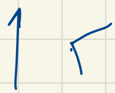
$$\vec{A} = \frac{\mu_0}{4\pi}\,\frac{\vec{M}\wedge\vec{e_r}}{r^2}$$

$$\vec{B} = \vec{\nabla}\wedge\vec{A} = \frac{\mu_0}{4\pi}\,\vec{\nabla}\wedge\left(\vec{M}\wedge\frac{\vec{e_r}}{r^2}\right)$$

$$= \frac{\mu_0}{4\pi}\,\vec{M}\left(\vec{\nabla}\cdot\frac{\vec{e_r}}{r^2}\right) - \frac{\vec{e_r}}{r^2}\left(\vec{M}\cdot\vec{\nabla}\right)$$



$$\vec{\nabla}\cdot\frac{\vec{e_r}}{r^2} = \frac{4}{r^2}\,\frac{\partial(\vec{e_r})}{\partial r} = 0$$

donc $\quad \vec{B} = -\frac{\vec{e_r}M}{r^2}\left[\cos\theta\,\frac{\partial}{\partial r} - \sin\theta\,\frac{1}{r}\frac{\partial}{\partial\theta}\right]$

$$= -\frac{M\mu_0}{4\pi}\left[-\cos\theta\cdot\frac{2}{r^3}\vec{e_r} - \frac{\sin\theta}{r^3}\vec{e_\theta}\right]$$

$$= \frac{M\mu_0}{4\pi r^3}\left[2\cos\theta\,\vec{e_r} + \sin\theta\,\vec{e_\theta}\right]$$

1.c $\quad \theta = \pi/2 \qquad B_e = \frac{M\mu_0}{4\pi R^3}\cdot 2 = \frac{M\mu_0}{2\pi R^2}$

$$\vec{B} = \frac{B_e}{2}\,\frac{R^3}{r^3}\left[2\cos\theta\,\vec{e_r} + \sin\theta\,\vec{e_\theta}\right]$$

$$\|\vec{B}\| = \frac{B_e R^3}{2r^3}\sqrt{4\cos^2\theta + \sin^2\theta}$$

$$\|\vec{B}\| = \frac{B_e R^3}{r^3}\sqrt{3\cos^2\theta + 1}$$

$$\|\vec{B}\| = \frac{B_e R^3}{n^3} \cdot 2 = \frac{2B_e R^3}{n^3} - \|\vec{B}\|$$

S_c

$$\rho = e(n_i - n_e)$$

$$\vec{j} = -e n_e \vec{v}_e$$

$$\frac{\partial n_e}{\partial t} + \vec{\nabla} \cdot (n_e \vec{v}_e) = 0$$

$$m_e n_e \left[\frac{\partial \vec{v}_e}{\partial t} + (\vec{v}_e \cdot \vec{\nabla}) \vec{v}_e \right] = -n_e e \, (\vec{E} + \vec{v}_e \wedge \vec{B})$$

$$\vec{\nabla} \cdot \vec{B} = 0 \qquad \vec{\nabla} \cdot \vec{E} = e(n_i - n_e) \qquad \vec{\nabla} \wedge \vec{B} = \mu_0 \vec{j} + \mu_0 \varepsilon_0 \frac{\partial \vec{E}}{\partial t} \qquad \vec{\nabla} \wedge \vec{E} = -\partial_t \vec{B}$$

$$\frac{\partial n_1}{\partial t} + \vec{\nabla} \cdot (n_0 \vec{v}_1) = 0 \qquad n_0 m \frac{\partial v_1}{\partial t} = -n_0 e \, (\vec{E}_1 + \vec{v}_e \wedge \vec{B}_0)$$

$$\vec{\nabla} \cdot \vec{B}_1 = 0 \qquad \vec{\nabla} \cdot \vec{E} = \frac{-e n_1}{\varepsilon_0} \qquad \vec{\nabla} \wedge \vec{B}_1 = -\mu_0 e n_0 \vec{v}_1 + \mu_0 \varepsilon_0 \frac{\partial \vec{E}_1}{\partial t} \qquad \vec{\nabla} \wedge \vec{E}_1 = -\partial_t \vec{B}_1$$

$$-i\omega n_1 + n_0 i\vec{k} \cdot \vec{v}_1 = 0 \qquad -i\omega \vec{v}_1 n_0 m = -n_0 e \, (\vec{E}_1 + \vec{v}_1 \wedge \vec{B}_0)$$

$$i\vec{k} \cdot \vec{B} = 0 \qquad i\vec{k} \cdot \vec{E}_1 = \frac{-e n_1}{\varepsilon_0} \qquad i\vec{k} \wedge \vec{B}_1 = -\mu_0 e n_0 \vec{v}_1 - i\omega \mu_0 \varepsilon_0 \vec{E}_1$$

$$i\vec{k} \wedge \vec{E}_1 = -i\omega \vec{B}_1$$

$$\mu_0 e m_0 \vec{v}_1 = -\left[i\vec{k}\wedge\vec{B}_1 + \frac{i\omega}{c^2}\vec{E}_1 \right]$$

$$= -\left[i\vec{k}\wedge\left[\frac{\vec{k}}{\omega}\wedge\vec{E}_1\right] + \frac{i\omega}{c^2}\vec{E}_1 \right]$$

$$= -i\left[(\vec{k}\cdot\vec{E}_1)\frac{\vec{k}}{\omega} - \frac{k^2}{\omega}\vec{E}_1 + \frac{\omega}{c^2}\vec{E}_1 \right]$$

$$= -i\omega\left[\frac{k^2}{\omega^2}\vec{E}_{1x} + \vec{E}_1\left(\frac{1}{c^2} - \frac{k^2}{\omega^2}\right) \right]$$

donc $$\vec{v}_1 = \frac{-i\omega}{\mu_0 e m_0}\left[\frac{k^2}{\omega^2}\vec{E}_{1x} + \vec{E}_1\left(\frac{1}{c^2} - \frac{k^2}{\omega^2}\right) \right]$$

$$-i\omega\vec{v}_1 m_0 m = -m_0 e\left(\vec{E}_1 + \vec{v}_1\wedge\vec{B}_0\right)$$

$$\frac{-\omega^2 m}{\mu_0 e}\left[\frac{k^2}{\omega^2}\vec{E}_{1x} + \vec{E}_1\left(\frac{1}{c^2} - \frac{k^2}{\omega^2}\right) \right] \qquad \frac{\omega^2 m\varepsilon_0}{\varepsilon_0\mu_0 m_0 e^2} = \frac{\omega^2}{\omega_p^2}c^2$$

$$= -m_0 e\left[\vec{E}_1 - \frac{i\omega}{\mu_0 e m_0}\left[\frac{k^2}{\omega^2}\vec{E}_{1x} + \vec{E}_1\left(\frac{1}{c^2} - \frac{k^2}{\omega^2}\right) \right]\wedge B_0\vec{e}_y \right]$$

Donc $$\frac{\omega^2 c^2}{\omega_p^2}\left[\frac{k^2}{\omega^2}\vec{E}_{1x} + \vec{E}_1\left(\frac{1}{c^2} - \frac{k^2}{\omega^2}\right) \right]$$

$$= \vec{E}_1 + \frac{i\omega}{\mu_0 e m_0}\frac{k^2}{\omega^2}B_0 E_{1x}\vec{e}_y + \frac{i\omega B_0 E_{1x}}{\mu_0 e m_0}\left(\frac{1}{c^2} - \frac{k^2}{\omega^2}\right)\vec{e}_y$$

$$- \frac{i\omega B_0}{\mu_0 e m_0} \left(\frac{1}{c^2} - \frac{h^2}{\omega^2} \right) E_{1y} \vec{e}_x$$

$$c^2 h^2 \vec{E}_{1x} + \omega^2 \vec{E}_1 \left(1 - \frac{h^2 c^2}{\omega^2} \right) = \omega_p^2 \vec{E}_1 + i\omega\omega_c \quad E_{1x} \vec{e}_y$$
$$- i\omega\omega_c \left(1 - \frac{h^2 c^2}{\omega^2} \right) E_{1y} \vec{e}_x$$

$$\frac{\omega_p^2}{\mu_0 e m_0} B_0 = \frac{n_0 e^2}{\varepsilon_0 m} \cdot \frac{B_0}{\mu_0 e n_0} = \omega_c \, c^2$$

d'où le système
$$c^2 h^2 E_{1x} + \omega^2 E_{1x} \left(1 - \frac{h^2 c^2}{\omega^2} \right) = \omega_p^2 E_{1x} - i\omega\omega_c \left(1 - \frac{h^2 c^2}{\omega^2} \right) E_{1y}$$

$$\Rightarrow \quad E_{1x} (\omega^2 - \omega_p^2) + i\omega\omega_c \left(1 - \frac{h^2 c^2}{\omega^2} \right) E_{1y} = 0$$
$$E_{1y} \left(\omega^2 - h^2 c^2 - \omega_p^2 \right) - i\omega\omega_c E_{1x} = 0$$
$$E_{1y} \left(\omega^2 - h^2 c^2 - \omega_p^2 \right) = 0$$

d'où
$$M = \begin{pmatrix} \omega^2 - \omega_p^2 & i\omega\omega_c \left(1 - \frac{h^2 c^2}{\omega^2} \right) & 0 \\ -i\omega\omega_c & \omega^2 - h^2 c^2 - \omega_p^2 & 0 \\ 0 & 0 & \omega^2 - h^2 c^2 - \omega_p^2 \end{pmatrix}$$

5.b $\vec{E}_1 = E_{1y}\vec{e}_y$ si $k \to 0$ alors $\omega \to \omega_{pe}$ valeur limite

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$E_{1y} = i E_{1x}$ $\qquad \vec{E} = \hat{E}_0\, e^{i(\vec{k}\cdot\vec{r} - \omega t)}$

6.a $\vec{j}\wedge\vec{B}$ $\qquad \vec{\nabla}\wedge\vec{B} = \mu_0\vec{j}$

$$\frac{1}{\mu_0}(\vec{\nabla}\wedge\vec{B})\wedge\vec{B} = -\frac{1}{\mu_0}\vec{B}\wedge(\vec{\nabla}\wedge\vec{B})$$
$$= -\frac{1}{2\mu_0}\left[\vec{\nabla}(B^2) - 2(\vec{B}\cdot\vec{\nabla})\vec{B}\right]$$

$$\boxed{\vec{j}\wedge\vec{B} = \vec{\nabla}\left(-\frac{B^2}{2\mu_0}\right) + \frac{1}{\mu_0}(\vec{B}\cdot\vec{\nabla})\vec{B}}$$

6.b $(\vec{t}\cdot\vec{\nabla})\vec{t} = \dfrac{\vec{n}}{R}$

$\vec{B} = B_t\vec{t}$: $\dfrac{1}{\mu_0}(B_t\vec{t}\cdot\vec{\nabla})B_t\vec{t}$

$$= \frac{1}{\mu_0}B_t(\vec{t}\cdot\vec{\nabla})B_t\vec{t}$$
$$= \frac{1}{\mu_0}B_t^2\frac{\vec{n}}{R_c} + \frac{1}{\mu_0}B_t\,\vec{t}\cdot(\vec{t}\cdot\vec{\nabla}B_t)$$

$$= \frac{B_t^2 \vec{m}}{\mu_0} + \frac{1}{\mu_0} B_t \vec{t} \cdot \vec{\nabla} B_t$$

$$= \frac{B_t^2 \vec{m}}{\mu_0} + \frac{1}{2\mu_0} \vec{t} \cdot \vec{\nabla} B^2$$

6.c

$$nm\left(\frac{\partial \vec{v}}{\partial t} + (\vec{v} \cdot \vec{\nabla})\vec{v}\right) = -\vec{\nabla} p - \vec{m} \cdot \left(\vec{\nabla} \frac{B^2}{2\mu_0}\right) + \frac{B^2}{\mu_0 R_c} \vec{m}$$

6.d $\vec{\nabla} \cdot \vec{B} = 0$

$$\frac{dx}{B_x} = \frac{dy}{B_y}$$ donc $x = cste$

$$\vec{\nabla} \frac{B^2}{2\mu_0}$$

$$= -\frac{x B_0}{\mu_0 e^2} + \cancel{\frac{x^2}{\mu_0 R_c}}$$