1. $\lambda \ll \mathcal{D}$ taille typique des instruments : on néglige la diffraction

2. rayon lumineux : trajectoire suivie par la lumière
ils sont perpendiculaires aux surfaces d'onde.

$* $ indépendants $\qquad\qquad \dfrac{d\vec{u}}{ds} = \vec{\nabla} n$

$*$ principe de retour inverse

3.a transparent : pas d'absorption (amplitude d'une onde plane conservée).

isotrope : ne dépend pas de la direction de propagation

b. $L(\mathcal{C}) = \int_{\mathcal{C}} n\,ds$: longueur que parcourrait la lumière dans le vide pour un même temps donné.

c. Le principe de Fermat énonce que le chemin optique entre deux points est extrémal.
Cela revient à extrémaliser $\int n\,ds$

$\int L\, ds = S$ de $\frac{\partial L}{\partial x} + \frac{d}{dt}\frac{\partial L}{\partial x} = 0$

$n(x,y,z)$ $ds = \sqrt{dx^2+dy^2+dz^2}$

$ds = dz\sqrt{1+\dot x^2+\dot y^2}$

$$L = n \cdot \sqrt{1+\dot x^2+\dot y^2}$$

$$\frac{\partial L}{\partial x} = \frac{d}{dz}\frac{\partial L}{\partial \dot x}$$

$$\Rightarrow \quad \frac{\partial n}{\partial x} = \frac{1}{\sqrt{1+\dot x^2+\dot y^2}} \cdot \frac{d}{dz}\left(\frac{1}{2\sqrt{1+\dot x^2+\dot y^2}}\, n\right)$$

$$\frac{d(n\vec{u})}{ds} = \vec{\nabla} n$$

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a. $\int_A^B n\,dl$ extremal soit $n\int_A^B dl$ extremal

↳ on retrouve bien une ligne droite.

$$dAB = A'B' - AB$$

$$= \| \vec{AA'} + \vec{AB} + \vec{BB'} \| - \| \vec{AB} \|$$

$$= \| (-\vec{dA} + \vec{dB}) + \vec{AB} \| - \| \vec{AB} \|$$

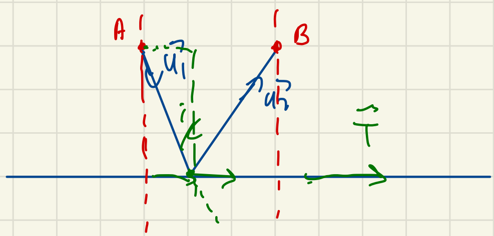
$$\| (\vec{dA} - \vec{dB}) + \vec{AB} \|^2$$

$$= \| AB \|^2 + 2 (\vec{dA} - \vec{dB}) \cdot \vec{AB} + \| \vec{dA} - \vec{dB} \|$$

$$dAB \simeq \sqrt{AB^2 + 2(\vec{dA} - \vec{dB}) \cdot \vec{AB} + \| \vec{dA} - \vec{dB} \|^2} - AB$$

$$\simeq AB \left(1 + \frac{(\vec{dA} - \vec{dB}) \cdot \vec{AB}}{AB^2} \right) - AB$$

$$= (\vec{dA} + \vec{dB}) \cdot \hat{u} \qquad \text{où } \vec{u} = \frac{\vec{AB}}{AB}$$



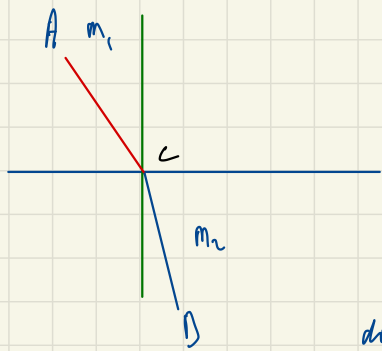
$$d(AB) = dAC + dCD$$

$$= \hat{u}_1 \cdot \vec{dC} - \hat{u}_2 \cdot \vec{dC}$$

$$= (\vec{u}_1 - \vec{u}_2) \cdot \vec{dC} = 0$$

$\vec{u}_1 \cdot \vec{T} = \sin i_1 \qquad \vec{u}_2 \cdot \vec{T} = -\sin i_2$

donc $\sin i_1 = \sin i_2$



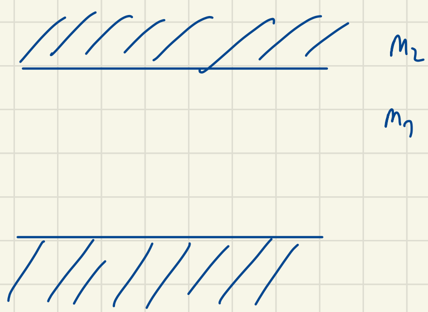
$m_1 \vec{u}_1 \cdot d\vec{C} - m_2 \vec{u}_2 \cdot d\vec{C} = 0$

donc $(m_1 \vec{u}_1 - m_2 \vec{u}_2) \cdot d\vec{C} = 0$

↳ $(m_1 \vec{u}_1 - m_2 \vec{u}_2) \perp$ au dioptre

donc $m_1 \vec{u}_1 \cdot \vec{T} = m_2 \vec{u}_2 \cdot \vec{T}$

d'où $m_1 \sin i_1 = m_2 \sin i_2$



α. réflexion totale donc $m_2 < m_1$

β. $\frac{\pi}{2} - i$ ce correspondants : réfl° totale. $\sin i_2 = \frac{m_1}{m_2} \sin i_1$

$\sin i_1 > \frac{m_2}{m_1}$

$i_{min} = \arcsin \frac{m_2}{m_1}$

$$\sin\left(\pi/2-\theta\right) > \frac{m_2}{m_1}$$

$$\Rightarrow \quad \cos\theta > \frac{m_2}{m_1}$$

$$\Rightarrow \quad \theta < \arccos\left(\frac{m_2}{m_1}\right)$$

$$\sin\theta = m_1 \sin\alpha$$

$$\cos\alpha \geq \frac{m_2}{m_1} \qquad \alpha_{max} = \arccos\frac{m_2}{m_1}$$

$$\text{donc} \quad \sin\theta_{max} = m_1 \sin\left(\arccos\left(\frac{m_2}{m_1}\right)\right)$$

$$\sin(\arccos(x))^2 + x^2 = 1$$

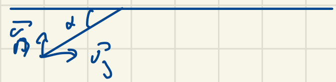
$$\sin(\arccos) = \sqrt{1-x^2}$$

$$\text{donc} \quad \sin\theta_{max} = m_1 \cdot \sqrt{1-\left(\frac{m_2}{m_1}\right)^2}$$

$$= m_1 \cdot \sqrt{\frac{m_1^2 \cdot m_2^2}{m_1^2}}$$

$$= m_1 \cdot \sqrt{2\Delta}$$

$$m^2(x) = m_1^2 \left(1 - 2\Delta\left(\frac{x}{x_1}\right)^2\right)$$

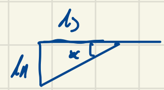


$$m \sin(\pi/2 - \alpha) = cte$$

$$\frac{d(m\vec{u})}{ds} = \vec{\nabla}(m) = \frac{d}{dx} m(x)\, \vec{e_x} \qquad ds = dy\sqrt{1+\dot{x}^2}$$

$$\frac{d}{ds}\left(m\left(\cos\alpha\, \vec{u}_y + \sin\alpha\, \vec{u}_x\right)\right) = \frac{d}{dx} m(x)\, \vec{e_x}$$

donc $\quad m\cos\alpha = cte = m_1\cos\theta_0$



donc $\quad \cos^2\alpha = \dfrac{\cancel{m_1^2}\cdot\cos^2\theta_0}{\cancel{m_1^2}\cdot\left(1-2\Delta\left(\frac{x}{x_1}\right)^2\right)} \qquad \dfrac{dx}{dy} = \tan\alpha$

$$\left(\frac{dx}{dy}\right)^2 = \tan^2\alpha = \frac{\sin^2}{\cos^2} = \frac{1-\cos^2}{\cos^2} = \frac{1}{\cos^2\alpha} - 1$$

$$= \frac{1-2\Delta\left(\frac{x}{x_1}\right)^2}{\cos^2\theta_0} - 1$$

$$= \left(1-\cos^2\theta_0\right) - 2\Delta\left(\frac{x}{x_1}\right)^2$$

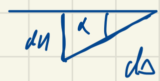
$$2\frac{dn}{dy}\frac{d^2n}{dy^2} = -\frac{4\Delta n}{n_1^2}\cdot\frac{dn}{dy}$$

donc $$\frac{d^2n}{dy^2} + \frac{4\Delta n}{n_1^2} = 0$$

$$k = \sqrt{\frac{4\Delta}{n_1^2}} = \frac{2}{n_1}\sqrt{\Delta}$$

donc $$\lambda = \frac{n\sqrt{\Delta}}{4\pi}$$

$$ds = dy\sqrt{1+n'^2}$$



$$\frac{d}{ds}\, m\sin\alpha = \frac{d}{dn}(m(n))$$

$$\frac{d}{ds}\left(m\cdot\left(\frac{dn}{ds}\right)\right) = \frac{d}{dn}\, m(n)$$

$$\frac{1}{\sqrt{1+n'^2}}\frac{d}{dy}\left(m\cdot\left(\frac{dn}{dy}\right)\cdot\frac{1}{\sqrt{1+n'^2}}\right) = \frac{d}{dn}\, m(n) = f(n)$$

a. charges libres : origines du champs exterieur $\vec{D}$ qui pénètre le matériaux.

charges liées : app de charges dans le matériaux vient un champ électrique.

b. $\vec{P} = \chi \varepsilon_0 \vec{E}$ où χ est le nombre adimique de dipôles

c. $\vec{\nabla} \cdot \vec{P} = -\rho_{liées}$, $\frac{\partial \vec{P}}{\partial t} = j_{liées}$

$\vec{\nabla} \cdot \vec{E} = \frac{\rho_{libre} + \rho_{liée}}{\varepsilon_0}$ => $\varepsilon_0 \vec{\nabla} \cdot \vec{E} + \vec{\nabla} \vec{P} = \rho_{libre}$

donc $\vec{\nabla}(\varepsilon_0 \vec{E} + \vec{P}) = \rho_{lib}$

$\vec{\nabla} \cdot \vec{D} = \rho_{libre}$, $D = \varepsilon_0 \vec{E} + \vec{P}$.

$j_{liée} + \vec{j}_e + \varepsilon_0 \frac{\partial \vec{E}}{\partial t} = 0$ donc $\partial_t(\varepsilon_0 \vec{E} + \vec{P}) = -j_{libre}$

$\partial_t \vec{D} = -j_{libre}$.

pas de charge libre $\vec{D}$ et $\vec{P}$ continue

$\rho = \vec{\nabla} \cdot \vec{P}$

Σ

$D = \varepsilon_0 E + P$, $\vec{P} = \chi \vec{D}$

a. $\vec{P} = \chi \varepsilon_0 \vec{E}$

$= \varepsilon \vec{E}$ $\qquad \varepsilon_0 \chi = \varepsilon$

a. $\Delta \vec{E} - \mu_0 \varepsilon \dfrac{\partial^2 \vec{E}}{\partial t^2} = 0$ $\qquad \vec{\nabla} \cdot \vec{D} = 0$ $\qquad \vec{\nabla} \wedge \vec{B} = \mu_0 \dfrac{\partial \vec{D}}{\partial t} = \mu_0 \varepsilon \dfrac{\partial \vec{E}}{\partial t}$

$\vec{\nabla} \wedge \vec{E} = -\dfrac{\partial \vec{B}}{\partial t}$ $\qquad \vec{\nabla} \cdot \vec{B} = 0$

$\Delta \vec{B} - \mu_0 \varepsilon \dfrac{\partial^2 \vec{B}}{\partial t^2} = 0$ $\qquad \varepsilon_0 \to \varepsilon$

b. $\vec{E}_0(k) = E_0 e^{iky} \vec{e}_y$

donc $(ik)^2 \vec{E} - \mu_0 \varepsilon (i\omega)^2 \vec{E} = 0$

donc $k^2 = \mu_0 \varepsilon \omega^2 = \mu_0 \varepsilon_0 \cdot \dfrac{\varepsilon}{\varepsilon_0} \omega^2 = \dfrac{1}{c^2} \varepsilon_r \omega^2$

soit

$n^2(\omega) = \varepsilon_r(\omega) = \dfrac{\varepsilon}{\varepsilon_0}$

$k = \pm \underline{n} \dfrac{\omega}{c}$ $\qquad n_r(\omega)$: relatif à la lumière (disp)

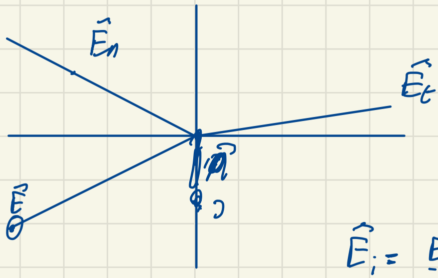
$n_i(\omega)$: absorption

transparent: $n_i(\omega) = 0$

$i\vec{k} \wedge \vec{E} = -i\omega \vec{B}$

$\vec{k} = \frac{1}{v_1}$

$\vec{k} \wedge \vec{B} = \frac{\omega}{c} \vec{E}$



$\vec{E}_i = \underline{E}_0\, e^{i(\vec{k}_i \cdot \vec{r} - \omega t)}$

$\vec{E}_r = \underline{E}_r\, e^{i(\vec{k}_r \cdot \vec{r} - \omega t)}$

$\underline{E}_i + \underline{E}_r = \underline{E}_t$

$e^{i(\vec{k}_i \cdot \vec{r} + \vec{k}_t \cdot \vec{r})}$

$(\vec{k}_i - \vec{k}_r) \cdot \vec{T} = 0$

$k_i = \frac{n_i \omega}{c}$

$\sin i_1 = \sin i_2$

$(\vec{k}_i - \vec{k}_t) \cdot \vec{T} = 0$

$n_1 \sin i_1 = n_2 \sin i_2$

$\underline{E}_i e^{i((k_i \cdot \vec{r}))} + \underline{E}_r e^{i(\vec{k}_r \cdot \vec{r})} = \underline{E}_t\, e^{i \vec{k}_t \cdot \vec{r}}$

$r_{\parallel} = \frac{E_r}{E_i}$

$R_{\parallel} = \frac{\|E_r\|^2}{\|E_i\|^2} = \|r_{\parallel}\|^2$

$$\Delta \vec{E} - \frac{n_i^2}{c^2}\frac{\partial \vec{E}}{\partial t^2} = 0$$

$x < a/2$:
$$A''(x)\, e^{i(\alpha y - \omega t)} + A(x)\cdot\alpha^2 e^{i(\alpha y - \omega t)} - \frac{n_i^2}{c^2}\omega^2 A(x)\, e^{i(\alpha y - \omega t)} = 0$$

donc $A''(x) + \alpha^2 A(x) - k_1^2 A(x) = 0$

$x > a/2$: $A'' + (k_2^2 - \alpha^2) A(x) = 0$

E continue donc $E(x = \frac{a}{2}^+, y, z, t) = E(x = \frac{a}{2}^-, y, z, t)$

$\Rightarrow A(a/2^+) = A(a/2^-)$: A continue en $a/2$

et $\frac{dA}{dx}$ continue via continuité de $\vec{B}$.

$A''(x) + (k_2^2 - \alpha^2) A(x) = 0$

$(k_j^2 - \alpha^2) > 0 \Rightarrow$ harmonique

$(k_j^2 - \alpha^2) < 0 \Rightarrow$ exponentielle.

A : $E_1 \cos(\beta x)$ $\quad E_2 \exp(\gamma x)$

$E_2 \exp(\gamma x)$

$\cos\left(\frac{\beta a}{2}\right) = \exp(\gamma a/2)$ $\quad -\beta \sin\frac{\beta a}{2} = \gamma \exp(\gamma a/2)$

$-\beta \tan\left(\frac{\beta a}{2}\right) = \gamma$

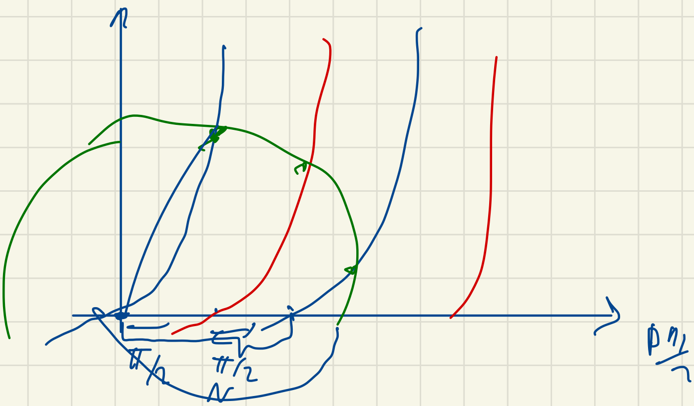
$\sin(\beta x)$ $\qquad$ $\exp(-\gamma x)$, $\quad -\exp(\gamma x)$

$$\beta \cos\left(\beta \frac{n_1}{2}\right) = -\gamma \exp(-\gamma n_1/2)$$

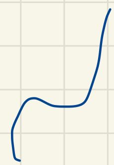
$$\sin\left(\beta \frac{n_1}{2}\right) = \exp(-\gamma n_1/2)$$

$$\beta \cot\left(\beta \frac{n_1}{2}\right) = -\gamma$$

$\gamma =$ x fnc



got β roots.



nodes

tj van mode paire $\qquad N = \left\lfloor \frac{R}{\pi/2} \right\rfloor + 1$

$$\frac{\beta n_1}{1} = m\pi$$

$$m = \frac{\beta n_1}{2\pi}$$

$$N = n_1 \cdot \frac{\sqrt{n_1^2 - n_2^2}}{\cancel{2}} \cdot \frac{k}{\pi} + 1$$

$$\boxed{N = \left\lfloor \frac{n_1}{\pi} \frac{\omega}{c} \sqrt{n_1^2 - n_2^2} \right\rfloor + 1}$$

$$\frac{n_1^{max}}{\pi} \cdot \frac{\omega}{c} \sqrt{n_1^2 - n_2^2} = 1$$

$$n_1^{max} = \frac{\pi c}{\omega \sqrt{n_1^2 - n_2^2}}$$

$$(p-1) = \frac{\omega n_1}{\pi c} \sqrt{n_1^2 - n_2^2}$$

donc si $\omega > \omega_c = \dfrac{\pi c (p-1)}{\sqrt{n_1^2 - n_2^2}\; n_1}$, pas d'existence