Statistical mechanics and kinetic theory of the 2D Euler and stochastic Navier-Stokes equations

F. BOUCHET – ENS-Lyon and CNRS (Lyon)

"Kinetic Theory and Fluid Mechanics" – March 2012 – Lyon

Collaborators

- Equilibrium statistical mechanics of ocean jets and vortices: A. Venaille (PHD student, now in post doc in Princeton)
- Equilibrium statistical mechanics of the Great Red Spot of Jupiter: J. Sommeria (LEGI-Coriolis, Grenoble)
- Random changes of flow topology in the 2D Navier-Stokes equations: E. Simonnet (INLN-Nice) (ANR Statocean)
- Asymptotic stability and inviscid damping of the 2D-Euler equations: H. Morita (Tokyo university) (ANR Statflow)

Collaborators

- Invariant measures of the 2D Euler and Vlasov equations: M. Corvellec (PHD student, INLN Nice, CNLS Los Alamos and ENS-Lyon)
- Kinetic theory of plasma and self-gravitating systems with stochastic forces: C. Nardini, T. Dauxois and S. Ruffo (ENS-Lyon and Florence)
- Instantons and large deviations for the 2D Navier-Stokes equations: J. Laurie (Post-doc ANR Statocean)
- Large deviations for systems with connected attractors: H. Touchette (Queen Mary Univ, London)

Outline

Introduction

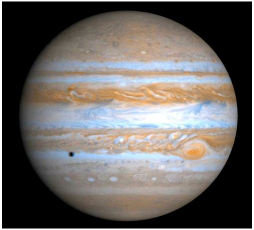
Self Organization of Geophysical Flows

The Great Red Spot of Jupiter

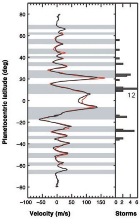
- Turbulent geophysical flows self-organize
- By contrast with usual geophysical flows

Towards a Kinetic Theory of Geophysical Flows

A kinetic theory of zonal jets on Earth and Jovian planet atmospheres



Jupiter atmosphere

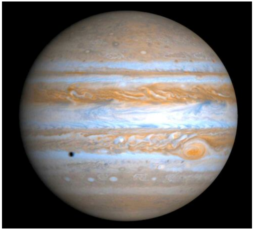


Jupiter Zonal wind (Voyager and Cassini, from Porco et al 2003)

How far are we to reproduce such phenomena in numerical simulations ? How to theoretically predict such velocity profile ?

Irreversibility : White Oval Mergers

A macroscopic irreversible for a microscopic time-reversible dynamics



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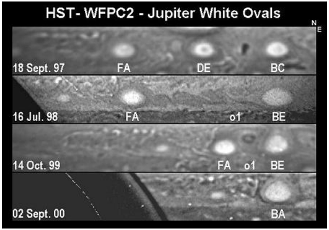

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The irreversible mergers of white ovals need no dissipative mechanisms

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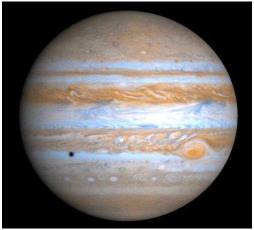

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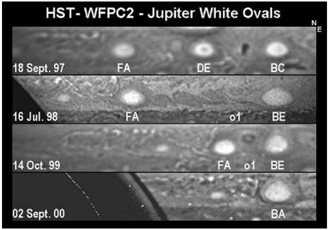

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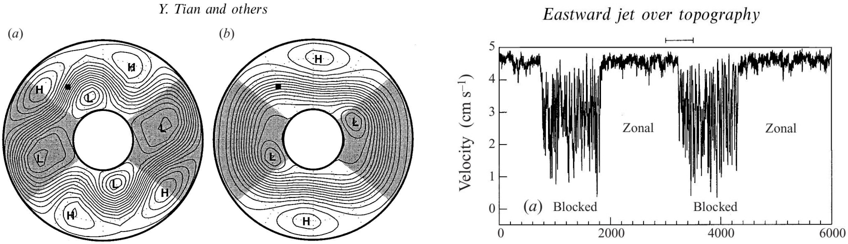
Phase Transitions in Rotating Tank Experiments

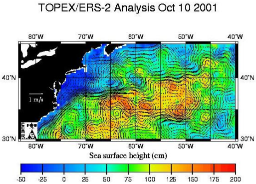
The rotation as an ordering field (Quasi Geostrophic dynamics)

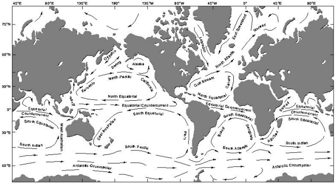
Transitions between blocked and zonal states



Y. Tian and col, J. Fluid. Mech. (2001) (groups of H. Swinney and M. Ghil)

The Physical Phenomena





Theoretical ideas:

- Self organization processes. Large number of degrees of freedom (turbulence).
- This has to be explained using statistical physics !!!

Mainly non-equilibrium statistical mechanics. We have to work out new theoretical concepts with such phenomena in mind

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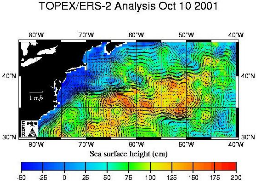

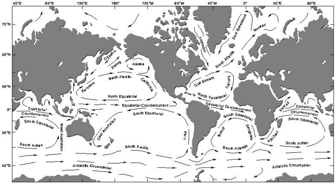

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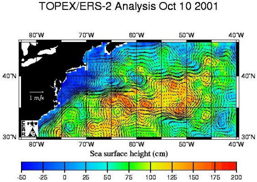

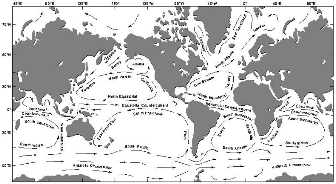

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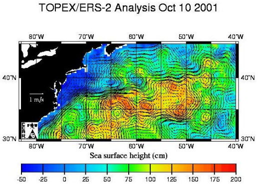

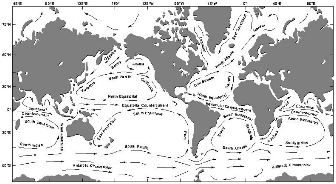

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The 2D Stochastic-Navier-Stokes (SNS) Equations

- The simplest model for two dimensional turbulence
- Navier Stokes equation with random forces

$$\frac{\partial \omega}{\partial t} + \mathbf{v}.\nabla \omega = \nu \Delta \omega - \alpha \omega + \sqrt{\sigma} f_s$$

where $\omega = (\nabla \wedge \mathbf{v}).\mathbf{e}_z$ is the vorticity, f_s is a random force, α is the Rayleigh friction coefficient.

- An academic model with experimental realizations (Sommeria and Tabeling experiments, rotating tanks, magnetic flows, and so on). Analogies with geophysical flows (Quasi Geostrophic and Shallow Water layer models)

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Soap film experiments

H. Kellay

Experiments in Thin Stratified Layers

J. Paret and P. Tabeling 3127

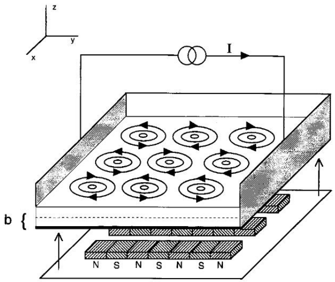

FIG. 1. The experimental set-up.

P. Tabeling

Electron Plasma Experiments

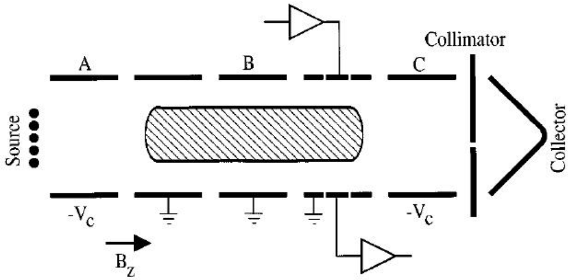

FIG. 1. Schematic of the cylindrical confinement geometry.

C. F. Driscoll

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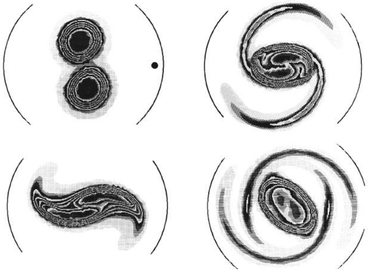

FIG. 8. $n(r,\theta,t)$ density (vorticity) plots of two symmetric vortices unstable to merger, at times 0, 16, 41 and 76 μs. The density between solid contours is 2.9×10^5 cm^{-3}. Here, the vortices have radii $r_{v2}=r_{v1}=0.25$ and radial positions $r_2=r_1=0.30$.

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Statistical Mechanics for 2D and Geophysical Flows

- Statistical hydrodynamics ? Very complex problems.
- Example: Intermittency in 3D turbulence ; phenomenological approach, simplified models (Kraichnan).
- It may be much simpler for 2D or geophysical flows: conservative systems.
- Statistical equilibrium: A very old idea, some famous contributions Onsager (1949), Joyce and Montgomery (1970), Caglioti Marchioro Pulvirenti Lions (1990), Robert (1990), Miller (1990), Robert et Sommeria (1991), Eyink and Spohn (1994), Kiessling and Lebowitz (1994), Bodineau and Guionnet (1999), Boucher, Ellis and Turkington (1999)

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Point Vortices and Mean-Field Behavior of N Particle Dynamics

Recent mathematical developments

- T. Gallay (2011) Relation between the dynamics of point vortices and the 2D Navier-Stokes Eq. in the limit of very large Reynolds numbers
- E. Caglioti, and F. Rousset, J. Stat. Phys. (2007), Large time stability of quasi-stationary states (QSS) (for QSS, see for example Y. Yamaguchi and co, Physica A, (2004))
- M. Hauray, Math. Models Methods Appl. Sci. (2007), M. Hauray, and P.E. Jabin, J. Stat. Mech. (2011)
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The Main Issues for Physicists

- What makes the dynamics of two-dimensional flows so peculiar?
- Why does the large scales of two dimensional flows self-organize?
- Can we predict the statistics of the large scales of two-dimensional flows?
- Are the Great Red Spot, the map of ocean currents, and the average velocity of Earth's and Jupiter's troposphere, statistical equilibria?
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The Main Mathematical Challenges

- Can we define microcanonical measures for the 2D Euler equations?
- Are those microcanonical measures invariant measures of the 2D Euler equation?
- Can we prove irreversible relaxation of the 2D Euler towards those equilibria?
- Can we treat close to equilibria dynamics in the framework of kinetic theory?
- What are the properties of the kinetic equations?
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- Conservative dynamics - Hamiltonian (non canonical) and time reversible
- Invariants:

Energy: $E[\omega] = \frac{1}{2}\int_{\mathscr{D}} d^2x\, \mathbf{v}^2 == -\frac{1}{2}\int_{\mathscr{D}} d^2x\, \omega\psi$

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Consequences of Multiple Invariants of 2D flows I)

Multiple steady solutions – Multiple stable steady solutions

- Any non degenerate minimum of a conserved quantity is a stable steady solution (think to mechanics and energy).
- Energy-Casimir functionals (Arnold 1966)
- Multiple invariants imply degeneracy of steady solutions to the 2D Euler Eq.:

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PV-Psi Relation for Jupiter's Great Red Spot

From Dowling and Ingersoll empirical analysis (1989)

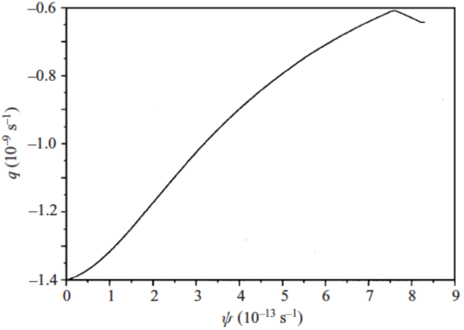
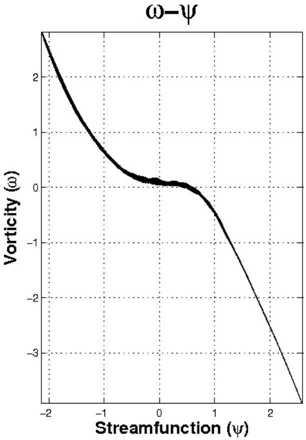


FIGURE 12. Potential vorticity q^*_{SW} versus stream function Ψ^*_{SW} from the determination of $q^*_{SW}(B^*_e)$ by Dowling & Ingersoll (1989) (their table 1) in the Great Red Spot (for $R = 2200$ km $q: 10^{-9}\ \mathrm{s}^{-1}$ and $\Psi: 10^{13}\ \mathrm{m\,s}^{-1}$). These SW potential vorticity and stream function are proportional to the QG ones in the QG limit. We observe that this function is in reasonable agreement with the tanh-like relation of the two-PV-level Gibbs states.

PV-Psi Relation for the 2D Navier-Stokes equations with stochastic forces



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- Many steady solutions of 2D Euler equations are attractors. Degeneracy: what does select f?
- f can be predicted using classical equilibrium statistical mechanics, or kinetic theory

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- Multiple invariants imply degeneracy of steady solutions to the 2D Euler Eq.:

$$\omega = \Delta\psi = f(\psi) \Rightarrow \mathbf{v}.\nabla\omega = (\nabla\psi \times \nabla\omega).\mathbf{e}_z = 0$$

- Many steady solutions of 2D Euler equations are attractors. Degeneracy: what does select f?
- f can be predicted using classical equilibrium statistical mechanics, or kinetic theory

Consequences of Multiple Invariants of 2D flows II)

- Fjorftof inequalities and energy dissipation
- Energy fluxes towards largest scales
- Equilibrium statistical mechanics gives non-trivial predictions (no Jeans paradox)

Other Models with Similar Properties

Hamiltonian PDE with degenerate Poisson brackets

- The main properties of the 2D Euler are related to its Hamiltonian structure : degenerate Poisson bracket with Casimir conservation laws
- Many models share the same structure
- Vlasov equations, Quasi-Geostrophic or Charney–Hasegawa–Mima models, Shallow-Water equations, Axisymmetric Euler equations, Primitive equations, 2D MHD, and so on.

Outline