alkali atoms, and for early fundamental studies of the properties of the condensates'.

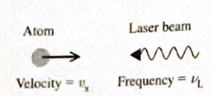

Fig. 11.1 In Doppler cooling, the laser frequency is tuned below the atomic resonance by δ. The frequency seen by an atom moving towards the laser is Doppler-shifted up by $\nu_0(v_x/c)$.

11.2 Laser cooling

The idea of using a laser to *cool* a gas of atoms is, at first sight, rather surprising: we would normally expect a powerful laser to produce a heating rather than a cooling effect. In fact, the technique only works in a very restricted range of conditions with the laser frequency close to resonance with an atomic transition. In the subsections that follow, we shall study the basic principles of laser cooling, the factors that determine the temperatures that are achieved, and the way in which the experiments are done.

11.2.1 Basic principles of Doppler cooling

The basic principles of laser cooling can be understood with fairly simple arguments that give the correct order of magnitude for the important parameters of the process. The more detailed analysis given in the next subsection reproduces the same basic results but with the numerical factors correctly evaluated.

Let us consider an atom moving in the $+x$-direction with velocity v_x as shown in Fig. 11.1. We assume that the atom interacts with a counter-propagating laser beam with its frequency $\nu_L \equiv c/\lambda$ tuned to near resonance with one of the transitions of the atom. We can then write:

$$\nu_L = \nu_0 + \delta, \tag{11.5}$$

where ν_0 is the atomic transition frequency and $\delta \ll \nu_0$. In the rest frame of the atom, the laser source is moving towards the atom, and its frequency is therefore shifted up by the Doppler effect. The Doppler-shifted frequency is given by:

$$\nu_L' = \nu_L\left(1+\frac{v_x}{c}\right) = (\nu_0+\delta)\left(1+\frac{v_x}{c}\right) \approx \nu_0 + \delta + \frac{v_x}{c}\nu_0, \tag{11.6}$$

where we assumed $v_x \ll c$. It is then apparent that if we choose

$$\delta = -\nu_0\frac{v_x}{c} = -\frac{v_x}{\lambda}, \tag{11.7}$$

we find $\nu_L' = \nu_0$. When this condition is satisfied, the laser will be in resonance with atoms moving in the $+x$-direction, but not with those moving away or obliquely, as depicted schematically in Fig. 11.2.

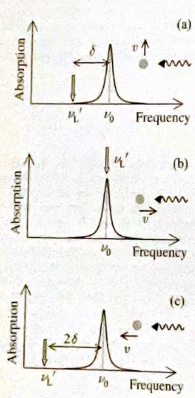

Fig. 11.2 Doppler-shifted laser frequency ν_L' in the rest frame of an atom moving with speed v. When the laser frequency is tuned below ν_0 by v/λ, the Doppler effect shifts the laser into resonance with the atoms if they are moving towards the laser (b), but not if they are moving sideways (a), or away (c) from the laser.

Now consider what happens after the atom has absorbed a photon from the laser beam. The atom goes into an excited state and then re-emits another photon of the same frequency by spontaneous emission in a random direction. This absorption–emission cycle is illustrated schematically in Fig. 11.3. Each time the cycle is repeated, there is a net change in the momentum of the atom of Δp_x in the x-direction, where:

$$\Delta p_x = -\frac{h}{\lambda}. \tag{11.8}$$

This follows from applying conservation of momentum to both the absorption and emission processes, with the photon momentum equal to h/λ. The momentum change on absorption is always in the $-x$-direction, but the recoil after spontaneous emission averages to zero, because the photons are emitted in random directions.

Equation 11.8 implies that repeated absorption–emission cycles generate a net frictional force in the $-x$-direction. If the laser intensity is large, the probability for absorption will be large, and the time to complete the absorption–emission cycle will be determined by the radiative lifetime τ. The frictional force exerted on the atom is then given by:

The existence of a light-induced mechanical force on an atom was first demonstrated by Frisch in 1933 by measuring the deflection of a sodium beam by light from a sodium lamp. See R. Frisch, *Z. Phys.* **86**, 42 (1933).

$$F_x = \frac{\mathrm{d}p_x}{\mathrm{d}t} \approx \frac{\Delta p_x}{2\tau} = -\frac{h}{2\lambda\tau}, \tag{11.9}$$

which corresponds to a deceleration given by

$$\dot{v}_x = \frac{F_x}{m} \approx -\frac{h}{2m\lambda\tau}. \tag{11.10}$$

The factor of two in the denominator of eqn 11.9 arises from the fact that, at high laser intensities, the population of the upper and lower levels equalize at a value close to $N_0/2$, where N_0 is the total number of atoms. When the atom is in the excited state (step 2 in Fig. 11.3), it can be triggered to undergo stimulated emission by other impinging laser photons. The stimulated photon is emitted in the same direction as the incident photon, and the photon recoil exactly cancels the momentum change due to absorption. This reduces the force in proportion to the number of atoms in the excited state. (See discussion of eqn 11.20.)

The number of cycles required to halt the atoms is given by:

$$\mathsf{N}_{\mathrm{stop}} = \frac{mu_x}{|\Delta p_x|} = \frac{mu_x\lambda}{h}, \tag{11.11}$$

where u_x is the initial velocity. (It is, of course, impossible to completely stop the atoms, and we are simply calculating here the conditions required to reduce the velocity to its minimum value, which is assumed to be very much less than u_x.) The minimum time for the laser to slow the atoms is given by:

$$t_{\mathrm{min}} \approx \mathsf{N}_{\mathrm{stop}} \times 2\tau = \frac{2mu_x\lambda\tau}{h}. \tag{11.12}$$

The distance travelled by the atoms in this time is given by:

$$d_{\mathrm{min}} = -\frac{u_x^2}{2\dot{v}_x} \approx \frac{m\lambda\tau u_x^2}{h}. \tag{11.13}$$

Typical values of the quantities calculated in eqns 11.9–11.13 are given in Example 11.1.

The Doppler cooling process stops working when the detuning required for cooling becomes comparable to the natural width $\Delta\nu$ of the transition line. In these conditions, the thermal energy of the atom will be roughly equal to $h\Delta\nu$, and therefore the minimum temperature will be given by

$$k_B T_{\mathrm{min}} \sim h\Delta\nu. \tag{11.14}$$

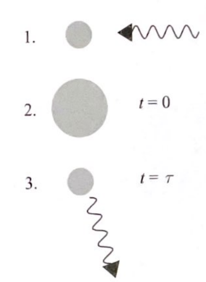

Fig. 11.3 An absorption–emission cycle. (1) A laser photon impinges on the atom. (2) The atom absorbs the photon and goes into an excited state. (3) After an average time equal to the radiative lifetime τ, the atom re-emits a photon in a random direction by spontaneous emission.

On recalling the relationship between the natural line width of the transition and its radiative lifetime (cf. eqn 4.30), we then find:

$$T_{\min} \sim \frac{\hbar}{k_B \tau}. \qquad (11.15)$$

This shows that the minimum temperature that can be achieved by the Doppler cooling mechanism is limited by the lifetime of the transition. The rigorous result for the minimum temperature given in eqn 11.37 differs only by a factor of two from eqn. 11.15.

The experimental implementation of the laser cooling process is complicated by the fact that the value of δ required to produce efficient cooling changes as the atoms slow down. In Section 11.2.5 we shall see how a carefully designed magnet can be used to maintain the cooling condition during the slowing process. Alternatively, the frequency of a tunable laser can be scanned in a programmed way to compensate for the deceleration of the atoms. This latter technique is called **chirp cooling** in analogy to the chirping sound made when an audio frequency is rapidly increased, for example, in bird song. Typical tunable lasers used for chirp cooling include dye lasers, Ti : sapphire lasers, and semiconductor diode lasers. In the first two cases the frequency is tuned by scanning an intracavity Fabry–Perot etalon, while the diode lasers can be tuned by varying the temperature of the semiconductor chip.

Example 11.1 A collimated beam of sodium atoms is emitted in the $+x$-direction from an oven at 600 °C and interacts with a counter-propagating laser beam tuned to near resonance with the D_2 line at 589 nm, which has a radiative lifetime of 16 ns. Estimate:

(a) the r.m.s. velocity and most probable velocity of the atoms in the beam as they leave the oven;

(b) the initial detuning required for efficient laser cooling;

(c) the frictional force exerted on the atoms by the laser and the deceleration it produces;

(d) the number of absorption–emission cycles required to bring the atoms to a near halt;

(e) the distance travelled during the laser cooling process.

Solution

(a) The velocity distribution of the atoms within the oven is given by the Maxwell–Boltzmann distribution (eqn 4.33), for which the r.m.s velocity is given by eqn 11.4 and the most probable velocity is given by:

d 11.16-
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$$v_{\mathrm{mp}} = \sqrt{\frac{2k_B T}{m}}. \qquad (11.16)$$

However, the velocity distribution within a collimated atomic beam is different because the atomic flux is proportional to the velocity of the atoms. The r.m.s velocity in the beam is given by:

$$v_{\mathrm{rms}}^{\mathrm{beam}} = \sqrt{\frac{4k_B T}{m}}, \qquad (11.17)$$

while the most probable velocity is given by:

$$v_{\mathrm{mp}}^{\mathrm{beam}} = \sqrt{\frac{3k_B T}{m}}. \qquad (11.18)$$

With $T = 873$ K and $m = 23\, m_H$, we find $v_{\mathrm{rms}}^{\mathrm{beam}} = 1120\ \mathrm{m\,s^{-1}}$ and $v_{\mathrm{mp}}^{\mathrm{beam}} = 970\ \mathrm{m\,s^{-1}}$

(b) The laser detuning required to cool an atom with velocity v_x is given by eqn 11.7. To instigate efficient cooling we need to tune the laser to the appropriate frequency for the most probable velocity in the beam (i.e. 970 $\mathrm{m\,s^{-1}}$). This gives $\delta = -1.6$ GHz.

(c) The frictional force is given by eqn 11.9 and the deceleration by eqn 11.10. With $\lambda = 589$ nm and $\tau = 16$ ns, we find $F_x \approx -3.5 \times 10^{-20}$ N and $\dot{v}_x \approx -9.1 \times 10^5\ \mathrm{ms^{-2}}$.

(d) The number of cycles is given by eqn 11.11 with u_x set by the most probable initial velocity within the beam, namely 970 $\mathrm{m\,s^{-1}}$ (cf. part(a)). This gives $N_{\mathrm{stop}} = 3.3 \times 10^4$.

(e) The distance travelled is given by eqn 11.13. On setting $u_x = 970\ \mathrm{m\,s^{-1}}$, we find $d_{\min} \approx 51$ cm.

11.2.2 Optical molasses

The results derived in eqns 11.9–11.13 should be considered only as order of magnitude estimations because a number of important processes have been neglected. In this subsection we shall reconsider the cooling process in more detail and derive a value for the limiting temperature that can be achieved.

Let us first consider a laser beam of optical intensity I and detuning $\Delta \equiv 2\pi\delta$ in angular frequency units interacting with an atom of velocity $+v_x$ with respect to the laser. As in eqn 11.9, the frictional force F_x is equal to the momentum change per absorption–emission cycle multiplied by the net rate of such cycles:

$$F_x = -\hbar k \times R(I, \Delta), \qquad (11.19)$$

where $k \equiv 2\pi/\lambda$ is the photon wave vector, and $R(I, \Delta)$ is the *net* absorption rate. $R(I, \Delta)$ is equal to the absorption rate minus the stimulated emission rate, and is given by:

$$R(I, \Delta) = \frac{\gamma}{2}\left(\frac{I/I_s}{1 + I/I_s + [2(\Delta + kv_x)/\gamma]^2}\right), \qquad (11.20)$$

where $\gamma \equiv 1/\tau$ is the natural linewidth in angular frequency units (cf. eqn 4.30), and I_s is the **saturation intensity** of the transition. It is apparent that at very high intensities the net absorption rate limits at $\gamma/2$, which, with $\gamma \equiv 1/\tau$, explains the factor of two in the denominator of eqn 11.9

The analysis of the cooling pro-
cess given here roughly follows th-
paper entitled 'Optical Molasses' b-
P. D. Lett *et al.* in *J. Opt. Soc. Am*
B **6**, 2084 (1989). The derivation
eqn 11.20 may be found, for exampl-
in Foot (2005) or Shen (1984).

We can understand the general form of eqn 11.20 by first noting that at low intensities, we can neglect the term in I in the denominator to find that the absorption rate is linearly proportional to the laser intensity as expected. In this low-intensity limit, the frequency dependence is then simply given by a Lorentzian shape (cf. eqn 4.29) with the frequency shift $(\Delta + kv_x)$ equal to the Doppler-shifted laser detuning in the rest frame of the atom. The need for the term in (I/I_s) in the denominator becomes most clearly apparent from considering the behaviour at high intensities at the line centre (i.e. with $(\Delta + kv_x) = 0$). An analysis of the transition rates using the Einstein coefficients quickly establishes the functional form of eqn 11.20. (See Exercise 11.4.)

The arrangement with a single laser beam shown in Fig. 11.1 works well when the laser detuning Δ is much larger than the linewidth. However, as the atoms cool down, it will eventually be the case that the value of Δ required for cooling becomes comparable to the linewidth γ. In these conditions, the atoms moving in the $-x$-direction will experience an accelerating force, and will reheat those moving in the $+x$-direction by collisions. To achieve very low temperatures we therefore need two laser beams as shown in Fig. 11.4. In this situation, the atom experiences separate forces from each laser, giving a net force of:

11.4 Two counter-propagating are used to produce the optical ses cooling effect.

$$F_x = F_+ + F_-, \tag{11.21}$$

where $F_\pm$ refers to the force from the laser beam propagating in the $\pm x$ direction, respectively. When the laser is tuned to the cooling condition given in eqn 11.7, $F_- \gg F_+$ for atoms moving in the $+x$ direction at high temperatures where $k|v_x| \gg \gamma$, and vice versa for those moving in the opposite direction. The two-beam arrangement is therefore able to cool atoms moving in both directions. However, when the atoms get very cold, so that $|v_x|$ is small, we have to analyse the net force more carefully. In the low-temperature limit where $|kv_x| \ll \Delta$, and $|kv_x| \ll \gamma$, the resultant force is given by (see Exercise (11.5)):

$$F_x(I,\Delta) = \frac{8\hbar k^2 \Delta}{\gamma}\left(\frac{I/I_s}{[1 + I/I_s + (2\Delta/\gamma)^2]^2}\right) v_x. \tag{11.22}$$

Irrespective of the direction of v_x, the force is of the form:

$$F_x = -\alpha v_x, \tag{11.23}$$

where α is the **damping coefficient**, given by:

$$\alpha = -\frac{8\hbar k^2 \Delta}{\gamma}\left(\frac{I/I_s}{[1 + I/I_s + (2\Delta/\gamma)^2]^2}\right). \tag{11.24}$$

is the name given to the k syrup drained from raw ng the refining processes. In d States the word is also reacle', and it gives a good of how the Doppler cooling ike a viscous medium for the oms.

When Δ is negative, α is positive, and the motion of the atom is damped in both directions. For this reason, the arrangement with two counter-propagating beams is called the **optical molasses**. At low intensities the damping force is largest when $\Delta = -\gamma/\sqrt{12}$, but this is not the frequency at which the lowest temperature is achieved, as we shall show below.

The limit to the temperature that is achieved is set by balancing the cooling effect of the damping force with the heating effect associated with the repeated absorption and emission of photons. The cooling rate is given by:

$$\left(\frac{dE}{dt}\right)_{\text{cool}} = F_x v_x = -\alpha v_x^2, \tag{11.25}$$

while the heating rate is given by (see eqn 11.34 below):

$$\left(\frac{dE}{dt}\right)_{\text{heat}} = \frac{D_p}{m}, \tag{11.26}$$

where D_p is the **momentum diffusion constant** defined in eqn 11.33. On setting the total change of energy equal to zero, we find:

$$-\alpha v_x^2 + \frac{D_p}{m} = 0, \tag{11.27}$$

which implies:

$$v_x^2 = \frac{D_p}{m\alpha}. \tag{11.28}$$

The temperature is then given by eqn 11.2 as:

$$\frac{1}{2}k_B T = \frac{1}{2}mv_x^2 = \frac{D_p}{2\alpha}. \tag{11.29}$$

We therefore obtain:

$$T = \frac{D_p}{\alpha k_B}. \tag{11.30}$$

It thus emerges that the limiting temperature is achieved by minimizing the ratio of D_p to α.

The momentum diffusion introduced into eqn 11.26 is associated with the fact that, even though the damping force reduces the average velocity to zero, the mean squared velocity is not zero. During each absorption–emission cycle, the atom absorbs and emits a photon with momentum $\hbar k$. An atom with zero mean velocity is equally likely to absorb a photon from the positive or negative travelling laser beams, and also to emit in either direction. The atom therefore performs a **random walk** in the x-direction, jolting backwards and forwards each time a photon is absorbed or emitted. If the random walk has N steps, where N is a large number, then the average value of the momentum will be zero, but the average of the square will be given by:

The momentum diffusion due to the random walk is similar to the diffusion of molecules in Brownian motion. The linear increase of $\langle p_x^2\rangle$ with the number of steps is reminiscent of a Poissonian process: see eqn A.10 in Appendix A. The extra factor of two in eqn 11.31 arises from the one-dimensional nature of the problem.

$$\langle p_x^2\rangle = 2N(\hbar k)^2. \tag{11.31}$$

On counting the interactions with both laser beams, we then have $N = 2Rt$ in time t, so that:

$$\frac{d\langle p_x^2\rangle}{dt} = 4\hbar^2 k^2 R. \tag{11.32}$$

The momentum diffusion coefficient D_p is defined by:

$$D_p = \frac{1}{2}\frac{\mathrm{d}\langle p_x^2\rangle}{\mathrm{d}t}. \tag{11.33}$$

The heating rate is then given by:

$$\left(\frac{\mathrm{d}E}{\mathrm{d}t}\right)_{\mathrm{heat}} = \frac{1}{2m}\frac{\mathrm{d}\langle p_x^2\rangle}{\mathrm{d}t} = \frac{D_p}{m} = \frac{2\hbar^2k^2R}{m}. \tag{11.34}$$

On substituting for R from eqn 11.20 in the limit where $|kv_x| \ll |\Delta|$, we then find:

$$D_p = \hbar^2k^2\gamma\left(\frac{I/I_s}{1 + I/I_s + (2\Delta/\gamma)^2}\right). \tag{11.35}$$

We finally substitute eqns 11.24 and 11.35 into eqn 11.30 to obtain:

$$T = -\frac{\hbar\gamma}{8k_{\mathrm{B}}}\frac{(1 + I/I_s + 4\Delta^2/\gamma^2)}{\Delta/\gamma}. \tag{11.36}$$

In the low-intensity limit with $I \ll I_s$, the minimum temperature is given by:

$$T_{\min} = \frac{\hbar\gamma}{2k_{\mathrm{B}}} \equiv \frac{\hbar}{2k_{\mathrm{B}}\tau}, \tag{11.37}$$

at $\Delta = -\gamma/2$. The temperature limit given in eqn 11.37 is called the **Doppler limit**. Through eqn 11.2, it corresponds to a minimum thermal r.m.s. velocity of

$$v_x^{\min} = \sqrt{\hbar/2m\tau}. \tag{11.38}$$

The Doppler temperature in eqn 11.37 puts a fundamental limit to the temperature that can be achieved by the Doppler cooling process in its simplest form.

Example 11.2 Calculate the lowest temperature that can be achieved by the Doppler cooling method using the D_2 line of sodium at 589 nm, which has a radiative lifetime of 16 ns. Calculate also the average velocity of the atoms at this temperature.

Solution

The minimum temperature for Doppler cooling is given by the Doppler limit temperature given in eqn 11.37. With $\tau = 16$ ns, this gives $T_{\min} = 240\ \mu$K. The corresponding minimum thermal velocity from eqn 11.38 with $m = 23m_{\mathrm{H}}$ is $0.29\ \mathrm{m\,s^{-1}}$.

11.2.3 Sub-Doppler cooling

Equation 11.37 appears to set a fundamental limit to the temperatures that can be achieved by laser cooling. However, careful experiments carried out in the 1980s led to the surprising conclusion that the temperatures that were being achieved could be *lower* than the Doppler limit. It transpires that laser cooling is one of the rare examples of an

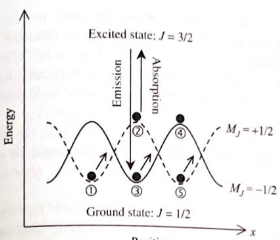

Fig. 11.5 Sisyphus cooling for a $J = 1/2 \to 3/2$ transition in an alkali atom. The atom is moving in the $+x$-direction, and interacts with two counter-propagating laser beams as in Fig. 11.4. The energies of the $M_J = \pm1/2$ sublevels of the $J = 1/2$ ground state vary sinusoidally with position in the interference pattern of the lasers. The laser frequency is tuned so that the atom can only make a transition to the excited state at the top of one of the potential hills. (Positions 2 and 4.) The atom in the excited state can re-emit to the same sublevel, or to the lower one. (Positions 3 and 5.) In the case of an atom following the path $1 \to 2 \to 3 \to 4 \to 5 \to \cdots$, the difference in the energy of the absorbed and emitted photons is taken from the total energy of the atom, leading to a cooling effect.

experiment that actually works better in the laboratory than the simple theory predicts.

The discrepancy can be explained by realizing that the Doppler cooling mechanism described in Sections 11.2.1 and 11.2.2 is too simplistic. The counter-propagating laser beams in an optical molasses experiment interfere with each other, and this leads to a new type of cooling mechanism called **Sisyphus cooling**.

The detailed mechanism of Sisyphus cooling is too complicated for our level of treatment, but the basic process can be understood with reference to Fig. 11.5. We consider an alkali atom in the $^2\mathrm{S}_{1/2}$ ground state moving in the $+x$-direction and making transitions to a $^2\mathrm{P}_{3/2}$ excited state under the influence of two counter-propagating resonant laser beams as shown in Fig. 11.4. The interference pattern of the lasers leads to a small periodic modulation of the energies of the ground state levels through the AC Stark effect. The light-induced shifts of the $M_J = \pm1/2$ magnetic sublevels differ in phase by 180° as shown in Fig. 11.5. As long as the atom stays in the same magnetic sublevel, it moves up and down potential hills, continually converting kinetic to potential energy and back again, but without change of the total energy. (Route $1 \to 2 \to 5 \to \cdots$ in Fig. 11.5.) However, by careful tuning of the laser, we can arrange that some of the atoms follow the route $1 \to 2 \to 3 \to 4 \to 5 \to \cdots$ in Fig. 11.5. In this case, the atoms are constantly losing energy, because they have to climb to the top of the potential hill, and then drop to to the valley again, just like Sisyphus.

Sisyphus cooling is named after the character in Greek mythology who was condemned to roll a stone up a hill forever, only for it to roll down again every time he got near the top. The mechanism of Sisyphus cooling is explained in more detail in Foot (2005). See also Cohen-Tannoudji and Phillips (1990). A brief discussion of the AC Stark effect may be found in Section 9.5.3.