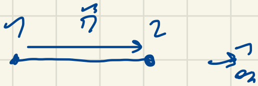
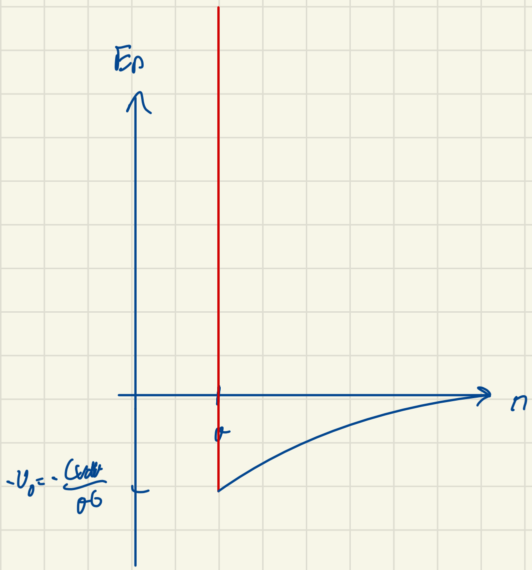
C2005

1. $U(r) = -\dfrac{C_{vdW}}{r^6}$



donc $\vec{F}_{2\to1} = -\vec{\nabla}U(r) = -\dfrac{6C_{vdW}}{r^7}\vec{e_r}$: c'est une force attractive.

2.



répulsion électrostatique des nuages électroniques

$\sigma \sim 10^{-10}\,m = 1\,Å$: les atomes ne rentrent pas en int^{n} :

force infinie dès contact.

3.a $F = U - TS$

$dF = dU - TdS + SdT$

$= TdS - pdV - TdS + SdT$

$= SdT - pdV$ donc $\boxed{p = -\frac{\partial F}{\partial V}}$

$$p = -\frac{\partial F}{\partial V} = p_{GP} - \frac{N^2}{V^2} B(T) k_B T$$

1.3.b $\left(p + \frac{a}{v^2}\right)(v - b) = k_B T$

$\frac{1}{v}$

$$p = \frac{N k_B T}{V} - \frac{N^2}{V^2} B(T) k_B T$$

$$= k_B T \left(\frac{N}{V} - \frac{N^2}{V^2} B(T) \right)$$

$v = \frac{V}{N} \gg 1$

$$p = \frac{k_B T}{v} - \frac{1}{v^2} B(T) k_B T$$

$$p \left(\frac{1}{\frac{1}{v} - \frac{1}{v^2} B(T)} \right) = k_B T$$

$$p \left(\frac{v}{1 - \frac{1}{v} B(T)} \right) = k_B T, \quad p v \left(1 + \frac{1}{v} B(T) \right) = k_B T$$

$$P = \frac{k_B T}{v} - \frac{1}{v^2} B(T)\, k_B T$$

$$= \frac{k_B T}{v} + \frac{1}{v^2} k_B T \left(b - \frac{a}{k_B T} \right)$$

$$= k_B T \left(\frac{1}{v} + \frac{b}{v^2} \right) - \frac{a}{v^2}$$

donc $$\left(P + \frac{a}{v^2} \right) \cdot \frac{v}{1 + \frac{b}{v}} = k_B T$$

$$\Rightarrow \boxed{\left(P + \frac{a}{v^2} \right) \cdot \left(v - b \right) = k_B T}$$

1.3.c $$B(T) = \frac{1}{2} \int_0^{+\infty} \left(1 - \exp\left(- \frac{U(r)}{k_B T} \right) 4\pi r^2 dr \right)$$

$$= \frac{1}{2} \int_0^{\sigma} 4\pi r^2 dr + \frac{1}{2} \int_0^{+\infty} \left(1 - e^{-C/r^6 k_B T} \right) 4\pi r^2 dr$$

$U_0 \ll k_B T \Rightarrow \frac{C}{r^6 k_B T} \ll 1$ donc $1 - e^{-C/r^6 k_B T}$

$$= \frac{C}{r^6 k_B T}$$

$$B(T) = \frac{1}{2}\left[\frac{4\pi}{3} r^3\right]_0^{\sigma} + \frac{1}{2}\int_{\sigma}^{\infty} \frac{4\pi C}{r^4 k_B T}\, dr \qquad \frac{1}{2}\frac{1}{r^2} = -\frac{1}{r^3}$$

$$= \frac{2\pi}{3}\sigma^3 + \frac{1}{2}\left[\frac{4\pi C}{3 r^3}\right]_{\sigma}^{\infty}$$

$$= \frac{2\pi}{3}\sigma^3 - \frac{2\pi C}{3\sigma^3 k_B T} \qquad u_0 = \frac{C}{\sigma^6}$$

$$\boxed{B(T) = \frac{2\pi}{3}\sigma^3\left(1 - \frac{u_0}{k_B T}\right)}$$

1.5.d $$\left(P + \frac{a}{v^2}\right)(v - b) = k_B T$$

$$v = \frac{V}{N} = \frac{V}{N_A} \qquad \left(P + \frac{N_A^2 a}{V}\right)\left(\frac{V}{N_A} - b\right) = k_B T$$

$$\left(P + \frac{a_{mol}}{V}\right)(V - bN_A) = N_A k_B T$$

$$\boxed{\left(P + \frac{a_{mol}}{V}\right)\left(V - b_{mol}\right) = RT}$$

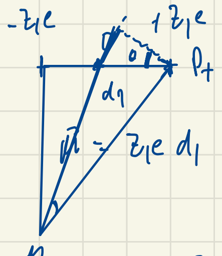
$a_{mol} = a N_a^2$

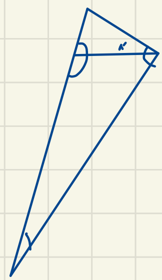
$b_{mol} = b N_a$

II.1.a d_1 a la dimension d'une longueur

$$-\frac{q}{4\pi\varepsilon_0 r}$$

$$\frac{d/2}{r}$$





$$V_{-Z_1 e} = \frac{-Z_1 e}{4\pi\varepsilon_0 r_-} \qquad V_{Z_1 e} = \frac{Z_1 e}{4\pi\varepsilon_0 r_+}$$

$\gamma = \frac{\pi}{2} - \alpha'$

$\alpha + \beta + \gamma = \pi$

$\alpha' + (\pi - \beta) + \frac{\pi}{2} - \alpha = \pi$

$\alpha' + \alpha + \gamma + \frac{\pi}{2} - \alpha = \pi$

$\alpha' + \pi - \alpha = \pi$, $\alpha' = \alpha$

$$\vec{P_+ M} = \vec{P_+ O} + \vec{OM}$$

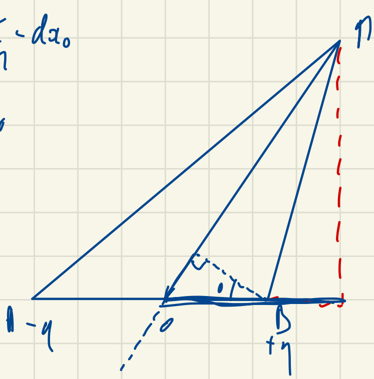
$$\|\vec{BM}\|^2 = y_0^2 + (x_0 - d/2)^2$$

$$= y_0^2 + x_0^2 + \frac{d^2}{4} - d x_0$$

$$= r^2 + \frac{d^2}{4} - d x_0$$

$$r^2 + \frac{d^2}{4} - 2\,\vec{BO}\cdot\vec{OM} = 2\vec{\mu}\cdot\vec{OM}$$

$$AO = r^2 + \frac{d^2}{4} + d x_0$$



$$V = \frac{Ze}{4\pi\varepsilon_0}\Big($$

$$\|BM\|^2 = BO^2 + OM^2 + 2\,\vec{BO}\cdot\vec{OM}$$

$$= \left(\frac{d}{2}\right)^2 + r^2 - \vec{d}\cdot\vec{OM}$$

$$\|AM\|^2 = \left(\frac{d}{2}\right)^2 + r^2 + \vec{d}\cdot\vec{OM} = r^2\left(1 + \frac{d^2}{4r^2} + \frac{\vec{d}\cdot\vec{OM}}{r^2}\right)$$

$$\text{donc} \quad V(M) = -\frac{z_1 e}{4\pi\varepsilon_0\|AM\|} + \frac{z_1 e}{4\pi\varepsilon_0\|BM\|}$$

$$= \frac{z_1 e}{4\pi\varepsilon_0}\left(\frac{1}{\|BM\|} - \frac{1}{\|AM\|}\right)$$

$$= \frac{z_1 e}{4\pi\varepsilon_0}\left(\frac{1}{r}\cdot\left(1 - \frac{1}{2}\frac{d^2}{4r^2} + \frac{1}{2}\frac{\vec{d}\cdot\vec{OM}}{r^2}\right) - \frac{1}{r}\left(1 - \frac{1}{2}\frac{d^2}{4r^2} - \frac{1}{2}\frac{\vec{d}\cdot\vec{OM}}{r^2}\right)\right)$$

$$= \frac{z_1 e}{4\pi\varepsilon_0}\cdot\left(\frac{\vec{d}\cdot\vec{OM}}{r^3}\right) = \frac{z_1 e\,\vec{d}\cdot\vec{OM}}{4\pi\varepsilon_0\,r^3}$$

$$\boxed{V = \frac{\vec{p}_1\cdot\vec{OM}}{4\pi\varepsilon_0\,OM^3}}$$

$$\vec{\nabla} = \partial_r\,\vec{e_r} + \frac{1}{r}\partial_\theta\,\vec{e_\theta}$$

$$V = \frac{\vec{\mu}_1 \cdot \vec{r}}{4\pi\varepsilon_0 r^3}$$

$$\vec{\nabla}(\vec{\mu}_1 \cdot \vec{r}) = (\vec{\mu}_1 \cdot \vec{e_r})\vec{e_r} + \frac{1}{r} \cdot \vec{\mu}_1 \cdot (r\vec{e_\theta})\vec{e_\theta}$$

$$= (\vec{\mu}_1 \cdot \vec{e_r})\vec{e_r} + (\mu_1 \cdot \vec{e_\theta})\vec{e_\theta} = \vec{\mu}_1$$

$$E = -\vec{\nabla} V = -\left(\frac{\vec{\nabla}(\mu_1 \cdot \vec{r}) \cdot 4\pi\varepsilon_0 r^3 - 3 \cdot 4\pi\varepsilon_0 r^2 \vec{e_r} \cdot (\mu_1 \cdot \vec{r})}{(4\pi\varepsilon_0 r^3)^2} \right)$$

$$= \frac{1}{4\pi\varepsilon_0 r^3} \left(\frac{3(\vec{\mu}_1 \cdot \vec{r})\vec{r}}{r^2} - \vec{\mu}_1 \right)$$