

Exploiting multiple resolutions to accelerate inverse problems in imaging

Elisa Riccietti
LIP-ENS Lyon

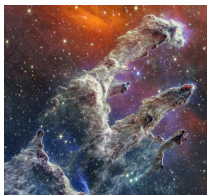
Joint work with Guillaume LAUGA (Univ. Cote d'Azur), Nelly
PUSTELNIK & Paulo GONCALVES (ENS Lyon)

GAMM ANLA 2025 - Bologna, 23 - 24 October 2025

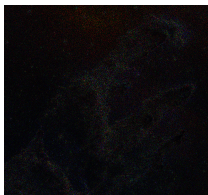


The context: image restoration problems

$$\hat{x} \in \arg \min_x \frac{1}{2} \|Ax - z\|_2^2 + \lambda \|Lx\|_*$$



$\bar{x}$



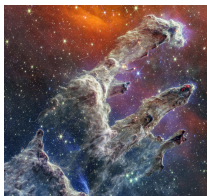
z



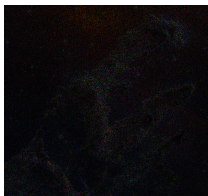
$\hat{x}$

The context: image restoration problems

$$\hat{x} \in \arg \min_x \frac{1}{2} \|Ax - z\|_2^2 + \lambda \|Lx\|_*$$



$\bar{x}$



z



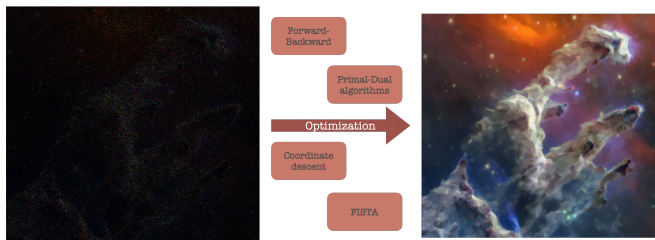
$\hat{x}$

More generally

$$\min_x f(x) + g(x)$$

- ▶ f differentiable with Lipschitz gradient
- ▶ g possibly non-smooth and often non-proximable

Focus on large-scale problems



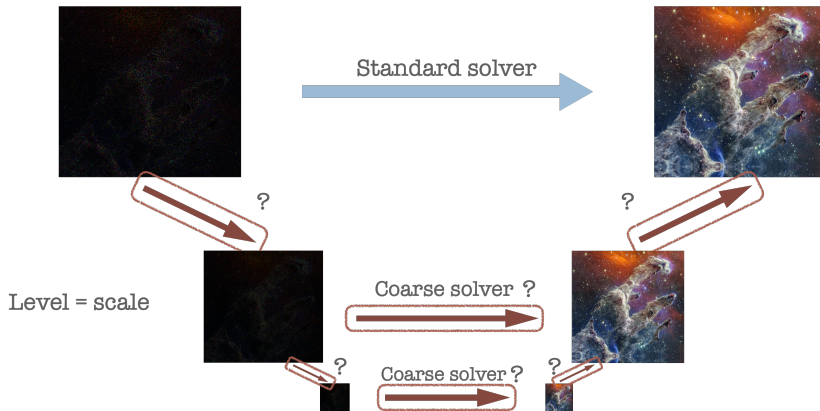
Prohibitive cost for large-scale problems: how to reduce this cost?

Existing acceleration strategies:

- ▶ **FISTA** [Beck & Teboulle, 2009] [Chambolle & Dossal, 2015],
- ▶ **Preconditioning** [Donatelli, 2019][Repetti et al., 2014],
- ▶ **Blocks methods** [Liu, 1996] [Chouzenoux et al., 2016] [Salzo, Villa 2022],

Alternative: Exploit the problem structure with a **multiresolution strategy**

The multilevel paradigm



The multilevel literature

CONTEXT	MULTILEVEL STRATEGY	CONVERGENCE GUARANTEES	PRACTICAL PERFORMANCE	REFERENCES
Linear PDEs	✓	✓	✓	[Hackbush 1985], [McCormick 1987], [Briggs 2000]
Smooth non-linear PDEs	✓	✓	✓	[Nash 2000], [Gratton 2008], [Calandra 2020]
Smooth imaging optimization	✓	✓	✗	[Javaherian 2017], [Plier 2021], [Fung 2020]
Non-smooth optimization	✓	✗ No convergence of the iterates	✗	[Parpas 2016], [Parpas 2017], [Ang 2023]
Non-smooth & non-proximable	✗	✗	✗	

Our contribution

IML FISTA: inexact multilevel FISTA

- ▶ A **general multilevel algorithm** with state-of-the-art convergence guarantees for image restoration that handles state-of-the-art non-proximable a-priori (TV, NLTV)
- ▶ Adaptation of IML FISTA to **multiple image restoration contexts** with state-of-the art practical performance
- ▶ Adaptation of IML FISTA to **radio-interferometric imaging** with state-of-the art practical performance

Our context

The problem:

$$\min_x F(x) := f(x) + \varphi(Lx)$$

- ▶ $\varphi \circ L$ non-smooth
- ▶ $\varphi \circ L$ non-proximable

The method: inexact FISTA [Aujol, Dossal, 2015]

$$x_{k+1} \approx_{\epsilon_k} \text{prox}_{\tau\varphi \circ L}(y_k - \tau \nabla f(y_k))$$

$$y_{k+1} = x_{k+1} + \alpha_k(x_{k+1} - x_k)$$

where $\alpha_k = \frac{t_k - 1}{t_{k+1}}$ and $t_k = \left(\frac{k-1+a}{a}\right)^d$.

Contribution: update y_k through a multilevel step

Outline

The multilevel step

The ingredients of the multilevel scheme

- The transfer operators

- The coarse model

Numerical experiments

- Hyperspectral images

- Radio-interferometric imaging

Outline

The multilevel step

The ingredients of the multilevel scheme

- The transfer operators

- The coarse model

Numerical experiments

- Hyperspectral images

- Radio-interferometric imaging

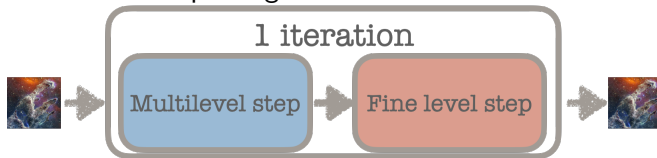
The multilevel step

- ▶ Classical proximal methods:

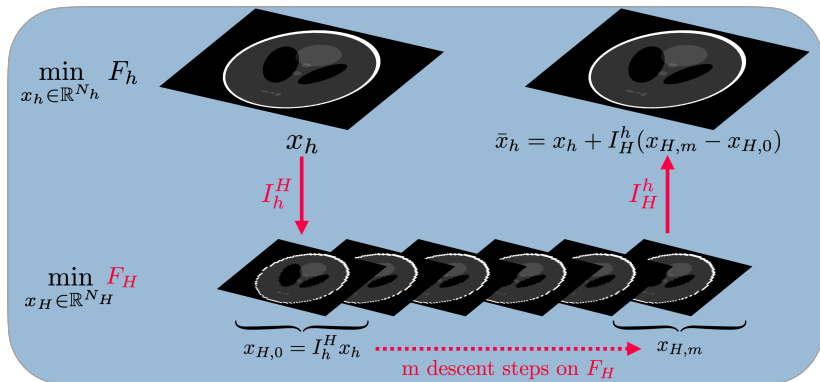
$$x_{k+1} = \text{prox}_{\tau\varphi \circ L}(x_k - \tau \nabla f(x_k))$$



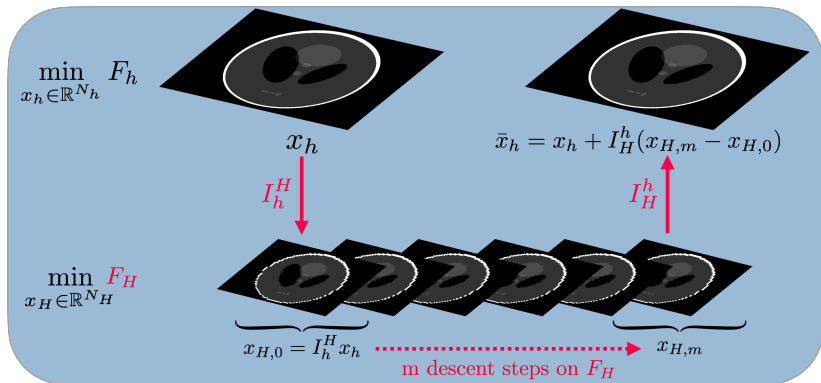
- ▶ The multilevel paradigm:



The multilevel step - two level case



The multilevel step - two level case



$I_h^H, I_H^h, F_H?$

Outline

The multilevel step

The ingredients of the multilevel scheme

- The transfer operators

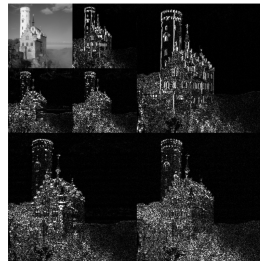
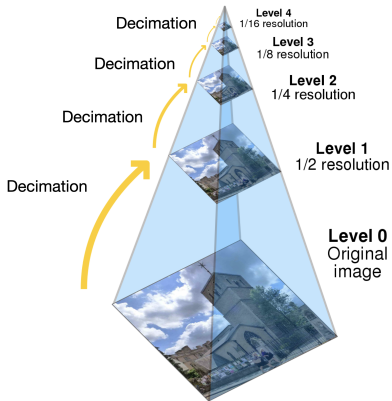
- The coarse model

Numerical experiments

- Hyperspectral images

- Radio-interferometric imaging

A hierarchy of images: I_h^H , I_H^h



Coarse model definition F_H

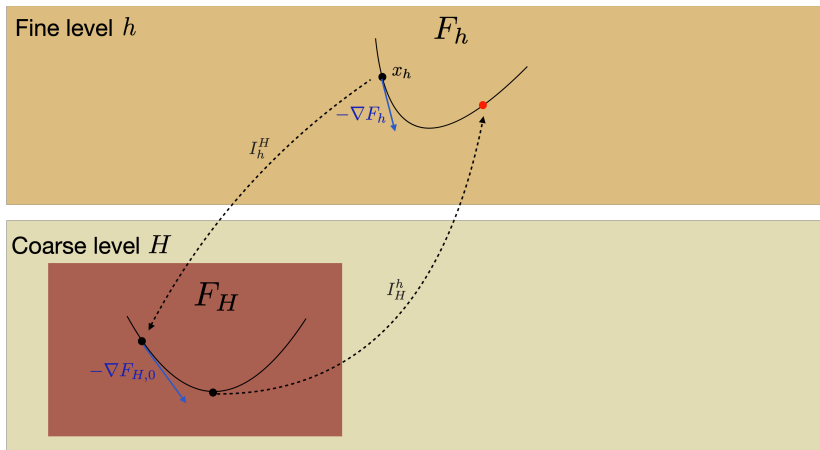
$$F_h(x) = F(x) = \frac{1}{2} \|Ax - z\|_2^2 + \varphi(Lx)$$
$$F_H(x) = \frac{1}{2} \|A_H x_H - z\|_2^2 + \varphi(L_H x_H)$$

Coarse model definition F_H

$$F_h(x) = F(x) = \frac{1}{2} \|Ax - z\|_2^2 + \varphi(Lx)$$
$$F_H(x) = \frac{1}{2} \|A_H x_H - z\|_2^2 + \varphi(L_H x_H)$$

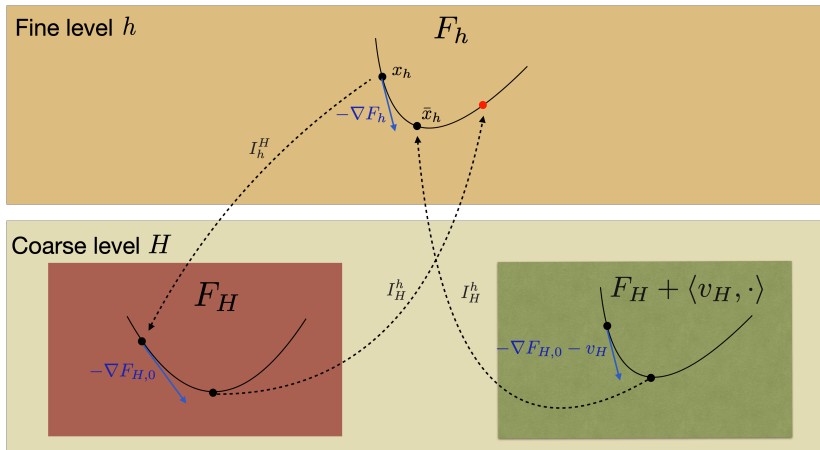
Is this model useful in minimizing F ?

Design of F_H in smooth context: First order coherence



$$v_H = v_H(\nabla F_h)$$

Design of F_H in smooth context: First order coherence



$$v_H = v_H(\nabla F_h)$$

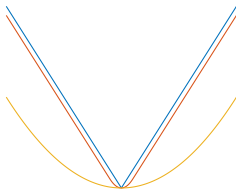
Coarse model for the non-smooth case: smoothing

Nonsmooth case $\rightarrow$ smoothing!

$$F_H \rightarrow {}^\gamma F_H$$

The Moreau envelope: ${}^\gamma F_H = \inf_{y \in \mathcal{H}} F_H(y) + \frac{1}{2\gamma} \|\cdot - y\|^2$

Illustration: Moreau envelope of l_1 -norm for $\gamma = 0.1$ and $\gamma = 1$



Non-smooth coarse model:

$$\tilde{F}_H = {}^\gamma F_H + \langle v_H({}^\gamma F_H), \cdot \rangle$$

IML FISTA

- The steps:

$$x_{k+1} \approx_{\epsilon_{h,k}} \text{prox}_{\tau\varphi \circ L}(\bar{y}_k - \tau \nabla f(\bar{y}_k))$$

$$y_{k+1} = x_{k+1} + \alpha_k(x_{k+1} - x_k)$$

- FISTA: $\bar{y}_k = y_k$
 - IML FISTA: $\bar{y}_k = ML(y_k)$ (i.e., $\min \tilde{F}_H$)
-
- IML FISTA recovers state-of-the-art **convergence guarantees**:

$$x_{k+1} \approx_{\epsilon_{h,k}} \text{prox}_{\tau\varphi \circ L}(\bar{y}_k - \tau \nabla f(\bar{y}_k) + e_{h,k})$$

Multilevel steps = bounded errors on the gradient

If $\sum_{k=1}^{\infty} k^2 \epsilon_{h,k} < \infty$, then

- The sequence $k^2(F_h(x_{h,k}) - F(x^*))_{k \in \mathbb{N}}$ belongs to $\ell_{\infty}(\mathbb{N})$

Outline

The multilevel step

The ingredients of the multilevel scheme

- The transfer operators

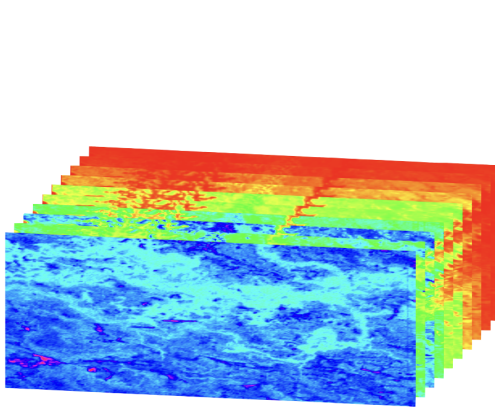
- The coarse model

Numerical experiments

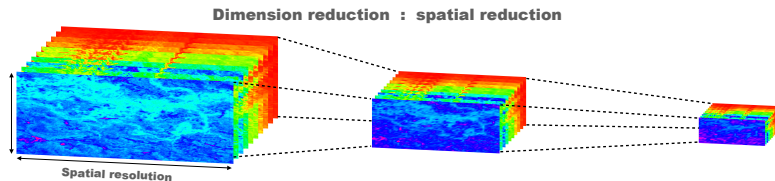
- Hyperspectral images

- Radio-interferometric imaging

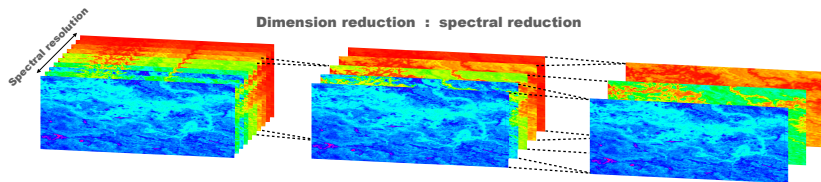
Hyperspectral images



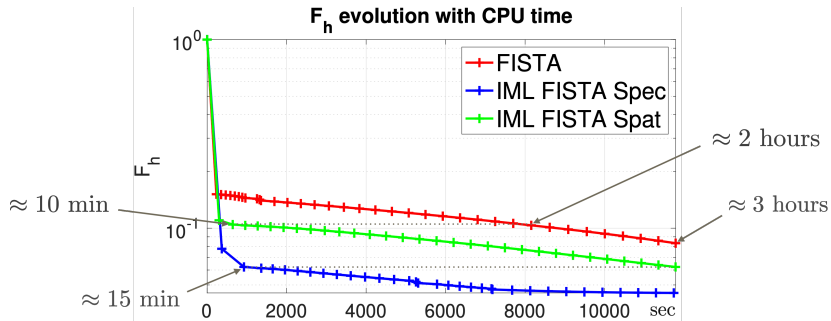
How to build the coarse approximations?



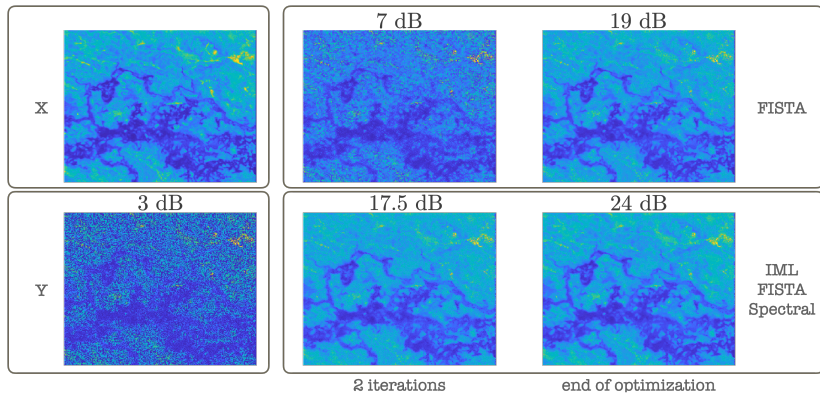
How to build the coarse approximations?



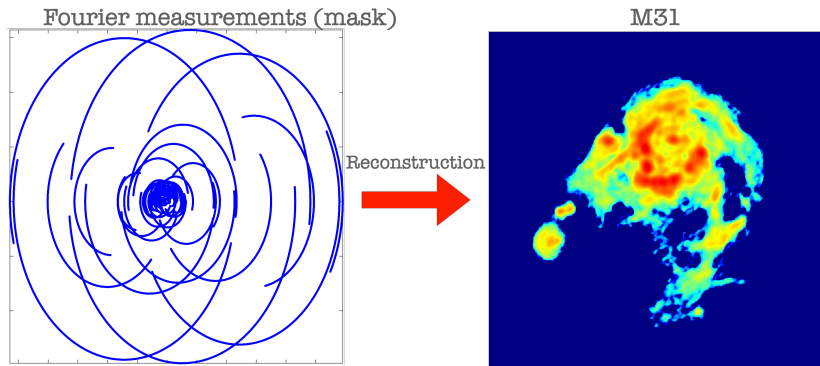
Objective function evolution



Reconstruction



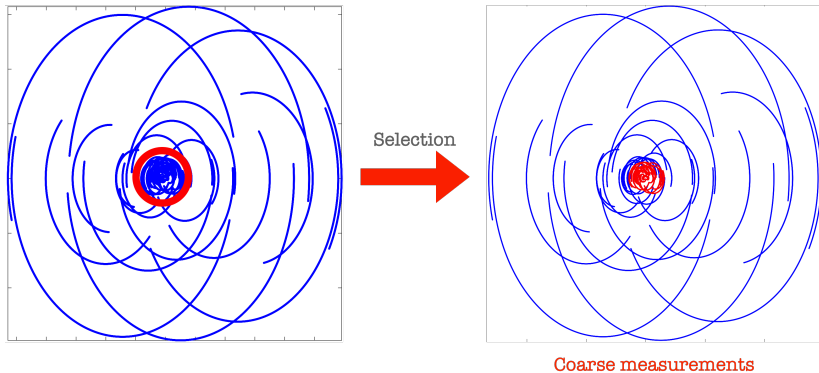
Radio-interferometric imaging



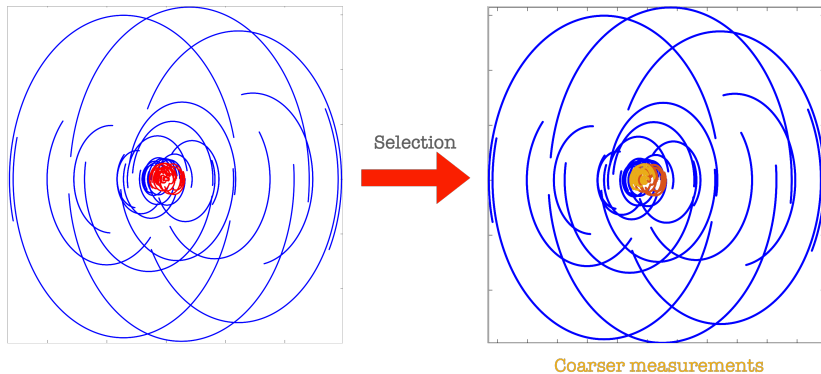
Dimension bottleneck: number of observations

Collaboration with Audrey Repetti and Yves Wiaux

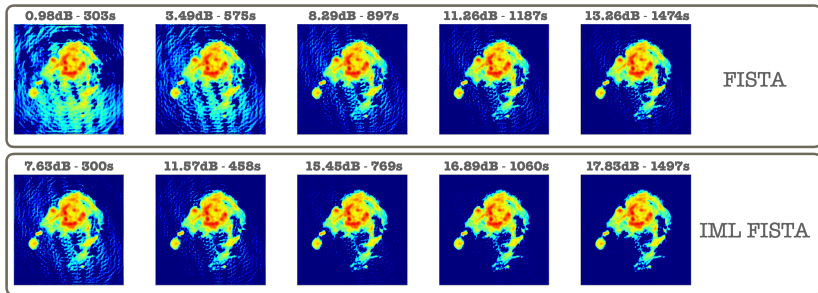
Radio-interferometric imaging



Radio-interferometric imaging

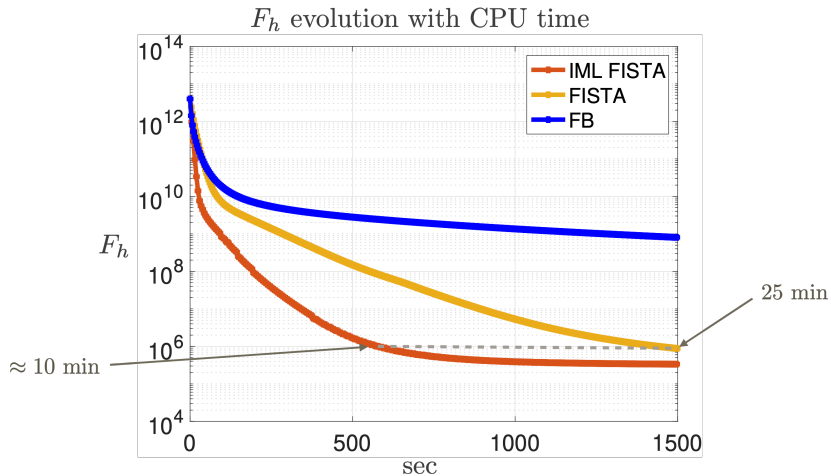


Radio-interferometric imaging



Reconstruction in log-scale of a region of the M31 galaxy

Radio-interferometric imaging



Conclusions and perspectives

Conclusions:

- ▶ We have proposed a **general multilevel framework for image restoration** with state-of-the-art convergence guarantees
- ▶ We have specialised the method in various contexts with really **good practical performance**

Perspectives:

- ▶ Extend the multilevel framework to **scientific machine learning**: multilevel Plug-and-Play methods, unrolled multilevel methods
- ▶ Extend the multilevel framework to **second-order methods**: proximal Gauss-Newton methods

Thank you for your attention!

A few references

- S. Gratton, A. Sartenaer, Ph. L. Toint. Recursive trust-region methods for multiscale nonlinear optimization, SIAM J. Opt., 19:414–444, 2008
- S. G. Nash. A multigrid approach to discretized optimization problems. Optimization Methods and Software, 14:99–116, 2000
- G. Lauga, E. Riccietti, N. Pustelnik, P. Gonçalves, IML FISTA: A Multilevel Framework for Inexact and Inertial Forward-Backward. Application to Image Restoration, SIAM Journal on Imaging Sciences 2024.
- A. Javaherian and S. Holman, A Multi-Grid Iterative Method for Photoacoustic Tomography, IEEE Transactions on Medical Imaging, 2017
- S. W. Fung and Z. Wendy, Multigrid Optimization for Large-Scale Ptychographic Phase Retrieval, SIAM Journal on Imaging Sciences, 13, 2020
- J. Plier, F. Savarino, M. Kočvara, and S. Petra, First-Order Geometric Multilevel Optimization for Discrete Tomography, in Scale Space and Variational Methods in Computer Vision, 2021
- V. Hovhannisyan, P. Parpas, and S. Zafeiriou, MAGMA: Multilevel Accelerated Gradient Mirror Descent Algorithm for Large-Scale Convex Composite Minimization, SIAM J. Imaging Sciences 2016
- P. Parpas, A Multilevel Proximal Gradient Algorithm for a Class of Composite Optimization Problems, SIAM J. Scient. Comp., 39 2017
- G. Lauga, E. Riccietti, N. Pustelnik, and P. Goncalves Multilevel FISTA for Image Restoration, IEEE ICASSP, 2023.