Superpolynomial Approximation

Édouard Bonnet

June 19th 2026, Vangelis' retirement

Coping with NP-hardness

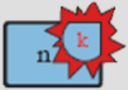
Approximation

D. Johnson

Approximate solution in polynomial time

Ideally: $(1+\varepsilon)$-approximation $\forall \varepsilon > 0$

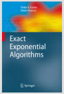
Parameterized Complexity



Solution in time $f(k)n^c$

minimize f and/or c

Exact Exponential Algorithms



F. Fomin & D. Kratsch

Solution in time $O^*(\lambda^n)$

minimize λ

Vangelis' early vision: these areas do not have to be *and* should not be studied in isolation.

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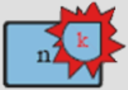

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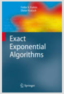

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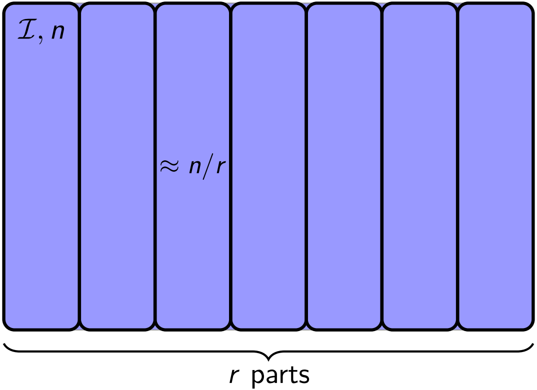
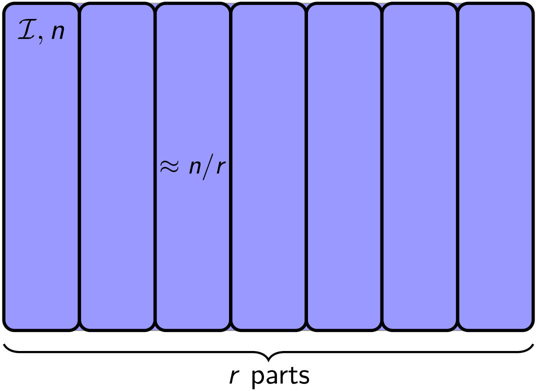
Scheme for monotone maximization subset problems



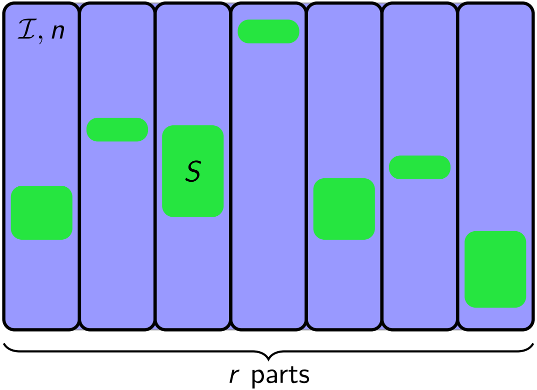
Split the instance into r parts of size n/r.

Scheme for monotone maximization subset problems



For each of the $r2^{n/r}$ part subsets, check feasibility.

Scheme for monotone maximization subset problems



Fix an optimal solution (green). By pigeon-hole, $|S| \geqslant \frac{\text{OPT}}{r}$.

Example of MAX INDEPENDENT SET

Exponential-Time Hypothesis (ETH):
$\exists\lambda > 1$ such that 3-SAT cannot be solved in time $O^*(\lambda^n)$

- Unless the ETH fails, no $2^{o(n)}$-time algorithm.
- Unless P $=$ NP, no polytime $n^{1-\varepsilon}$-approx algorithm $\forall\varepsilon > 0$.

Previous slide: r-approximation in time $2^{O(n/r)}$
(essentially optimal under the ETH)

Let's talk about

"***On Subexponential*** *and FPT-time* ***Inapproximability***"
with Bruno, Eunjung, Vangelis

APETH

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To propagate hardness, we crucially need linear reductions

$\rightarrow$ Sparsification

Sparsification

Theorem (Impagliazzo–Paturi–Zane '01)

ETH implies its strengthening to instances with $O(n)$ clauses.

Turing reduction creating $2^{o(n)}$ sparse formulas

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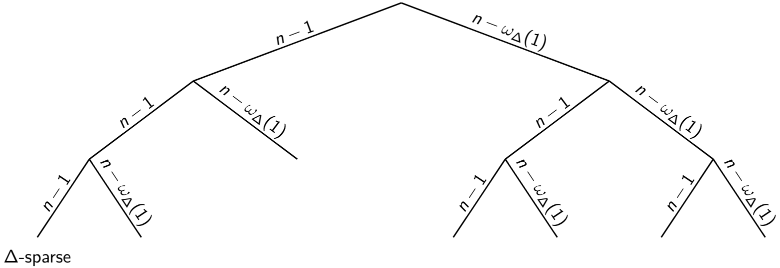
For our purpose, a $(n-1, n-\omega_\Delta(1))$-branching is sufficient



Sparsification: Max Independent Set

Pick a vertex v of degree Δ

Add v to solution:
$n \leftarrow n - \Delta - 1$

Exclude v from the solution:
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The ratio is not altered as the branching is exhaustive

Consequence of these approximation preserving sparsifier

+ classic linear reductions on bounded-degree instances

Theorem (B., Escoffier, Kim, Paschos '13/'15)

MAX INDEPENDENT SET, MIN VERTEX COVER, MIN SET COVER, MIN DOMINATING SET, *and their bounded-degree versions are all APETH-equivalent.*

Antibes, 2013: Between my first two conference talks

Détends-toi, mon garçon.

The importance of APETH

2017: GapETH is introduced; it is APETH($\textsc{Max 3-SAT}$)

Two related best papers at STOC

Guruswami–Lin–Ren–Sun–Wu '24

Bafna–Minzer–Vyas–Yun '25

ETH $\Leftrightarrow$ *GapETH* remains an important open question

Thank you all!

Thank you, Vangelis!