Geometric machines

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Outline

1. Origins

2. Model

3. Computational aspects

4. Singularities

Origins

- Geometrical mosaic, frieze
- Particle trajectories
- Cellular automata algorithms based on geometrical constructions

regular movements	$\rightsquigarrow$	lines
encouters	$\rightsquigarrow$	rules

Model

- Simple
- Minimal
- Deterministic

Meta-signal $\equiv$ state and velocity
Rule $\equiv$ Meta-signals $\rightarrow$ Meta-signals

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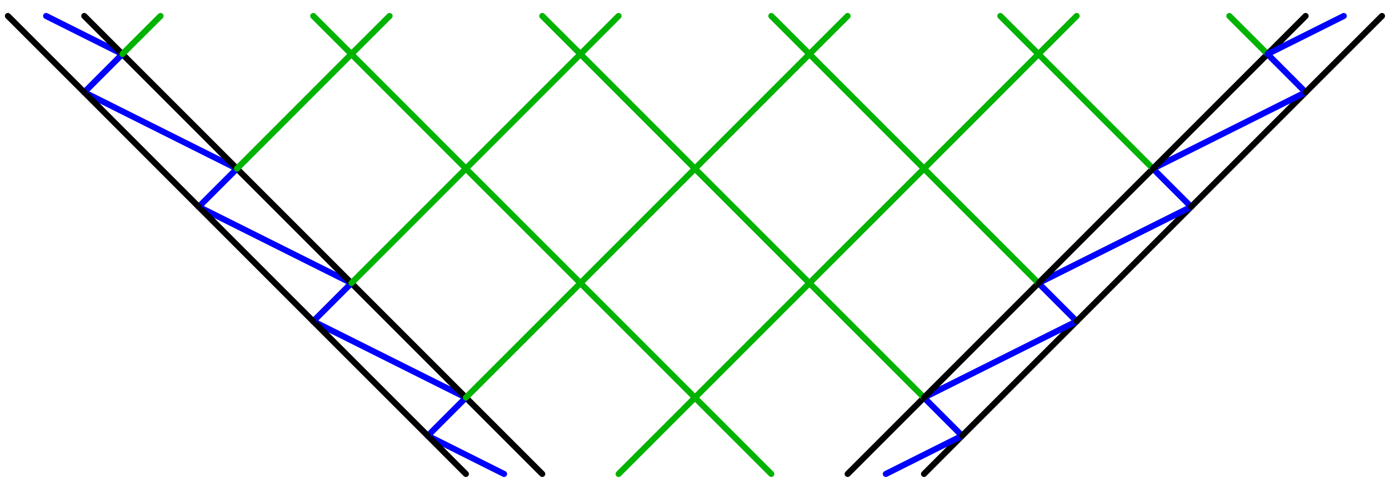
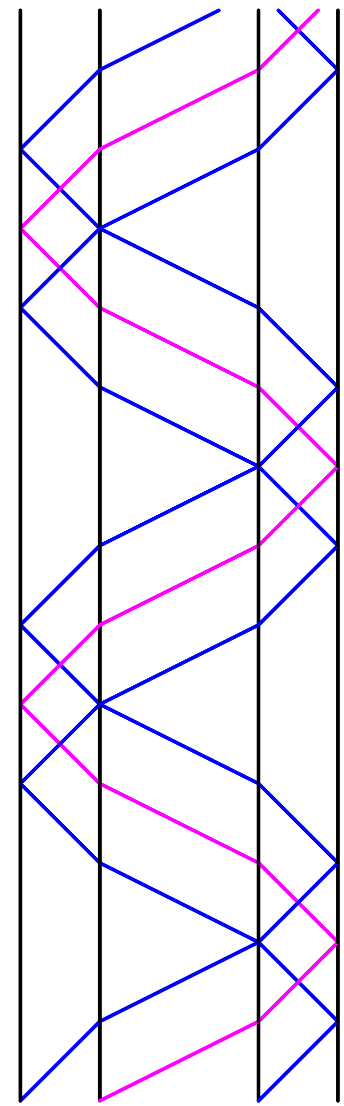
Space-time diagram & configuration





Properties

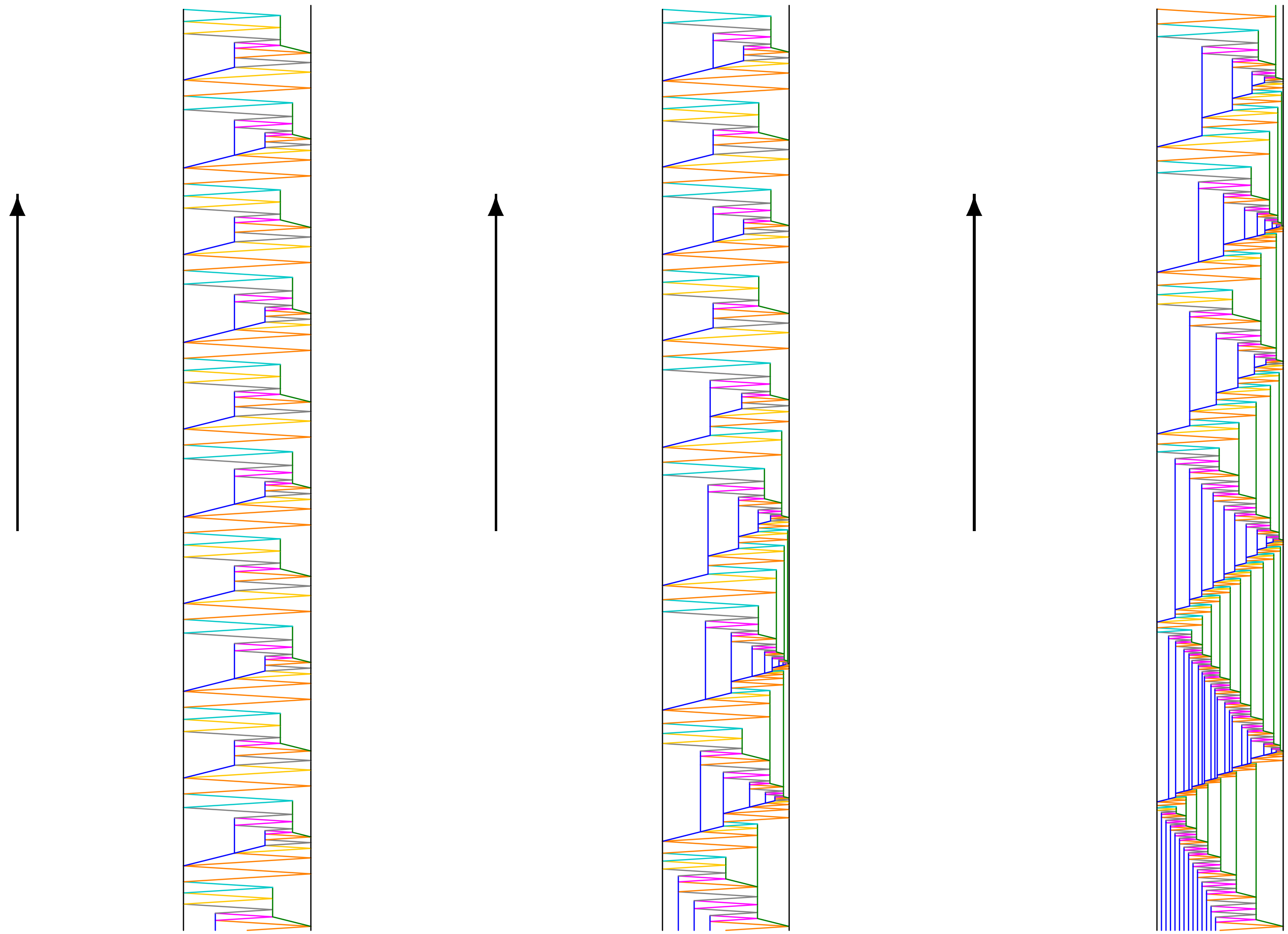
- local
- light cone
- uniform in space and time
- finite number of states
- continuous space
- continuous time

$\rightsquigarrow$ Zeno paradox?

Computation universal (Turing)

Simulate any 2-counter automaton

```
beg:  B++
      A--
      A != 0  beg1
      B != 0  imp
beg1: A--
      A != 0  beg
pair: B--
      A++
      B != 0  pair
      A != 0  beg
imp:  B--
      A++
      A++
      B != 0  imp1
      A != 0  beg
imp1: B--
      A++
      A++
      A++
      B != 0  imp1
      A != 0  beg
```



$A=1\quad B=0$ $\qquad A=3\quad B=0$ $\qquad A=13\quad B=0$

$\rightsquigarrow$ Highly unpredictable

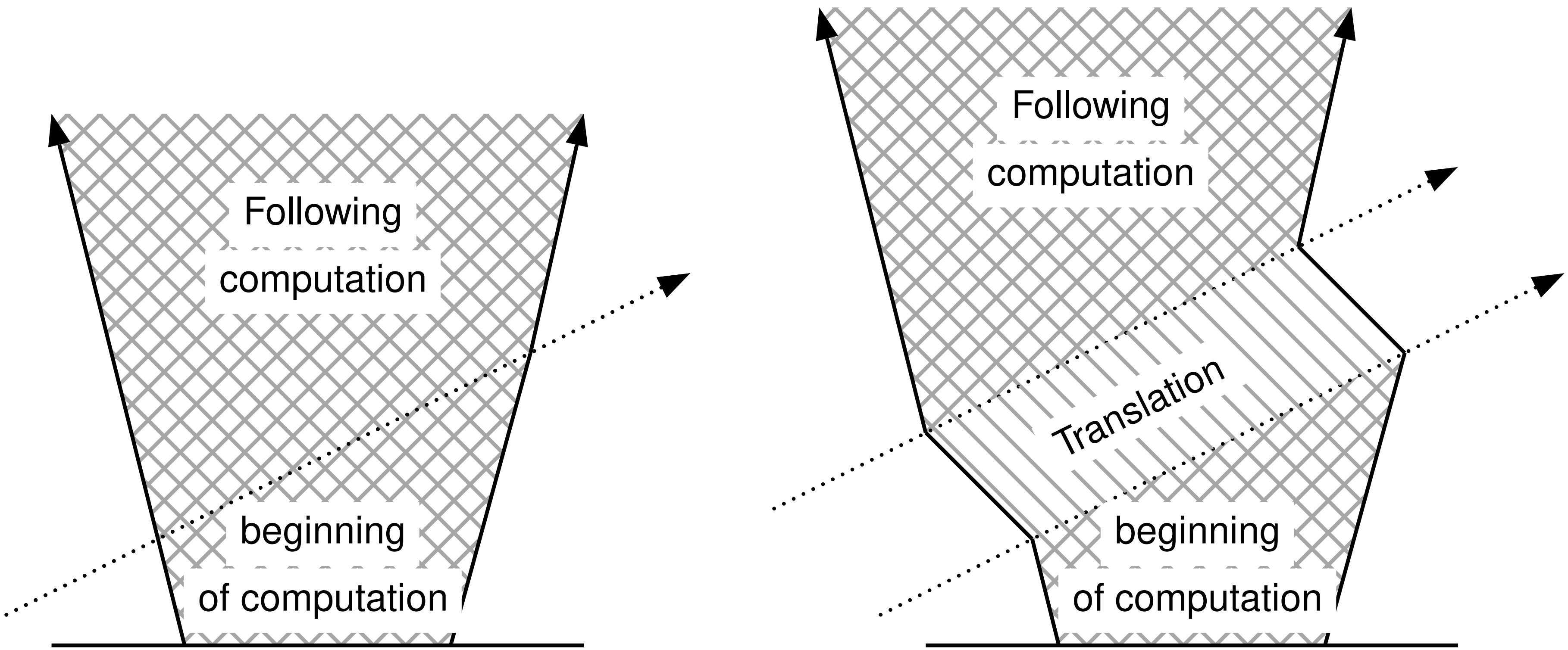
Geometrical constructions

- Static
  - Translations
  - Homotecy
- Dynamic
  - Freezing
  - Translations
  - Homotecy

$\rightsquigarrow$ Construction with these “operators”

Freezing the computation

Freeze then restore



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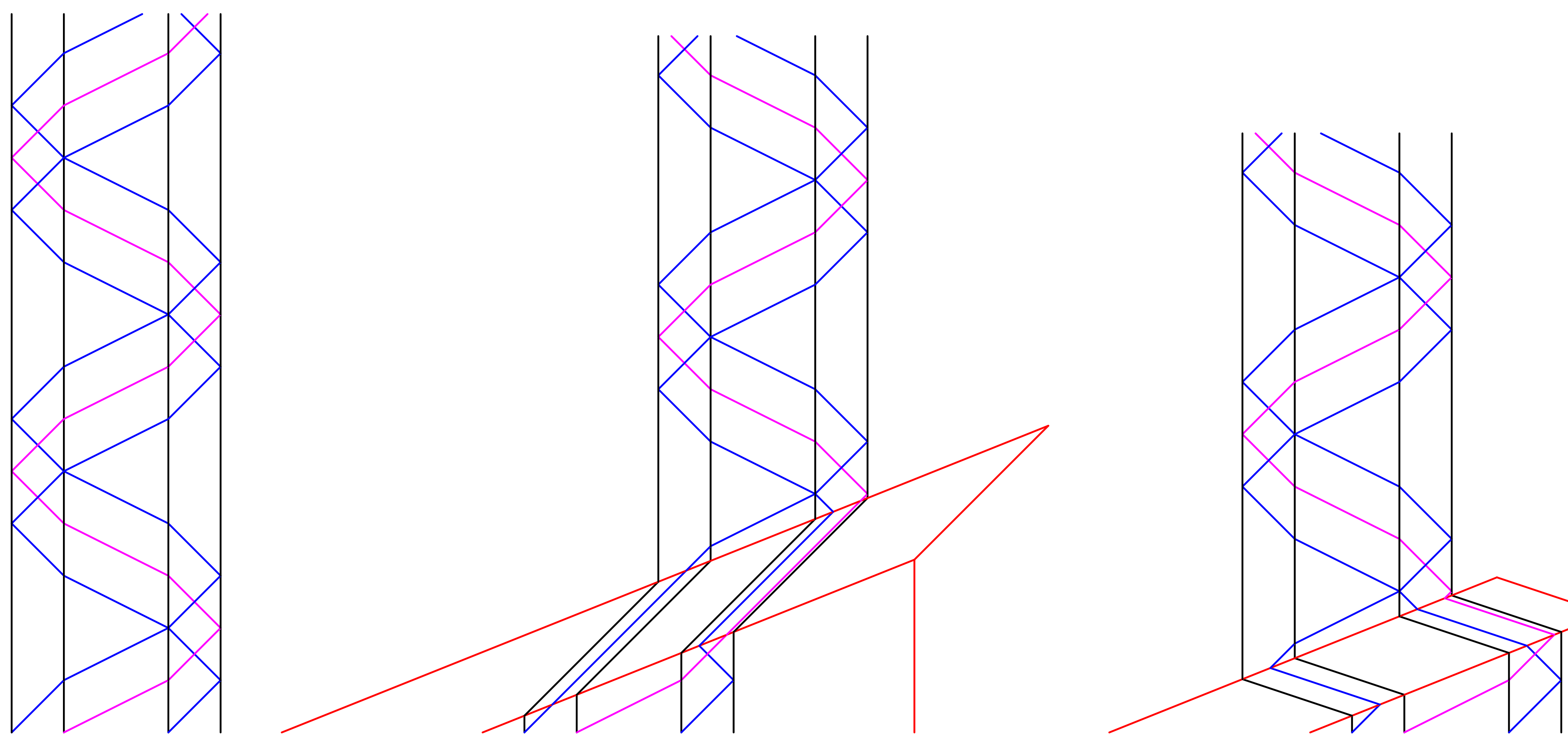
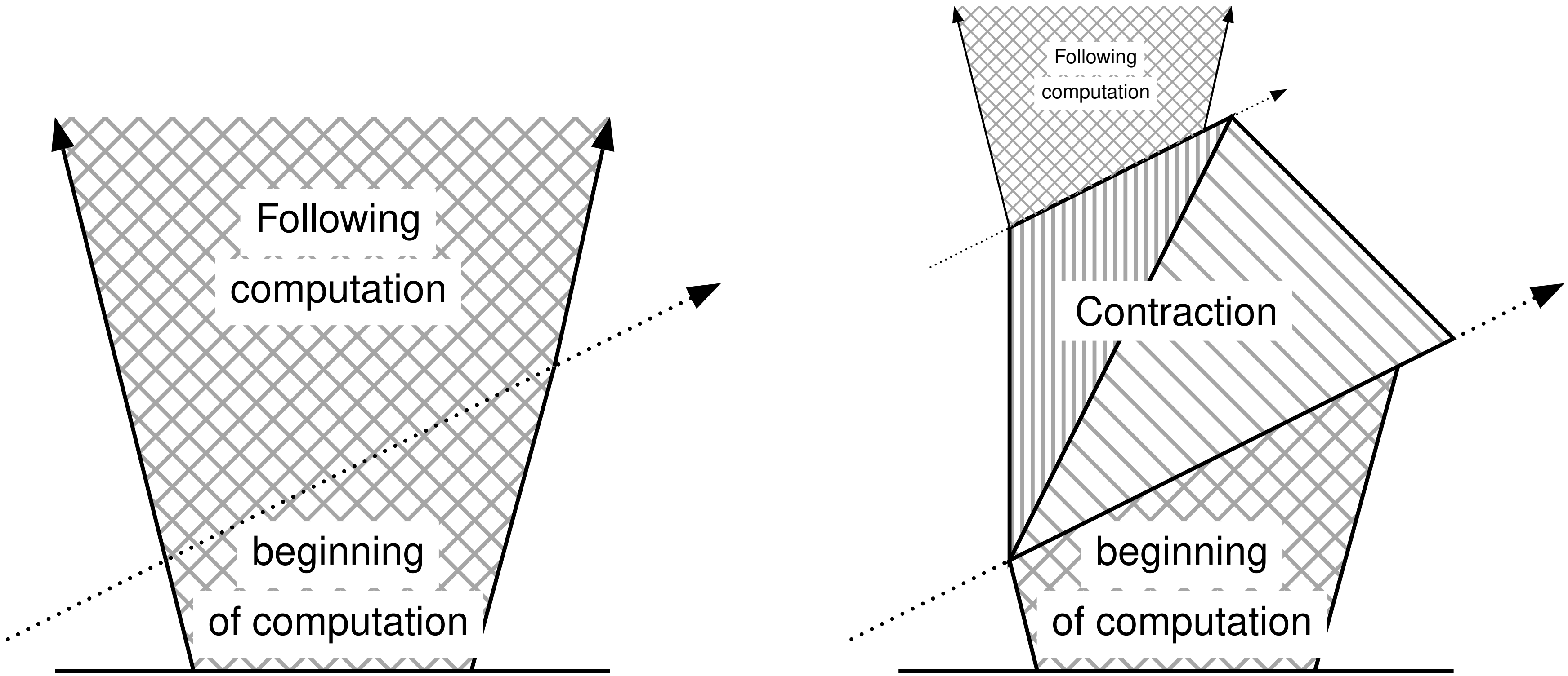
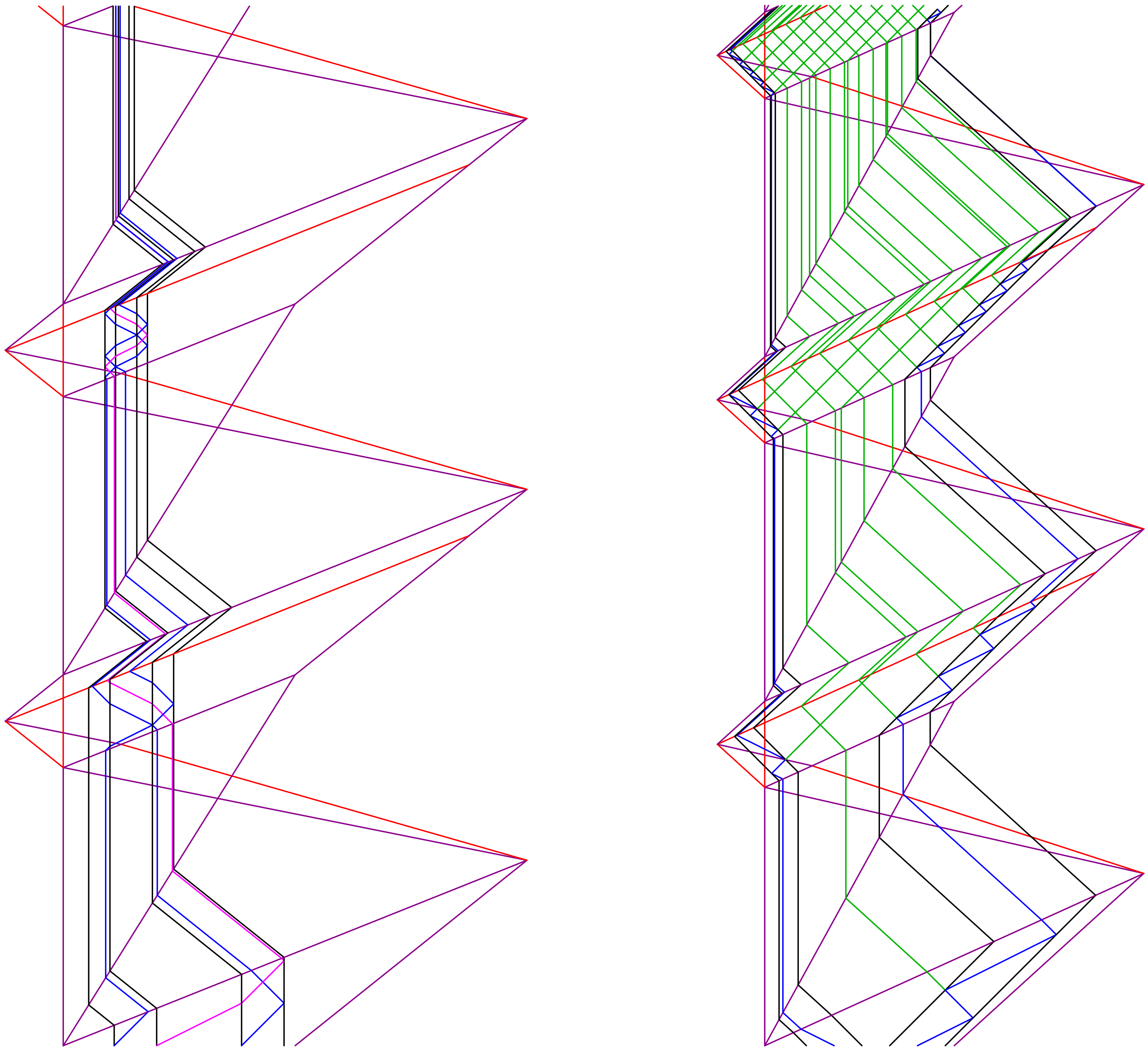


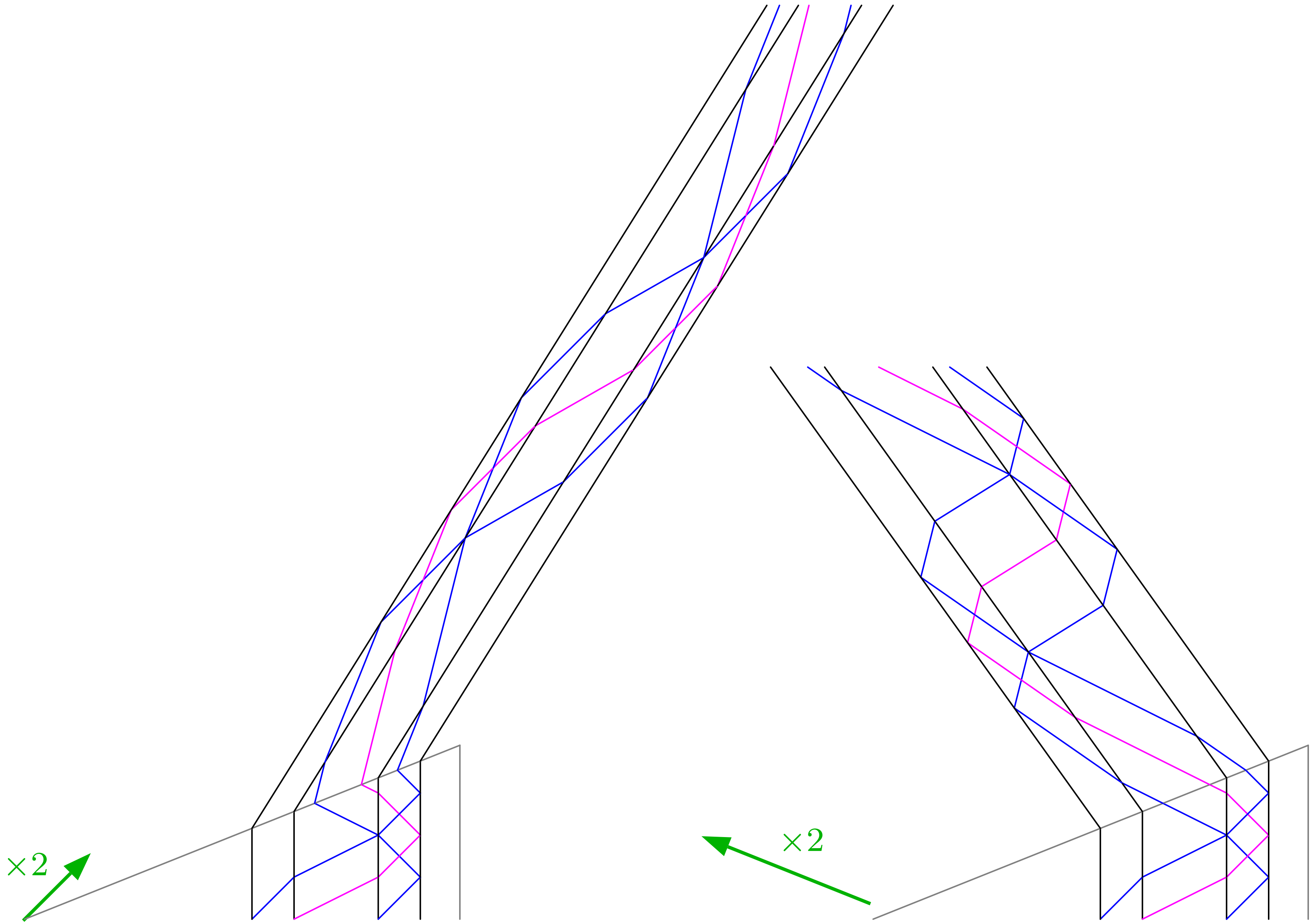
Contraction to a ribbon



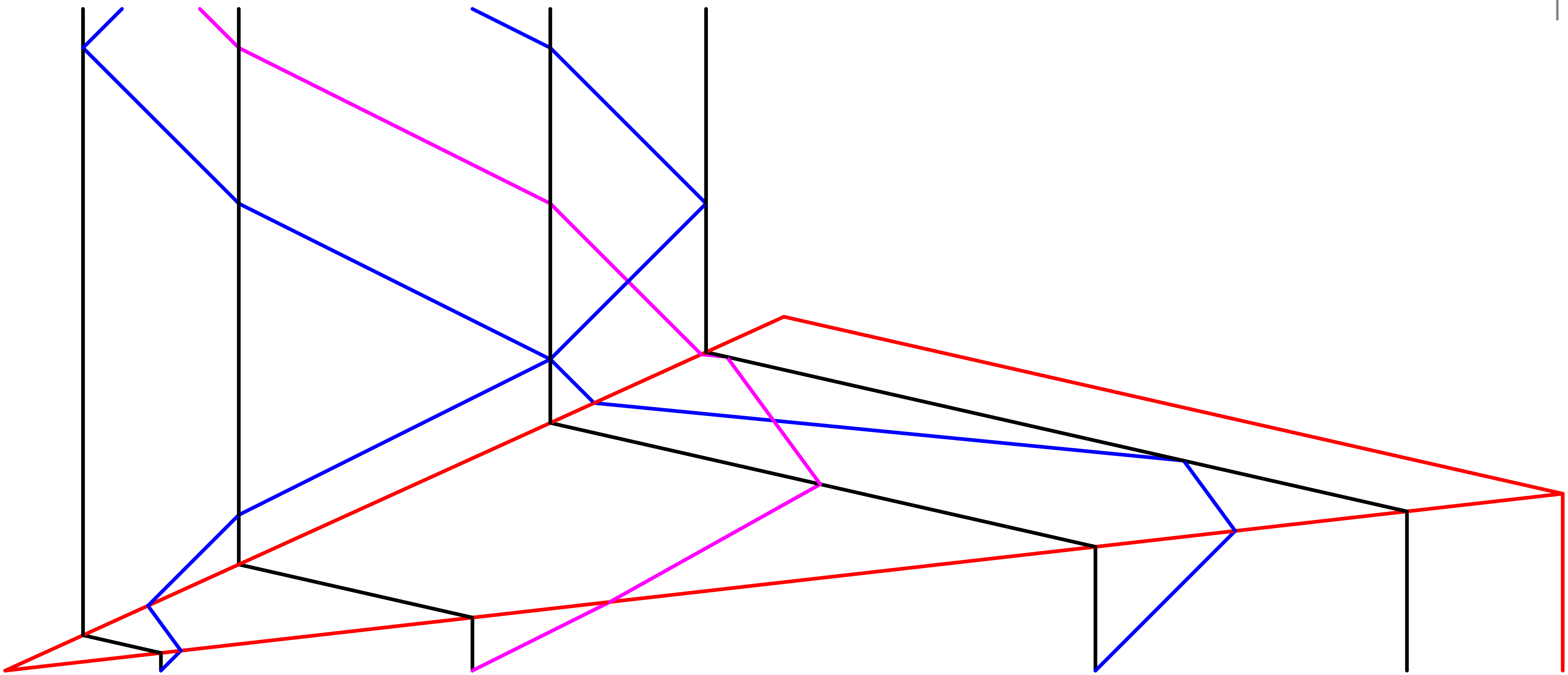
Contraction to a ribbon



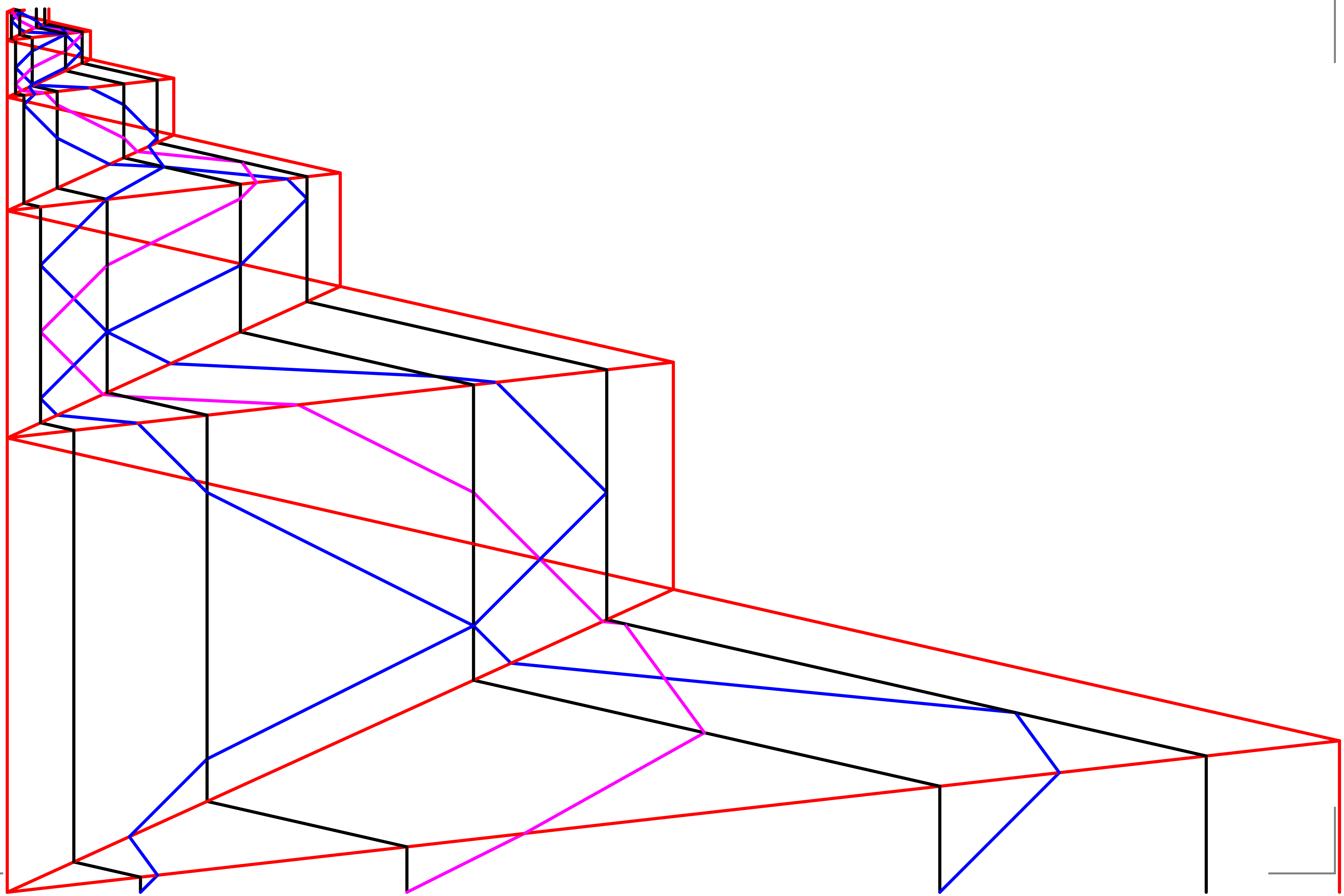
Distortion



Continuous contraction



Contraction to a square



Isolated singularities

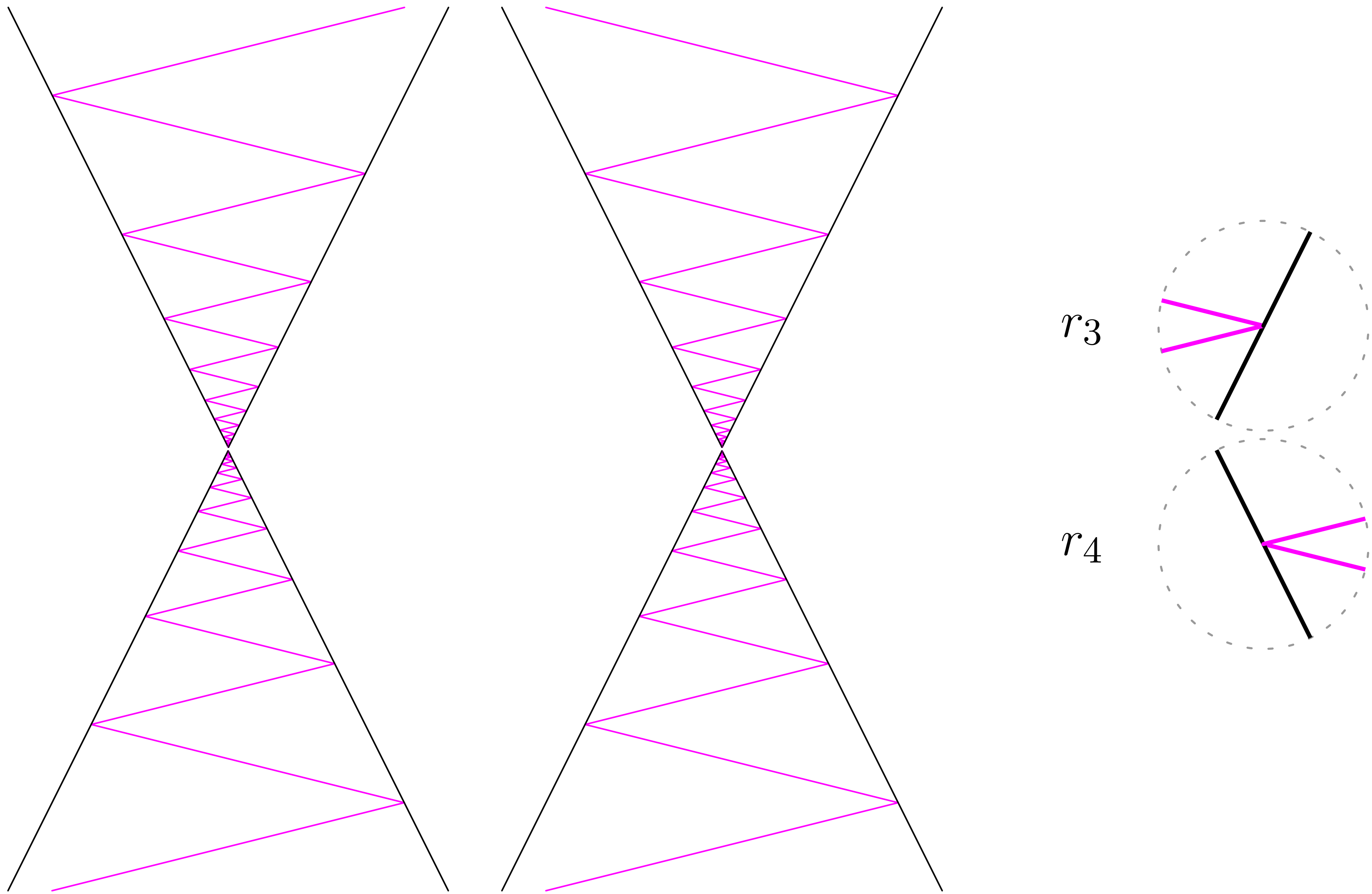
Topological definition of space-time diagrams

It is not a trace of an execution anymore

Rather an extension

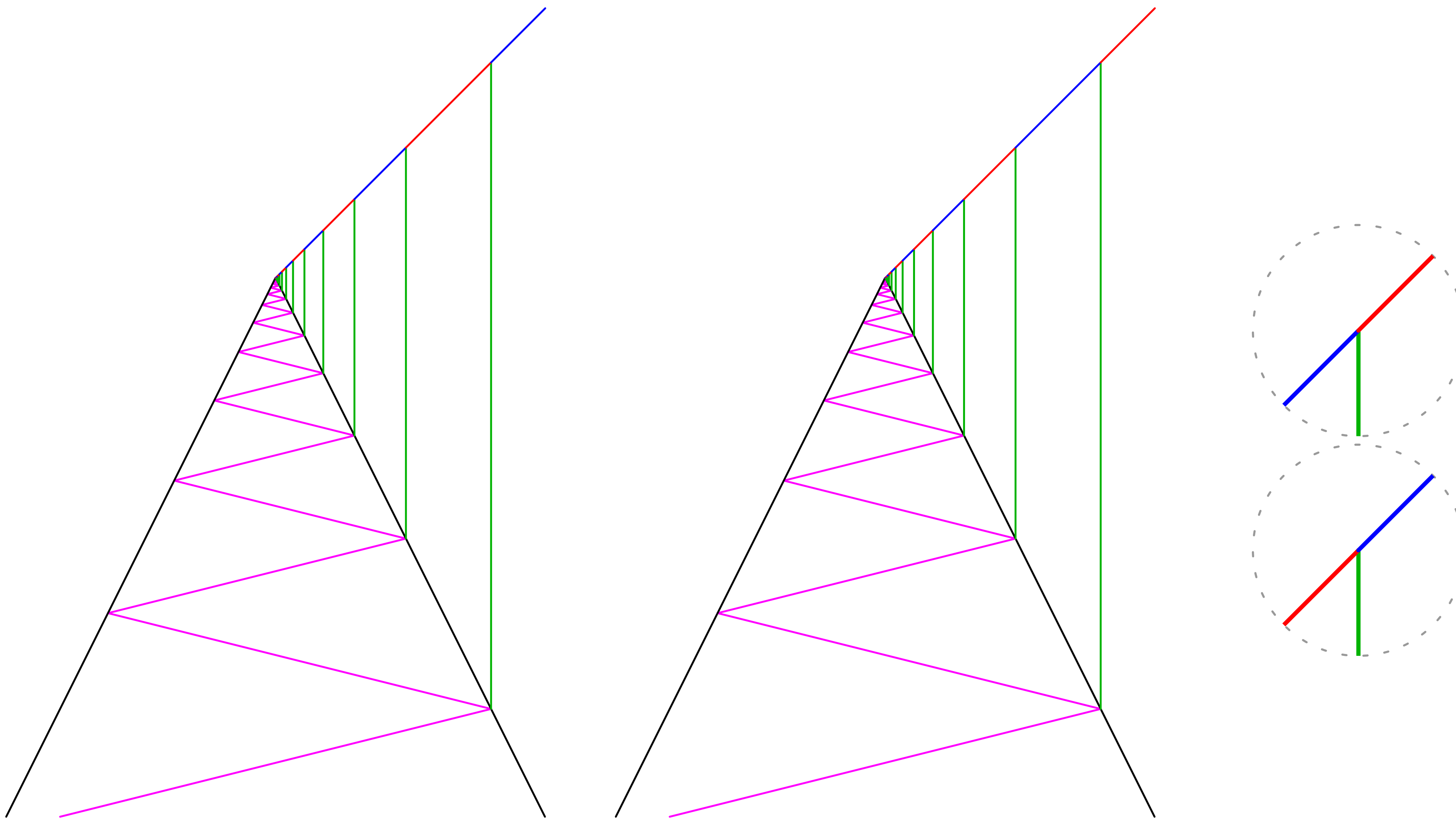
Topological criteria of accumulated values around singularities

Non determinism



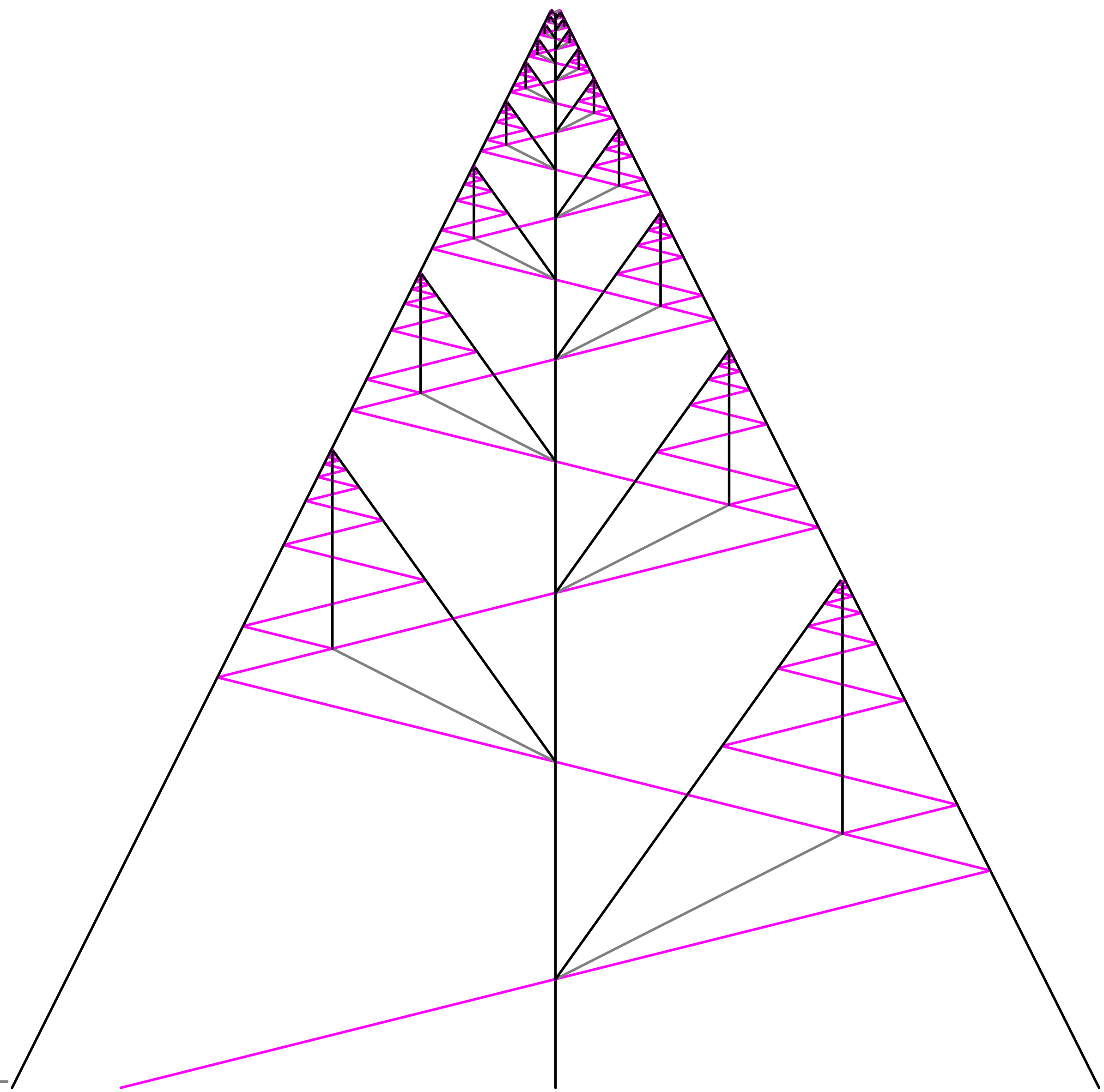
Infinitely many possibilities

Non determinism



Finitely many possibilities

Second order isolated singularities



Decidability

Instance

M: rational geometrical machine,
c_0: finite initial configuration,

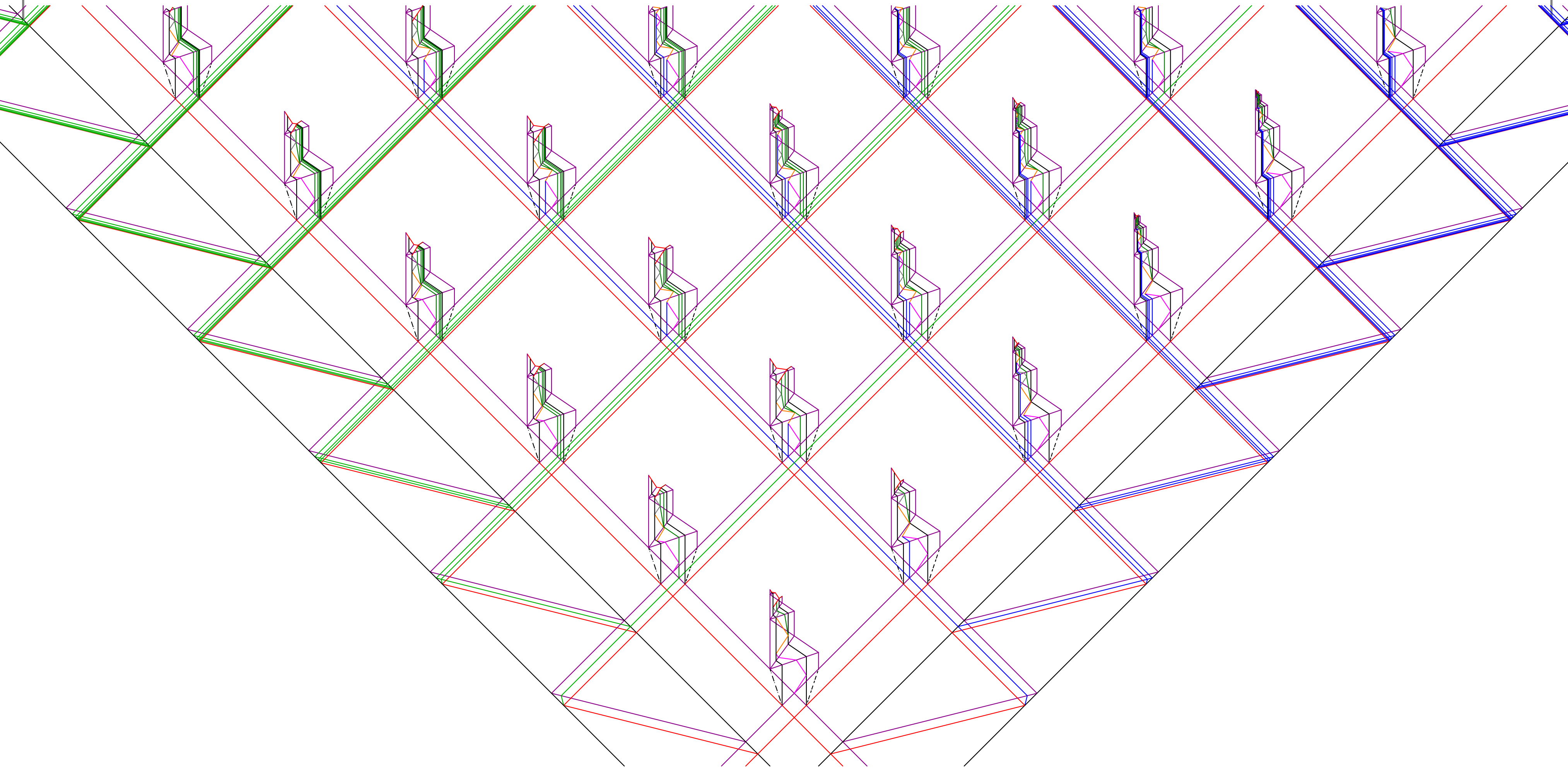
Question

Will there be any singularity?

Highly undecidable (Σ_2^0 in the arithmetical hierarchy)

Reduction: Accumulation iff there is an initial value for which the 2 counter automaton does not stop

Reduction



Un-isolated singularities