C2016

$\hat{H} \to \hat{H} - e\phi$ $\qquad \partial_t \psi \to (\partial_t \psi - e\phi\psi) = \mathcal{D}_t \psi$

$\hat{p} \to \hat{p} - e\hat{A}$ $\qquad (-i\hbar\vec{\nabla}) \to (-i\hbar\vec{\nabla} - e\vec{A})^2 = -i\hbar\vec{\mathcal{D}}$

$$i\hbar\partial_t \psi = \frac{1}{2m} \cdot (-i\hbar\vec{\nabla})^2 \psi$$

Donc $\quad i\hbar\mathcal{D}_t \psi = \frac{1}{2m}(-i\hbar\vec{\mathcal{D}})^2 \psi$

42. Cela revient à montrer que $\mathcal{D}'_t \psi' = (\mathcal{D}_t \psi)'$

$$(\mathcal{D}_t \psi)' = G\mathcal{D}_t \psi = e^{ie\alpha(\vec{r},t)/\hbar}(i\hbar\partial_t\psi \cdot e\phi\psi)$$

$$= i\hbar\partial_t\psi' \cdot i\hbar\psi \cdot \partial_t e^{ie\alpha/\hbar} - e\phi\psi'$$

$$= i\hbar\partial_t\psi' + e\psi'\partial_t\alpha - e\phi\psi'$$

$$= i\hbar\partial_t\psi' - e(\phi - \partial_t\alpha)\psi'$$

$$\boxed{(\mathcal{D}_t\psi)' = \mathcal{D}'_t\psi'}$$

$$G\vec{\mathcal{D}}\psi = e^{ie\alpha/\hbar} \cdot (-i\hbar\vec{\nabla} - e\vec{A})^2 \psi$$

$-\hbar^2\Delta \cdot$

$$\vec{\mathcal{D}}'\psi' = \left(-i\hbar\vec{\nabla} - e\vec{A} + e\vec{\nabla}\alpha\right)^2 e^{ie\alpha/\hbar}\psi$$

$$\left(-i\hbar\vec{\nabla} - e\vec{A} + e\vec{\nabla}\alpha\right)\left(e^{ie\alpha/\hbar}\psi\right) = -i\hbar\left[\frac{ie}{\hbar}\psi\vec{\nabla}\alpha + e^{ie\alpha/\hbar}\vec{\nabla}\psi\right.$$

$$= e^{ie\alpha/\hbar}\left[\vec{\mathcal{D}}\right]\psi$$

donc en procédant à nouveau. $\boxed{\vec{\mathcal{D}}'^2\psi' = G\vec{\mathcal{D}}^2\psi}$

43. ψ et ψ' représente la même sol° physique car ils diffèrent d'un facteur de phase.

Même sol° physique car $\begin{cases} \vec{E}' = \vec{E} \\ \vec{B}' = \vec{B} \end{cases}$

44. $$i\hbar\left(\partial_t - e\phi\right)\psi = \frac{1}{2m}\left[-i\hbar\vec{\nabla} - e\vec{A}\right]^2\psi$$

$$i\hbar\partial_t\psi - i\hbar e\phi\psi = \frac{1}{2m}\left[-i\hbar\vec{\nabla} - e\vec{A}\right]\left(-i\hbar\vec{\nabla}\psi - e\vec{A}\psi\right]$$

$$= \frac{1}{2m}\left[-\hbar^2\Delta\psi + 2i\hbar e\left(\vec{A}\cdot\vec{\nabla}\right)\psi + e^2A^2\psi\right]$$

$$-i\hbar\partial_t\psi^* + i\hbar e\phi\psi^* = \frac{1}{2m}\left[-\hbar^2\Delta\psi^* - 2i\hbar e\left(\vec{A}\cdot\vec{\nabla}\right)\psi^* + e^2A^2\psi^*\right]$$

$i\hbar\partial_t\psi\psi^*$

$$= \frac{1}{2m}\left[\hbar^2 \psi \Delta\psi^* - \hbar^2 \psi^* \Delta\psi + 2i\hbar e (\vec{A}\cdot\vec{\nabla})(\psi\psi^*)\right]$$

$$= \frac{1}{2m}\left[\hbar^2 \left(\vec{\nabla}\left[\psi\vec{\nabla}\psi^* - \psi^*\vec{\nabla}\psi\right] + 2i\hbar e (\vec{A}\cdot\vec{\nabla})\psi\psi^*\right.\right.$$

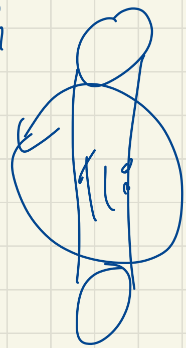
$\partial_t \rho$

46. $\vec{\nabla}\wedge\vec{A} = B\vec{u}_z$

$\vec{A}_\varphi = \frac{\pi B}{2}\vec{e}_\varphi$

$\downarrow$

$\vec{A} =$



donc $\iint_{(S)} \vec{\nabla}\wedge\vec{A} = \oint_{(C)} \vec{A}\cdot d\vec{l} = \iint B\cdot d\vec{S}$

donc $A\cdot 2\pi a = \pi a^2 B$

d'où $A_\varphi = \frac{\pi B}{2} = \frac{aB}{2}$

47. $\hat{H}\psi = E\psi$

$\varphi_0 = \frac{2\pi\hbar}{|e|}$

$\varphi = B\pi a^2$

$$\hat{H} = \frac{1}{2m}\left(-\frac{i\hbar}{a}\frac{\partial}{\partial\varphi} + |e|A_\varphi\right)^2$$

$$\varphi/\varphi_0 = \frac{B\pi a^2 |e|}{2\pi\hbar} = \frac{Ba^2|e|}{2} = a|e|A_\varphi$$

$$\hat{H} = \frac{1}{2ma}\left(-i\hbar\frac{\partial}{\partial\varphi} + \varphi/\varphi_0\right)$$

$$\frac{1}{2ma^2}\hbar\left(-i\frac{\partial}{\partial\varphi} + \varphi/\varphi_0\right)^2\psi = E\psi$$

$$-i\hbar\frac{\partial^2\varphi}{\partial\varphi^2} + \left(\frac{\phi}{\phi_0}\right)^2\varphi = 2ma^2E\,\varphi$$

$$-i\,\partial_\varphi\psi + \frac{\phi}{\phi_0}\psi = \alpha\,\psi$$

$$\partial_\varphi\psi + i\left(\frac{\phi}{\phi_0}-\alpha\right)\psi = 0$$

$\psi = A\,e^{-i\left(\frac{\phi}{\phi_0}-\alpha\right)\cdot\varphi}$ donc $\ell = \frac{\phi}{\phi_0}-\alpha$ entien

et $E = \dfrac{\hbar^2}{2ma^2}\cdot\alpha^2 = \dfrac{\hbar^2}{2ma^2}\cdot\left(\dfrac{\phi}{\phi_0}-\ell\right)$ $\alpha =$

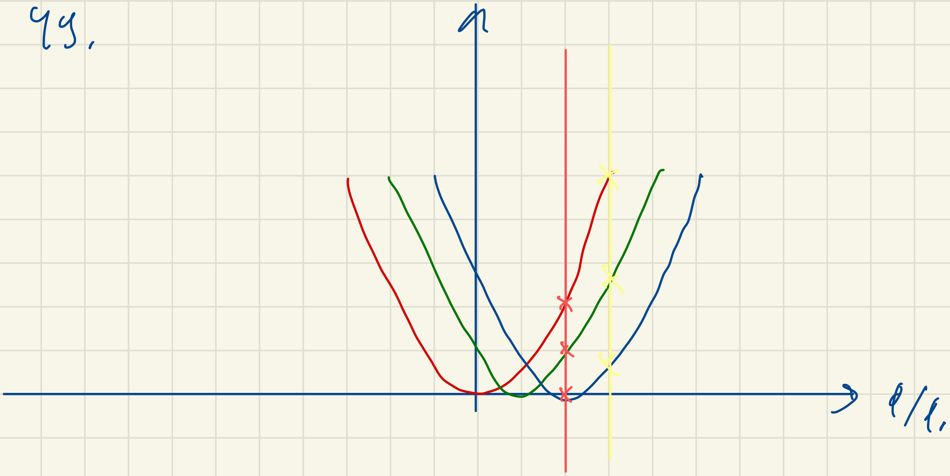
$$\boxed{E_\ell = \frac{\hbar^2}{2ma^2}\left(\frac{\phi}{\phi_0}+\ell\right)}$$

$\psi = A\cdot e^{i\ell\varphi}$ donc $\displaystyle\int_0^{2\pi} a\cdot d\varphi\cdot|\psi|^2 = \int_0^{2\pi} a\,d\varphi\cdot A^2 = 1$

donc $A = \dfrac{1}{\sqrt{2\pi a}}$

$\psi_\ell = \dfrac{1}{\sqrt{2\pi a}}\cdot e^{i\ell\varphi}$, $\boxed{E_\ell = \dfrac{\hbar^2}{2ma^2}\left(\dfrac{\phi}{\phi_0}+\ell\right)}$

44.



On obtient bien le même spectre. $B \to -B \Rightarrow \ell \to -\ell$: même spectre également.

51. $\vec{e}_\varphi \cdot (\vec{A}' + \vec{\nabla}\alpha) = 0 \Rightarrow \frac{Br}{2} + \frac{1}{r}\partial_\varphi \alpha = 0$

$$\partial_\varphi \alpha = -\frac{Br^2}{2}$$

$$\Rightarrow \alpha = -\frac{Br^2}{2}\varphi + C$$

donc $\vec{A}'' = \partial_r \alpha = -Br\varphi\, \vec{e}_r$

$\vec{A}''$ augmente

$$\boxed{\vec{A}'' = -Br\varphi\, \vec{e}_r}$$

$$\oint_C \vec{A}\cdot d\vec{l} = \vec{0} \neq \iint \vec{B}\cdot d\vec{S}$$

$$\oint_C \vec{A}\cdot d\vec{l} = \int_0^a + Br \cdot dr \cdot 2\pi = \left[\frac{r^2}{2} B \cdot 2\pi\right]_0^a = a^2 B\pi$$

$$= \pi a^2 B = \phi$$

Le potentiel ne doit pas être multivalué en un point:

il faut éviter la branche de coupure.

53. $\psi' = e^{iq\alpha/\hbar}\,\psi(\varphi)$

$$\psi'(\varphi+2\pi) = e^{iq(\alpha(2\pi+\varphi)-\alpha(\varphi))/\hbar}\,\psi'(\varphi)$$

$$= e^{i\,2\pi\phi/\phi_0}$$

63. $V_d' = V_0 - L\theta A = V_0\left(1 - \frac{L\theta A}{V_0}\right)$

$PV = nRT$

$$V_g = V_0 + L\theta A = V_0\left(1 + \frac{L\theta A}{V_0}\right)$$

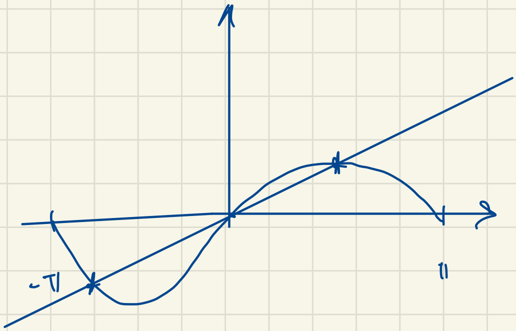
Donc $P_d - P_g = \frac{nRT}{V_0}\left(\frac{1}{1-\frac{L\theta A}{V_0}} - \frac{1}{1+\frac{L\theta A}{V_0}}\right)$

$$= \frac{nRT}{V_0}\cdot 2L\theta A = a\theta A$$

$$\boxed{a = \frac{2n_0 R L A^2}{V_0}}$$

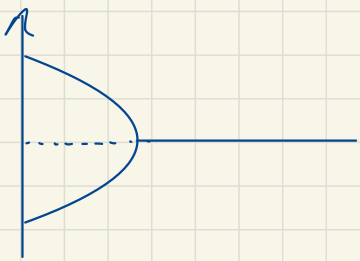
$m_0 = 1$ mole. $\qquad a = \dfrac{2RLA^2}{V_0^2}$

$$mg\sin\theta = aT\theta \qquad a$$



$$\frac{aT_c}{mg} = 1$$

$$T_c = \frac{mg}{a}$$



$$\sin\theta_0 = \frac{aT}{mg}\theta_0 = \frac{T}{T_c}\theta_0 \quad ; \quad \sin\theta_0 = \theta_0 - \frac{\theta_0^3}{6} + o(\theta_0^3)$$

$$\theta_0 - \frac{1}{6}\theta_0^3 = \frac{(T-T_c) + T_c}{T_c}\,\theta_0$$

$$-\frac{1}{6}\theta_0^3 = \frac{T-T_c}{T_c}\,\theta_0$$

donc $\quad \theta_0 = \sqrt{\dfrac{6}{T_c}} \cdot \left(|T-T_c|\right)^{1/2}$, $\beta = 1/2$

$mg \sin \alpha + h$