A 2011

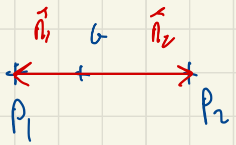
1.1 On effectue à DL à l'ordre 2 du potentiel $V_1(r)$:

$$V_1(r) = V_1(r_0) + \underbrace{\frac{dV_1}{dr}\Big|_{r=r_0}}_{=0 \text{ car eq}} (r-r_0) + \frac{1}{2}\frac{d^2V_1}{dr}\Big|_{r=r_0} (r-r_0)^2$$

donc $$\boxed{k = \frac{d^2V_1(r)}{dr}\Big|_{r=r_0}}$$

1.2 E_L est l'énergie de liaison. Si le système possède une E méca positive alors la liaison est brisée.

2.



$\vec{r}_1 = -r_1\vec{e}_y \quad \vec{r}_2 = r_2\vec{e}_y$

P_1 subit $\vec{F}_{2\to1} = -\vec{\nabla} V(r)$

$$= k(r-r_0)\vec{e}_y$$

$$\vec{F}_{1\to2} = -k(r-r_0)\vec{e}_y \text{ par act° réciproques}$$

Ainsi PFD dans le ref barycentrique supposé galiléen considéré :

$$\begin{cases} m_1\ddot{\vec{r}}_1 = \vec{F}_{2\to1} \\ m_2\ddot{\vec{r}}_2 = \vec{F}_{1\to2} \end{cases} \qquad \begin{cases} m_1\ddot{r}_1 = -k(r-r_0) \\ m_2\ddot{r}_2 = -k(r-r_0) \end{cases}$$

donc $\ddot{r} = -\left[\frac{1}{m_1}+\frac{1}{m_2}\right]k(r-r_0)$

$\Rightarrow m\ddot{r} = -k(r-r_0)$

Finalement on obtient bien l'éq° différentielle avec $m = \frac{m_1 m_2}{m_1 + m_2}$

3. $E = -E_L + \frac{1}{2} k (r - r_0)^2$ $\quad \omega_0 = \sqrt{\frac{k}{m}}$

$$= -E_L + \frac{1}{2} \omega_0^2 m (r - r_0)^2 = -E_L + \frac{1}{2} \omega_0^2 m \Delta r^2$$

or $\Delta r^2 = \frac{\Delta p^2}{q^2}$ donc $E = -E_L + \frac{1}{2} \omega_0^2 m \frac{(\Delta p)^2}{q^2}$

4. $\langle P_{ray} \rangle = \frac{\mu_0 \omega^4 p_0^2}{12 \pi c}$

$$\frac{dE}{dt} = \frac{-\mu_0 \omega_0^4 p_0^2}{12\pi c} \Rightarrow \frac{dE}{dt} + \frac{m \omega_0^2 \Delta p^2}{2q^2} \cdot \frac{\mu_0 \omega_0^2 \cancel{p_0^2}\, 2q^2}{12\pi c\, m\, \cancel{\Delta p^2}}$$

$$\Rightarrow \frac{dE}{dt} + E + E_L \cdot \frac{1}{\tau_E} = 0$$

avec $\boxed{\tau_E = \frac{\mu_0 \omega_0^2 q^2}{6 \pi c m}}$

$\dot{r} = \omega_0 r$

5.1 $T/\tau_E \ll 1$ soit $\boxed{\omega_0 \tau_E \gg 1}$

$\dot{r} = \omega_0^2 \Delta r^2$

5.2 TEm : $\frac{dE_m}{dt} = \left\langle \frac{m}{\tau} \dot{r}^2 \right\rangle = \frac{m}{\tau} \cdot \frac{1}{2} \omega_0^2 \Delta r^2$

$$= \frac{1}{\tau} \cdot \frac{m \omega_0^2 \Delta p^2}{2q^2} = \frac{1}{\tau} \cdot (E + E_L)$$

donc $\boxed{\tau = \tau_E}$

7. a $\ell^* = \frac{1}{n^* \sigma^*}$ or $P = n^* k_B T$

Donc $\ell^* = \frac{k_B T}{P \sigma_c}$

7.b

$Z_c = \frac{\ell^*}{v^*}$ AN: $Z_c =$

8.1 $m_0 \ddot{\xi}_2 = -k(\xi_2 - \xi_0)$

$m_c \ddot{\xi}_0 = k(\xi_2 - \xi_0) - k(\xi_0 - \xi_1) = k(\xi_2 + \xi_1 - 2\xi_0)$

$m_0 \ddot{\xi}_1 = k(\xi_0 - \xi_1)$

8.2 $m_c \xi_0 + m_0(\xi_1 + \xi_2) = 0$

8.3 $\xi_0 = -\frac{m_0}{m_c}(\xi_1 + \xi_2)$

donc $m_0 \ddot{\xi}_1 = -k\left(\left(\frac{m_0}{m_c} + 1\right)\xi_1 + \xi_2\right)$

$$\boxed{\ddot{\xi}_1 + k\left(\frac{1}{m_c} + \frac{1}{m_0}\right)\xi_1 = -\frac{k}{m_c}\xi_2}$$

9.a En regime libre, les états sont décrits par une superposit° d'oscillations aux modes propres.

$$m\ddot{\vec{r}} = q\dot{\vec{r}} \wedge \vec{B} \qquad \vec{B} = B\vec{e}_z$$

$$m\ddot{x} = qB\dot{y} \qquad m\dot{x} = qBy + m\dot{x}_0$$

$$m\ddot{y} = -qB\dot{x} \qquad m\dot{y} = qBx + m\dot{y}_0$$

$$z = x + iy$$

$$m\ddot{z} = qB(\dot{y} - i\dot{x})$$

$$= -qBi(i\dot{y} + \dot{x})$$

$$= -qBi\dot{z}$$

$$\dot{z} = \dot{z}_0 \, e^{-\frac{qBi t}{m}}$$

$$\dot{x} = \dot{x}_0 \cos\left(\frac{qB}{m} t\right)$$

$$\dot{y} = -\dot{y}_0 \sin\left(\frac{qB}{m} t\right)$$

9.b

S : $\xi_0 = 0$

$\xi_1 = -\xi_2$

A : $\xi_1 = \xi_2$

$$\xi_0 = -\frac{m_0}{m_c}\left(\xi_1 + \xi_2\right)$$

$$= \frac{-2m_0}{m_c}\xi$$

$$\omega_A = \sqrt{h\left(\frac{2}{m_c} + \frac{1}{m_0}\right)}$$

$$\omega_S = \sqrt{\frac{h}{m_0}}$$

$$\omega_A^2 = \frac{h(2m_0 + m_c)}{m_c m_0}$$

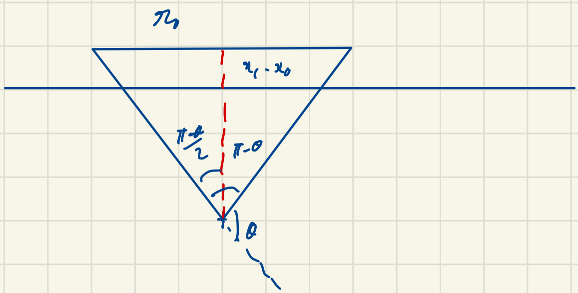
$$\omega_S^2 = \frac{h}{m_0}$$ donc $$\frac{\omega_A}{\omega_S} = \frac{(2m_0 + m_c)}{m_c} = \frac{2m_0}{m_c} + 1$$

$$= 2 \cdot \frac{16}{12} + 1$$

$$= \frac{8}{3} + 1 = \frac{11}{3}$$

donc $$\omega_S = \sqrt{11/3} \cdot \omega_A$$

10. $x_1 m_0 + x_2 m_0 + x_0 m_c = 0$



$\tan\left(\frac{\pi-\theta}{2}\right) = \frac{x_0}{x_1 - x_0}$ et $x_0 = -2x_1 \frac{m_0}{m_c}$

$\tan\left(\frac{\pi-\theta}{2}\right) = \frac{x_0}{x_1\left(1+2\frac{m_0}{m_c}\right)}$

$\tan(\pi/2 - \theta) \sim \frac{\sin(\pi/2-\theta)}{\cos(\pi/2-\theta)}$

$\sim \frac{\cos(\theta)}{\sin(\theta)}$

$\sim \frac{1}{\theta/2}$

donc $\boxed{\theta/2 \sim \frac{x_1(1+2(m_0/m_c))}{x_0}}$

10.2 $E_c = m_0 \dot{x}_1^2 + \frac{1}{2} m_c \dot{x}_0^2$

$= m_0 \dot{x}_1^2 + \frac{1}{2} m_c \cdot 4 \dot{x}_1^2 \frac{m_0^2}{m_c^2}$

$= \dot{x}_1^2 \left[m_0 + 2\frac{m_0^2}{m_c} \right] = m_0 \dot{x}_1^2 \left(1 + \frac{2 m_0}{m_c} \right)$

$E_{pe^\circ} = \frac{1}{2} \Gamma \theta^2$ $\quad E_c + E_p = \text{cste} \Rightarrow 2 m_0 \dot{x}_1 \ddot{x}_1 \left(1 + \frac{2m_0}{m_c}\right) + \Gamma \dot{\theta} \theta = 0$

$\theta = \frac{2x_1}{n_0}\left(1 + 2\, m_0/m_c \right)$

$$0 = 2m_0\ddot{x}_1\left(1+\frac{2m_1}{m_c}\right) + 4\Gamma\,\frac{x_1}{n_0^2}\left(1+\frac{2m_0}{m_c}\right)^2 = 0$$

$$\ddot{x}_1 + \frac{2\Gamma}{m_0 n_0^2}\cdot\left(1+\frac{2m_0}{m_c}\right)x_1 = 0$$

$$\omega_0 = \sqrt{\frac{2\Gamma}{m_0 n_0^2}\left(1+\frac{2m_0}{m_c}\right)}$$

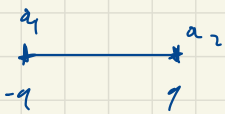
$$\omega = \sqrt{\frac{2\Gamma}{n_0^2}\left(\frac{1}{m_1}+\frac{2}{m_c}\right)}$$

12.1 $\lambda \sim 1000\ \text{nm} \gg 10^{-10}\ \text{m}$

12.2 $\frac{v\Delta}{E} \sim \frac{v}{c} \ll 1$ pour des p non relativistes

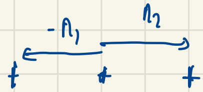
12.3 $\vec{F} = q\vec{E}(P_2) - q\vec{E}(P_1) \simeq \vec{0}$ pour unif de $\vec{E}$.

$P_1 = -q\vec{E}\cdot\dot{x}_1$



$P_2 = +q\vec{E}\cdot\dot{x}_2$

donc $P = P_1 + P_2 = \vec{E}\,q(\dot{x}_2 - \dot{x}_1) = \vec{E}\cdot\frac{d\vec{p}}{dt}$



$\vec{r}_1 = -r_1 \vec{e_x}$

$r = r_1 + r_2$

$m_1 \ddot{\vec{r}}_1 = -k(r - r_0) - qE \Rightarrow m_1 \ddot{r}_1 = k(r - r_0) + qE$

$m_1 \ddot{\vec{r}}_2 = k(r - r_0) + qE \qquad m_1 \ddot{r}_2 = k(r - r_0) + qE$

$$\ddot{\xi} + \frac{1}{\tau}\dot{\xi} + \omega_0^2 \xi = \frac{q}{m} E_0 \cos(\omega t)$$

$$\xi = \frac{\frac{q}{m} E_0}{j\frac{1}{\tau} + (\omega_0^2 - \omega_0^2)}$$ donc $$\dot{\xi} = \frac{j\omega \frac{q}{m} E_0}{j \cdot \frac{1}{\tau} + (\omega_0^2 - \omega^2)}$$

Ainsi $P_{abs} = \frac{1}{2} \mathrm{Re}(E \cdot \xi^*)$

$$E^* \dot{\xi} = \frac{j\omega \frac{1}{m} E_0^2}{j/\tau + (\omega_0^2 - \omega^2)} = \frac{j\omega \frac{q}{m} E_0^2 \left((\omega_0^2 - \omega^2) - j/\tau \right)}{\frac{1}{\tau^2} + (\omega_0^2 - \omega^2)^2}$$

$$\frac{1}{2} \mathrm{Re}(E^* \dot{\xi}) = \frac{\frac{\omega}{\tau} \frac{q^2}{m} E_0^2}{2\left(1/\tau^2 + (\omega_0^2 - \omega^2)^2 \right)} = \frac{\tau q^2}{2m\left(1 + \tau^2(\omega_0^2 - \omega^2)\right)}$$

$\approx$ [illegible] (ω)