B., Geniet, Kim, Thomassé, Watrigant '20 & '21)

The following classes have bounded twin-width, and $O(1)$-sequences can be computed in polynomial time.

- *Bounded rank-width or clique-width graphs,*
- *every hereditary proper subclass of permutation graphs,*
- *posets of bounded antichain size,*
- *unit interval graphs,*
- *K_t-minor free graphs,*
- *map graphs,*
- *subgraphs of d-dimensional grids,*
- *K_t-free unit d-dimensional ball graphs,*
- *$\Omega(\log n)$-subdivisions of all the n-vertex graphs,*
- *strong products of two bounded twin-width classes, one with bounded degree,*
- *(given) first-order transductions of the above.*

Twin-width bounds of graph classes

Graphs on surfaces

Theorem (Hliněný & Jedelský '22; Král & Lamaison '22)

The class of planar graphs has twin-width at most 8, and at least 7.

A matching upper bound of 7 is being written up by H & J.

Theorem (Hliněný & Jedelský '22)

The class of 1-planar graphs has twin-width at most 16.

Theorem (Hliněný & Jedelský '22)

The class of map graphs has twin-width at most 38.

Theorem (Král, Pekárková & Štorgel '23)

The class of graphs of genus at most g has twin-width $\Theta(\sqrt{g})$.

Theorem (B., Kim, Thomassé & Watrigant; B. & Déprés '22)

The class of K_t-minor-free graphs has twin-width $2^{2^{2^{O(t)}}}$, and $2^{\Omega(t)}$.

Subdivisions

Theorem (Bergé, B. & Déprés '21)

For every graph G, any $(\geqslant 2\log n)$-subdivision of G has twin-width at most 4.

Theorem (Ahn, Chakraborti, Hendrey & Oum '23)

Full understanding of $(\geqslant 2)$-subdivisions of twin-width 0, 1, 2, or 3 as which minors the original graph excludes.

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As one consquence:

Theorem (Ahn, Chakraborti, Hendrey & Oum '23)

The class of grids has twin-width 4.
Already the 7×7 grid has twin-width 4.

Random graphs and graphs of largest twin-width

Theorem (Ahn, Chakraborti, Hendrey, Kim & Oum '22)

Almost surely $tww(G(n, \frac{1}{2})) = \frac{n}{2} - \frac{\sqrt{3n \log n}}{2} \pm o(\sqrt{n \log n})$.

Theorem (Ahn, Chakraborti, Hendrey, Kim & Oum '22)

For any $p \in [\frac{726 \ln n}{n}, \frac{1}{2}]$, $tww(G(n, p)) = \Theta(n\sqrt{p})$.

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Let $P(q)$ be the Paley graphs on q vertices.

Theorem (Ahn, Hendrey, Kim & Oum '21)

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$tww(P(q)) = \frac{q-1}{2}$.

Open Question

Is there an n-vertex graph of twin-width at least $\frac{n}{2}$*?*

Are these classes of bounded twin-width?

Polynomial expansion $\equiv$ truly sublinear separators:
$\exists \varepsilon > 0$ s.t. every n-vertex graph in the (closure of the) class has balanced separators of size $O(n^{1-\varepsilon})$.

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Is every class with polynomial expansion of bounded twin-width?

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Is every $K_{t,t}$-free H-induced-minor-free class of bounded twin-width?

It is known that every $K_{t,t}$-free H-induced-minor-free class has polynomial expansion, and the converse does not hold.

Bounded-degree graphs

Theorem (B., Geniet, Kim, Thomassé & Watrigant '20; B., Nesetril, Ossona de Mendez, Siebertz & Thomassé '21)

Every class of bounded twin-width has growth $n!2^{O(n)}$ as a labeled class; has growth $2^{O(n)}$ as an unlabeled class.

$\rightarrow$ the class of subcubic graphs has unbounded twin-width

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$\to$ the class of subcubic graphs has unbounded twin-width

Open Question

Find an explicit construction of bounded-degree graphs with increasing twin-width.

First-order graph model checking

Given a first-order sentence $\varphi \in FO[\{E_{(2)}\}]$, e.g.,

$$\exists x_1 \cdots \exists x_k \forall x (x = x_1 \vee \cdots \vee x = x_k) \vee (E(x, x_1) \vee \cdots \vee E(x, x_k))$$

$G \models \varphi$ is true if φ holds in G.

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Theorem (B., Kim, Thomassé & Watrigant '20)

Given a graph G, a d-sequence of G, and a first-order sentence φ, $G \models \varphi$ can be decided in time $f(d, \varphi)|V(G)|$.

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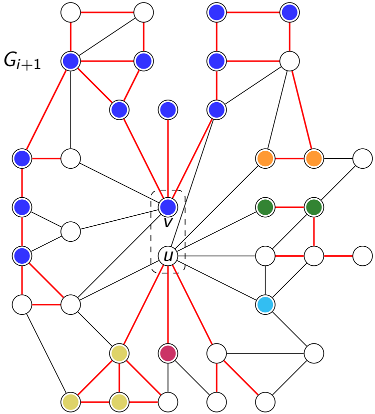
Theorem (B., Kim, Thomassé & Watrigant '20)

For every FO interpretation $\varphi(x, y)$ of a monadic lift of a class $\mathcal{C}$ of bounded twin-width, $\varphi(\mathcal{C})$ has bounded twin-width.

Special cases with better running times

special cases like k-INDEPENDENT SET in time $d^{O(k)}|V(G)|$



How hard is computing twin-width?

Theorem (Bergé, B. & Déprés '21)

It is NP-complete to decide if the twin-width is at most 4.

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Twin-width at most 1 is in polynomial-time; in linear-time.

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Open Question

What about twin-width at most 2? at most 3?

Theorem (Bergougnoux, Gajarský, Guspiel, Hlinený, Pokrývka & Sokolowski '23)

$K_{t,t}$-free classes of twin-width at most 2 have bounded treewidth.

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Open Question

Is there an algorithm that given a graph G and an integer d, either

- *provides an $f(d)$-sequence of G, or*
- *correctly report that $tww(G) > d$*

in time $|V(G)|^{g(d)}$?

Twin-width parameterized by larger parameters

Theorem (Balabán, Ganian & Rocton '24)

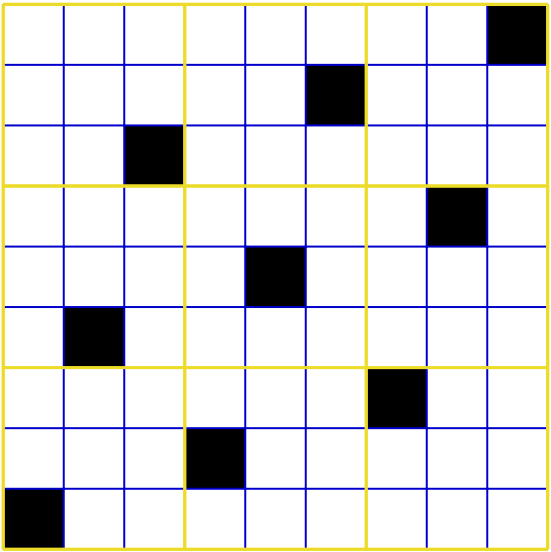
FPT algorithm for twin-width w.r.t. feedback edge number, FPT 2-approximation for twin-width w.r.t. vertex integrity.

Open Question

Is there an FPT (XP) $f(OPT)$-approximation algorithm for twin-width parameterized by pathwidth, treewidth, rank-width?

Unconditional parameterized algorithms

k-grid permutation

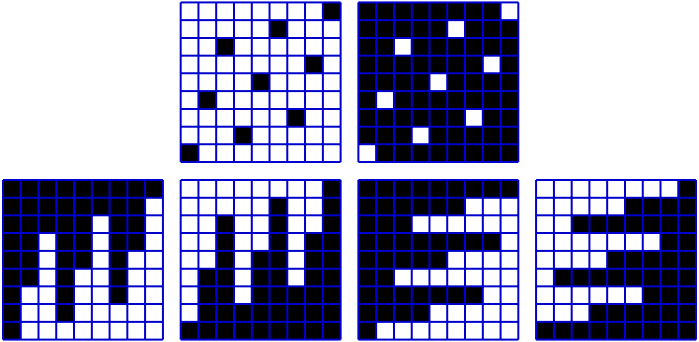


Here with $k = 3$, it has every 3-permutation as subpermutation

The 6 minimal families of unbounded twin-width

Theorem (B., Giocanti, Ossona de Mendez, Simon, Thomassé & Toruńczyk '21)

$\exists f, g$ s.t., given an $n \times n$ adjacency matrix $Adj_{\prec}(G)$, in time $g(k)n^{O(1)}$ one can find an $f(k)$-sequence of $(G, \prec)$ or one of the six following encodings of a k-grid permutation submatrix:



Semi-induced matching/antimatching, and 4 half-graphs or ladders

Ordered binary structures

Theorem (B., Giocanti, Ossona de Mendez, Simon, Thomassé & Toruńczyk '21)

Let $\mathscr{C}$ be a hereditary class of ordered graphs. There is an FPT $f(OPT)$-approximation for twin-width on $\mathscr{C}$, and the following are equivalent.

(1) $\mathscr{C}$ *has bounded twin-width.*

(2) $\mathscr{C}$ *is dependent.*

(3) $\mathscr{C}$ *contains $2^{O(n)}$ ordered n-vertex graphs.*

(4) $\mathscr{C}$ *contains less than $\sum_{k=0}^{\lfloor n/2 \rfloor} \binom{n}{2k} k!$ ordered n-vertex graphs, for some n.*

(5) $\mathscr{C}$ *does not include one of 25 hereditary ordered graph classes with unbounded twin-width.*

(6) *FO-model checking is fixed-parameter tractable on $\mathscr{C}$ (assuming $FPT \neq AW[*]$).*

Twin-width win-win

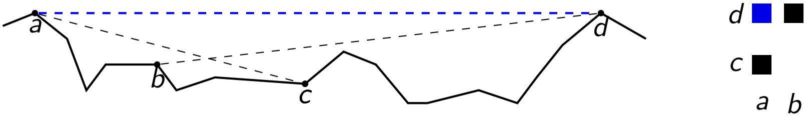
Goal: compute FO-definable parameter p in FPT time in $\mathcal{C}$.

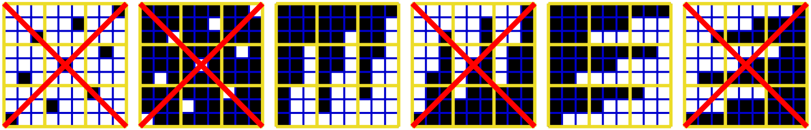
Show that $\exists f$ non-decreasing, such that $\forall G \in \mathcal{C}$ an $f(p(G))$-sequence of G can be computed in FPT time

- Width $> f(k)$: report $p(G) > k$
- Width $\leqslant f(k)$: use FO model checking algorithm

Visibility graphs of 1.5D terrains

Order along x-coordinates





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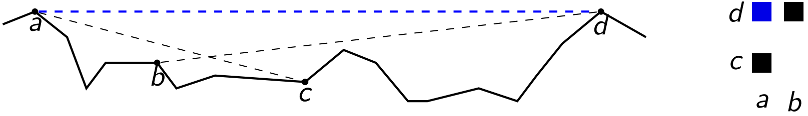

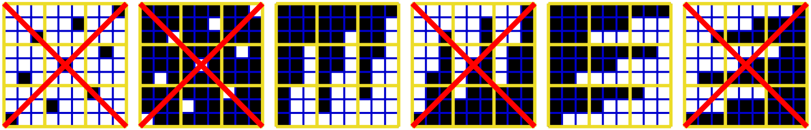

k-Biclique and k-Ladder are FPT in this class

Matrix Multiplication

Theorem (B., Giocanti, Ossona de Mendez & Thomassé '23)

Given two $n \times n$ $\mathbb{F}_q$-matrices A, B of twin-width at most d, one can compute AB in time $O_{d,q}(n^2 \log n)$.

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Consequence of:

- $O_{d,q}(n^2 \log n)$ time $f(\mathrm{OPT})$-approximation for twin-width of ordered binary structures,
- FO+MOD interpretations preserve bounded twin-width,
- squaring is an FO+MOD interpretation, and

Theorem (Gajarský, Pilipczuk, Przybyszewski & Toruńczyk '22)

Given an FO(+MOD) interpretation $\varphi(x_1, \ldots, x_k)$ and a binary structure G with a d-sequence, a data structure can be computed in time $O_{d,\varphi}(n^{1+\varepsilon})$ that answers queries "does $\varphi(v_1, \ldots, v_k)$ hold in G?" in time $O_{d,\varphi}(1/\varepsilon)$.

Matrix Multiplication

Theorem (B., Giocanti, Ossona de Mendez & Thomassé '23)

Given two $n \times n$ $\mathbb{F}_q$-matrices A, B of twin-width at most d, one can compute AB in time $O_{d,q}(n^2 \log n)$.

Consequence of:

- $O_{d,q}(n^2 \log n)$ time $f(\mathrm{OPT})$-approximation for twin-width of ordered binary structures,
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- squaring is an FO+MOD interpretation, and

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$\varphi_1(r, c)$ holds in $\widehat{M}$ $\equiv$ there is a 1-entry at row r, column c of M^2.

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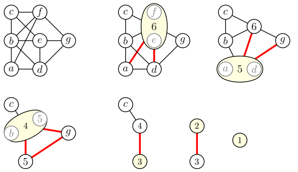
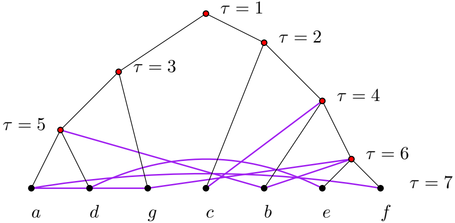
- $O_{d,q}(n^2 \log n)$ time $f(\text{OPT})$-approximation for twin-width of ordered binary structures,
- a d-sequence can be turned into a twin-decomposition of width d in time $O_d(n^2)$, and
- $q^{O(d)}n$-time algorithm for the twin-decomposition of M^2.





Approximation algorithms

Balanced contraction sequences

Theorem (B., Geniet, Kim, Thomassé & Watrigant '21)

For every d, there is a D such that every n-vertex graph with twin-width at most d iteratively admits $\frac{n}{D}$ disjoint pairs that can be contracted in a D-sequence.

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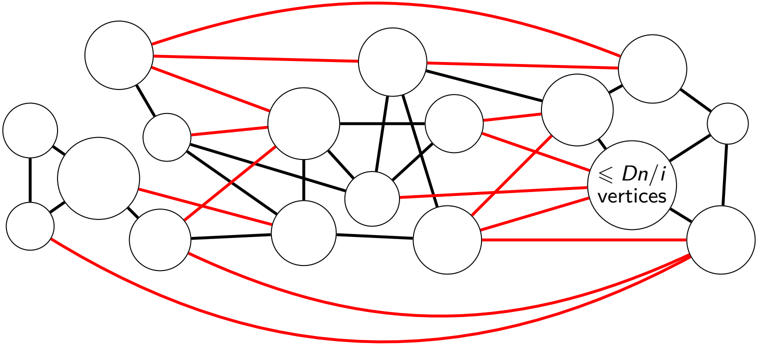
Consequence: We can turn a d-sequence into a *balanced* D-sequence $\mathcal{S}$, i.e., such that $\forall \mathcal{P}_i \in \mathcal{S}, \forall P \in \mathcal{P}_i$, $|P| \leqslant D\frac{n}{i}$



Approximating Max Independent Set

In general graphs: an $n^{1-\varepsilon}$-approximation or $r(n)$-approximation in time $\exp(\frac{n^{1-\varepsilon}}{r(n)^{1+\varepsilon}})$ are unlikely

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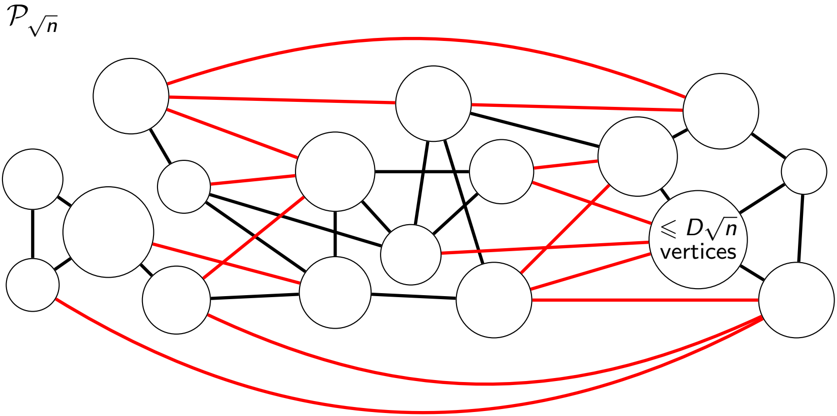

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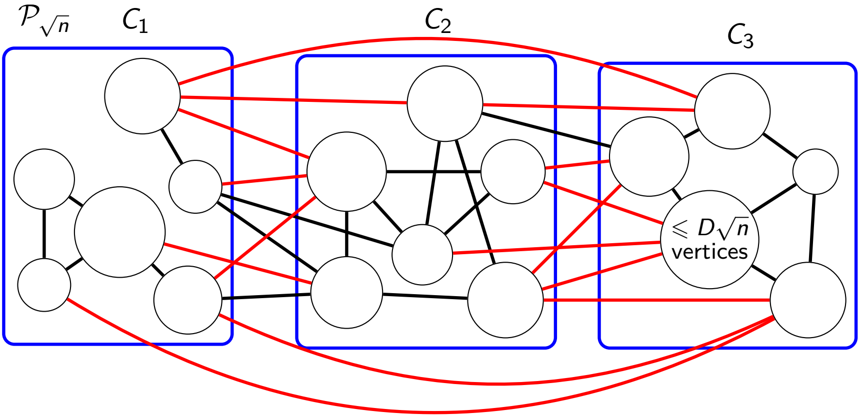

$D+1$-color the red graph of $G/\mathcal{P}_{\sqrt{n}}$ in polynomial time

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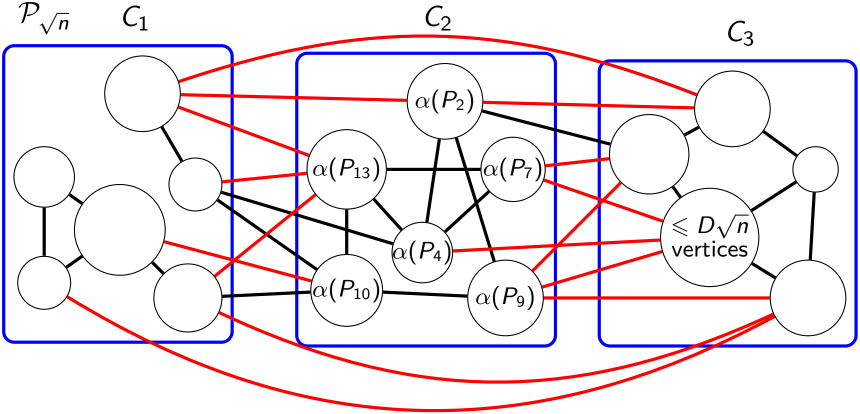

Solve MIS in $G[P_j]$ for every $P_j \in \mathcal{P}_{\sqrt{n}}$ in $2^{O(D\sqrt{n})}$ time

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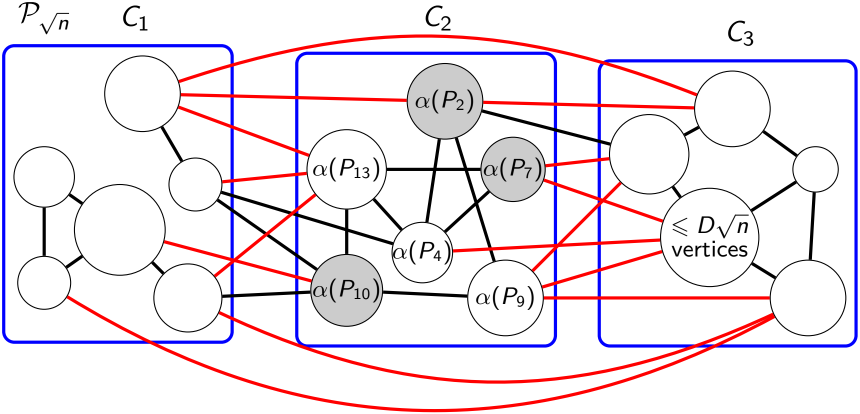

Solve weighted MIS in $G/\mathcal{P}_{\sqrt{n}}[C_i]$, $\forall i \in [D+1]$ in $2^{O(\sqrt{n})}$ time

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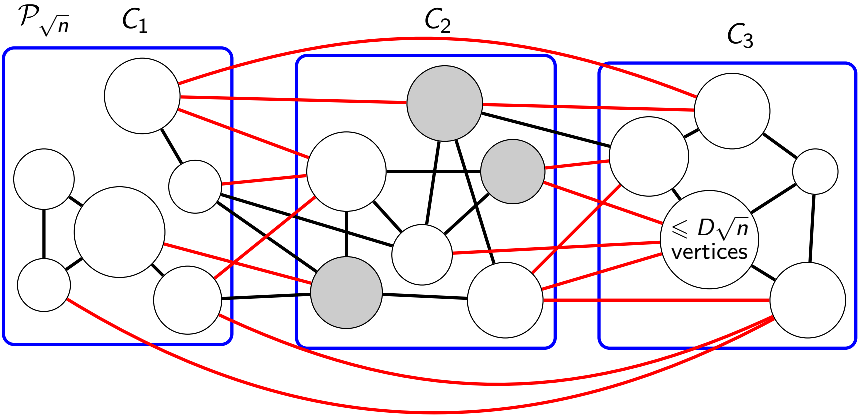

A heaviest such solution is a $(D+1)$-approximation

Approximating MIS given a d-sequence

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MIS *can be* $O_d(1)$*-approximated in time* $2^{O_d(\sqrt{n})}$.

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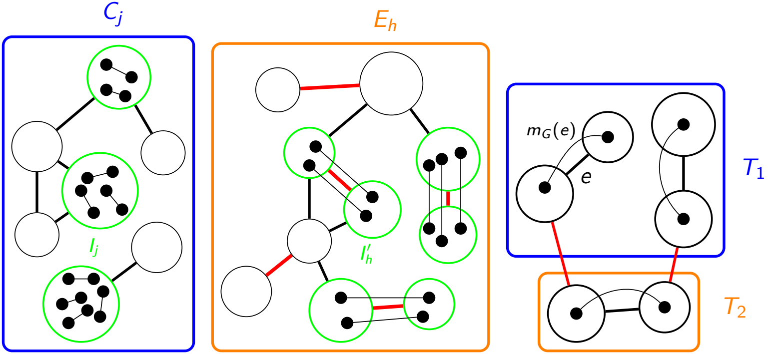
Setting $q := \log \frac{\varepsilon \log n}{O_d(1)}$

Theorem (Bergé, B., Déprés & Watrigant '23)

MIS *can be* n^{ε}*-approximated in polynomial time.*

Coloring, Max Induced Matching

Similar results for these problems



Better approximation algorithms

Open Question

Does MIS *admit a PTAS on graphs of bounded twin-width given with* $O(1)$*-sequences?*

Any constant factor approximation would imply a PTAS for MIS.

Open Question

Is there a constant factor approximation for Coloring *on graphs of bounded twin-width given with* $O(1)$*-sequences?*

This constant has to be at least $4/3$.

Open questions

FPT/XP approximation of twin-width (parameterized by larger parameters)

Find an explicit family of bounded-degree graphs with unbounded twin-width (counting-free argument)

Is every $K_{t,t}$-free induced-minor-free class of bounded twin-width?

Better than n^{ε}-approximation for MIS given $O(1)$-sequences?

More unexpected uses of the FO model checking algorithm on bounded twin-width (like [HJLMPSS '23] for Directed Multicut with three terminal pairs parameterized by cutset size)

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Thank you for your attention!