Les plasmons de surface

3.1 $\frac{\partial \rho}{\partial t} + \vec{\nabla} \cdot \vec{j} = 0$

donne $\frac{\partial \sigma}{\partial t} + \vec{\nabla} \cdot \vec{j}_s = 0$

et ainsi $\partial_t \underline{\sigma} = -\partial_z j_s$

$= -j_{sM}\, ik\, e^{i(kz-\omega t)}$

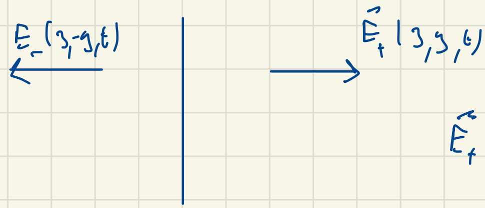
et donc $\underline{\sigma} = -j_{sm}\, ik \frac{e^{i(kz-\omega t)}}{-i\omega}$

$\underline{\sigma} = + j_{sm}\, k\, e^{i(kz-\omega t)}$

3.2 $\underline{\sigma}$ ne varie que selon z donc $(O, \vec{e}_y, \vec{e}_z)$ plan de symétrie donc, par inv par transl° selon x et par de Curie :

$\vec{E} = E_y \vec{e}_y + E_z \vec{e}_z$ f° de y, z et t.

3.3



$\vec{E}_+ = -\vec{\underline{E}}_-$

et $E_+ - E_- = \frac{\sigma}{\varepsilon_0}$ donc finalement $y \to 0 \Rightarrow E(y=0, z, t) = \frac{\sigma(z,t)}{2\varepsilon_0}$

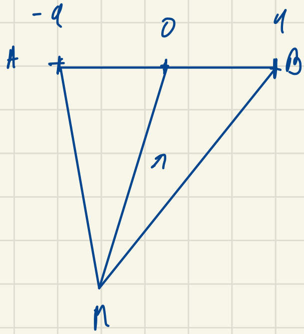
3.4 $\Delta \underline{E}_y - \frac{1}{c^2} \frac{\partial^2 \underline{E}_y}{\partial t^2} = 0 \Rightarrow \partial_y^2 \underline{E}_y - k^2 \underline{E}_y + h^2 \underline{E}_y = 0$

$\Rightarrow \frac{\partial^2 \underline{E}_y}{\partial y^2} = (k^2 - h^2) \underline{E}_y \Rightarrow E$

On peut supposer $E_y = E_{0y}\, e^{i(k_y + k_y \cdot y - \omega t)}$

donc $\Delta E_y - \frac{1}{c^2}\frac{\partial^2 E_y}{\partial y^2} = 0$, $\quad (M\cdot\vec{\nabla})\vec{B}$

$M \cdot \partial_y B = \int dB$



$$V(M) = \frac{q}{4\pi\varepsilon_0 BM} - \frac{q}{4\pi\varepsilon_0 AM}$$

$$= \frac{q}{4\pi\varepsilon_0}\Big($$

$$\|BM\|^2 = BO^2 + OM^2 + 2\vec{BO}\cdot\vec{OM}$$

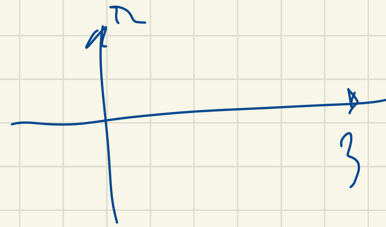
$$V(M) = \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{OM}\cdot\left(1 - \frac{1}{2}\frac{\vec{BO}\cdot\vec{OM}}{OM^2}\right) - \left(1 - \frac{1}{2}\frac{\vec{AO}\cdot\vec{OM}}{OM^2}\right)\right)$$

$$= \frac{q}{4\pi\varepsilon_0}\left(\frac{(\vec{AO}+\vec{OB})\cdot\vec{OM}}{OM^3}\right)$$

$$\boxed{V(M) = \frac{q\,\vec{AB}\cdot\vec{OM}}{4\pi\varepsilon_0 r^3} = \frac{\vec{p}\cdot\vec{OM}}{4\pi\varepsilon_0 OM^3}}$$

$u = \frac{dr}{ds}$

$\frac{d}{ds} = \frac{dy}{ds} \cdot \frac{d}{dy}$



$n = n(z)$

$ds = \sqrt{dx^2 + dy^2}$

$\frac{d}{ds}\left(n\, u_1 \right)$

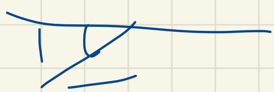
$n_1 \frac{\cos(\theta_0)}{n}$

$$\frac{d}{dy}\left(n \frac{dr}{ds}\right) = \frac{dz}{ds}\frac{d}{dz}\left(n \frac{dz}{ds} \frac{d^2 r}{dz^2} \right)$$

$$= \frac{(n_1 \cos(\theta_1))^2}{n} \frac{d}{dz}\left(\frac{dr}{ds} \right) = d$$

$n \cos\alpha = n_1 \cos\theta_0$

$$\boxed{\tan\alpha = \frac{dr}{dy}}$$



$\tan^2\alpha = \frac{1}{\cos^2\alpha} - 1$

$\hat{u} = \frac{dr}{ds}$

$\left(\frac{dr}{dy}\right)^2 =$

$\hat{r} = \hat{x} e_x + y \hat{e}_y$

$$\sigma(\omega) = i\,\varepsilon_0 \frac{\omega_p^2}{\omega}$$

Dans le métal : $m_e \dot{v} = -e\vec{E} - \frac{m_e}{\tau} v$ $\quad \vec{E} =$

$$-m_e (i\omega)\,\underline{v} = -e\,\underline{\vec{E}} - \frac{m_e}{\tau} v$$

donc $$\underline{\vec{v}} = \frac{e}{m_e\left(i\omega - \frac{1}{\tau}\right)} \vec{E}$$

$$\vec{j} = -ne\vec{v} = \frac{ne^2}{m_e\,(1/\tau - i\omega)} = \frac{\tau n e^2}{m_e\,(1 - i\omega\tau)}$$

$$\underline{\sigma} = \frac{ne^2\tau/m_e}{1 - i\omega\tau} = \boxed{\frac{\sigma_0}{1 - i\omega\tau} = \underline{\sigma}}$$

σ_0 : conductivité statique $= \frac{ne^2\tau}{m_e}$

$\omega\tau \gg 1$: $$\sigma = i\frac{ne^2\tau}{m_e\omega\tau} = i\frac{ne^2}{m_e\omega} = i\varepsilon_0\frac{\omega_p^2}{\omega}$$

On retrouve le cadre du plasma $\quad \tau_{cu} = 10^{-14}\,s$

électroneutralité :

$$\frac{\partial \rho}{\partial t} + \vec{\nabla}\cdot\vec{j} = 0$$

$$\vec{j} = \sigma\vec{E}, \quad \frac{\partial \rho}{\partial t} + \sigma\vec{\nabla}\cdot\vec{E} = \frac{\partial \rho}{\partial t} + \frac{\sigma}{\varepsilon_0}\rho = 0$$

$$\rho = \rho_0 e^{-t/\tau}, \quad \tau = \frac{\varepsilon_0}{\sigma}$$

métal : ondes magnétiques cas $\omega \ll \omega_p$

$\omega\tau \ll 1$: ondes et $\rho = 0$

$\frac{1}{\tau} < \omega < \omega_p$: ondes et $\rho \neq 0$

$\omega > \omega_p$: pas ondes et $\rho \neq 0$: plasma (car $\omega\tau \gg 1$)

effet de peau :

$$\vec{\nabla}\cdot\vec{E} = 0 \qquad \vec{E} = E_0 f(x) e^{-i\omega t}\vec{e_y}$$

$$\vec{\nabla}\cdot\vec{B} = 0$$

$$\vec{\nabla}\wedge\vec{E} = -\frac{\partial \vec{B}}{\partial t} \Rightarrow \frac{\partial \vec{B}}{\partial t} = E_0 f'(x) e^{-i\omega t}$$

$$B = E_0 \frac{1}{i\omega} f'(x) e^{-i\omega t}$$

$$\vec{\nabla}\wedge\vec{B} = \mu_0\vec{j} = \mu_0\sigma\vec{E}$$

$$E_0 \frac{1}{i\omega} f''(x) e^{-i\omega t} = \mu_0\sigma\vec{E}$$

$$f''(x) = (\mu_0 \sigma i\omega) f(x)$$

ou bien $\quad \partial_t(\vec{\nabla} \wedge \vec{B}) = \vec{\nabla} \wedge (-\vec{\nabla} \wedge \vec{E})$ $\quad$ car $\vec{\nabla} \cdot \vec{E} = 0$

$$= -\Delta \vec{E}$$

donc $\quad -\Delta \vec{E} = \mu_0 \sigma \, \partial_t \vec{E}$

soit $\quad \Delta \vec{E} + \mu_0 \sigma \, \partial_t \vec{E} = 0$ $\quad$ eq. de diff

$$E_0 e^{i\omega t} \left(f''(x) - \mu_0 \sigma i\omega f(x) \right) = 0$$

on suppose $\sigma \sim \sigma_0$ cad: $\underline{\omega \tau \ll 1}$

$$e^{i\pi/4}$$

$$f = e^{-x/\delta} \quad , \quad \underline{\delta}^2 = \frac{i}{\mu_0 \sigma_0 \omega}$$

$$\underline{\delta} = \frac{1}{\sqrt{2}} \frac{(1+i)}{\sqrt{\mu_0 \sigma_0 \omega}}$$

$$f(x) = e^{-ix/\delta} \cdot e^{-x/\delta}$$