Statistical mechanics and kinetic theory of the 2D Euler and stochastic Navier-Stokes equations

"Kinetic Theory and Fluid Mechanics" – March 2012

F. BOUCHET – ENS-Lyon and CNRS

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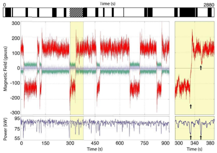
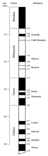
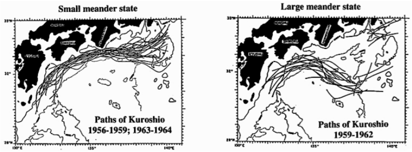
Outline

Random Transitions in Turbulence

A huge number of turbulent flows have a bistable or multistable behavior







VKS experiment   Earth   Kuroshio current

Other examples :

- Turbulent convection, Van Karman and Couette turbulence.
- Multistability in the atmosphere, weather regimes, and so on.

The 2D Stochastic-Navier-Stokes (SNS) Equations

- The simplest model for two dimensional turbulence.
- Navier Stokes equation with a random force

$$\frac{\partial \omega}{\partial t} + \mathbf{u}.\nabla \omega = \nu \Delta \omega - \alpha \omega + \sqrt{\sigma} f_s \quad (1)$$

where $\omega = (\nabla \wedge \mathbf{u}).\mathbf{e}_z$ is the vorticity, f_s is a random force, α is the Rayleigh friction coefficient.

- An academic model with experimental realizations (Sommeria, Tabeling, Ecke experiments, rotating tanks, magnetic flows, soap films, and so on). Analogies with geophysical flows.

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Freidlin–Wentzell theory or Onsager Machlup Formalism

Classical Large Deviations for SDE or SPDE

$$dx = f(x)dt + \sqrt{\nu}dW$$

- Hypothesis: the deterministic dynamics has isolated attractors. Large deviation results:

$$P(X) \sim \exp\left(-\frac{V(X)}{\nu}\right) \text{ to mean } \lim_{\nu\to 0} \nu \log P = -V$$

$$\text{with } V(X) = \inf_{t>0} \inf_{\{x(t)|x(0)\in 0 \text{ and } x(t)=X\}} L[x]$$

$$\text{and } L[x] = \frac{1}{2}\int_0^t ds\,(\dot{x} - f(x))^2$$

- Because of the connected attractors, the 2D Navier–Stokes Eq. do not fulfill this hypothesis

Non-Equilibrium Phase Transitions and Large Deviations in the 2D Stochastic Navier-Stokes Equations

1. Non-equilibrium phase transitions
   - Experiments
   - Random changes of flow topology in the 2D stochastic Navier–Stokes Eq. (F. B., E. Simonnet and H. Morita)
2. Large deviations and path integrals
   - Introduction to path integrals and large deviations
   - 2D turbulence attractors
   - Instantons for the 2D stochastic Navier–Stokes equations (F.B. and J. Laurie)
3. Large deviations in dynamical systems with connected attractors
   - A toy model with connected attractors
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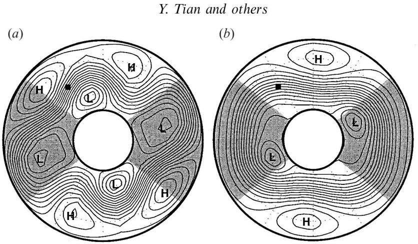
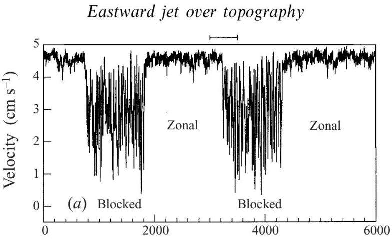
Non-Equilibrium Phase Transitions in Real Flows

Rotating tank experiments (Quasi Geostrophic dynamics)

Transitions between blocked and zonal states:

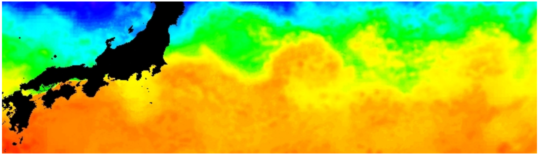




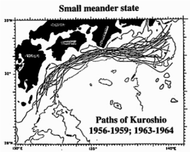
Y. Tian and col, J. Fluid. Mech. (2001) (groups of H. Swinney and M. Ghil)

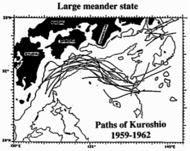
Non-Equilibrium Phase Transitions in Real Flows

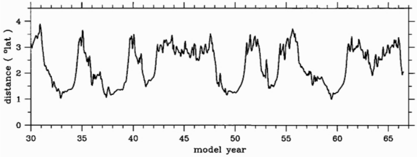
The Kuroshio current bistability (two layer Quasi-Geostrophic or primitive equations dynamics)



See surface temperature of the pacific ocean, east of Japan



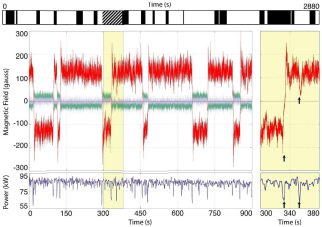




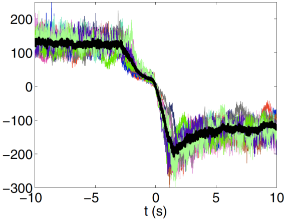
Kuroshio paths and bistability timeseries

Random Transitions in Turbulence Problems

Magnetic Field Reversal (Turbulent Dynamo, MHD Dynamics)



Magnetic field timeseries



Zoom on reversal paths

(VKS experiment)

In turbulent flows, transitions from one attractor to another often occur through a predictable path

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2D Stochastic Navier-Stokes Eq. and 2D Euler Steady States

$$\frac{\partial \omega}{\partial t} + \mathbf{u}.\nabla\omega = \nu\Delta\omega - \alpha\omega + \sqrt{2\alpha} f_s \tag{2}$$

- Time scale separation: magenta terms are small.
- At first order, the dynamics is nearly a 2D Euler dynamics. The flow self organizes and converges towards steady solutions of the Euler Eq.:

$$\mathbf{u}.\nabla\omega = 0 \text{ or equivalently } \omega = f(\psi)$$

  where the Stream Function ψ is given by: $\mathbf{u} = \mathbf{e}_z \times \nabla\psi$.
- Steady states of the Euler equation will play a crucial role. Degeneracy : what does select f ?

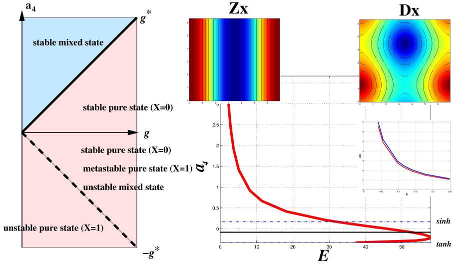
Steady States of Euler Eq. as Maxima of Variational Problems

Energy-Casimir Variational Problems

$$S(E) = \max_{\omega} \left\{ \int_{\mathscr{D}} d\mathbf{r}\, s(\omega) \,\middle|\, \frac{1}{2}\int_{\mathscr{D}} d\mathbf{r}\, \frac{\mathbf{v}^2}{2} = E \right\}.$$

- Numerical results : Z. Yin, D. C. Montgomery, and H. J. H. Clercx, Phys. Fluids (2003).
- Maxima: $\omega = \Delta\psi = \left(s^{'}\right)^{-1}(\beta\psi)$ (stable steady states of the Euler Eq.).
- In the following, normal form analysis with $s(\omega) = -\frac{\omega^2}{2} + a_4\frac{\omega^4}{4} + \dots$
- Geometry parameter $g = E(\lambda_1 - \lambda_2) \propto (L_x - L_y)$.

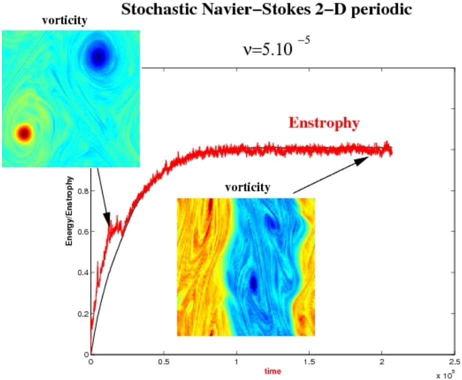
Steady States for the 2D-Euler Eq. (doubly periodic)



Bifurcation analysis : degeneracy removal, either by the domain geometry (g) or by the nonlinearity of the vorticity-stream function relation (f, parameter a_4).

Derivation: normal form for an Energy-Casimir variational problem.
A general degeneracy removal mechanism.

Numerical Simulation of the 2D Stochastic NS Eq.



Very long relaxation times. 10^5 turnover times.

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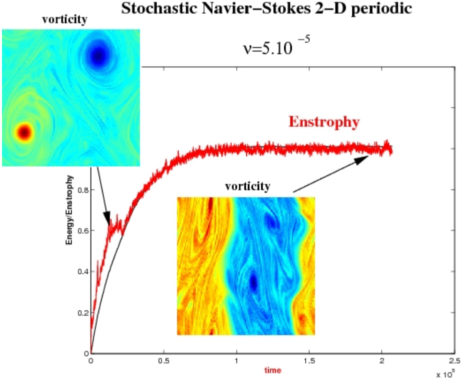

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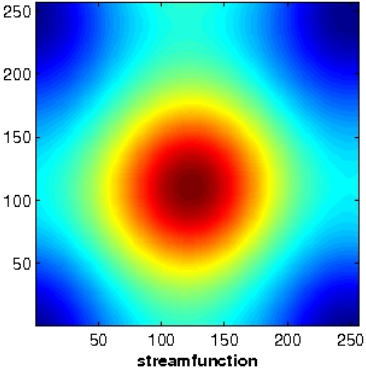
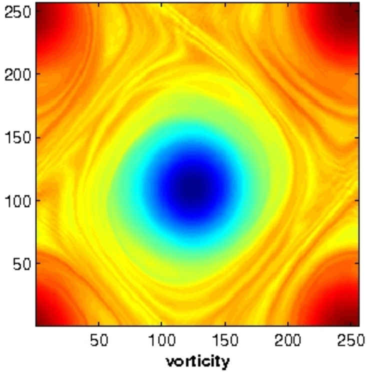
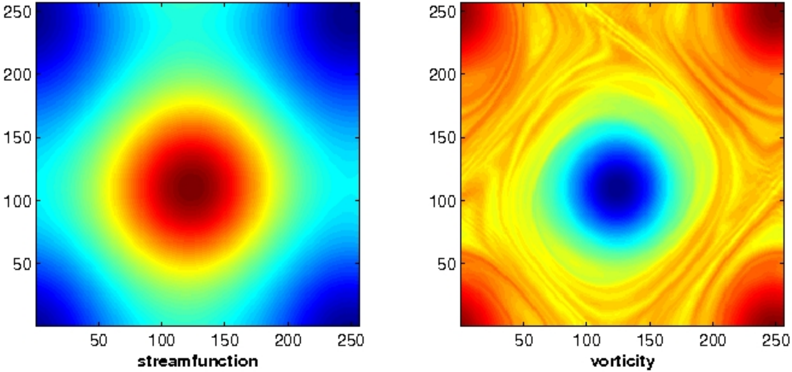
Out of Equilibrium Stationary States: Dipoles





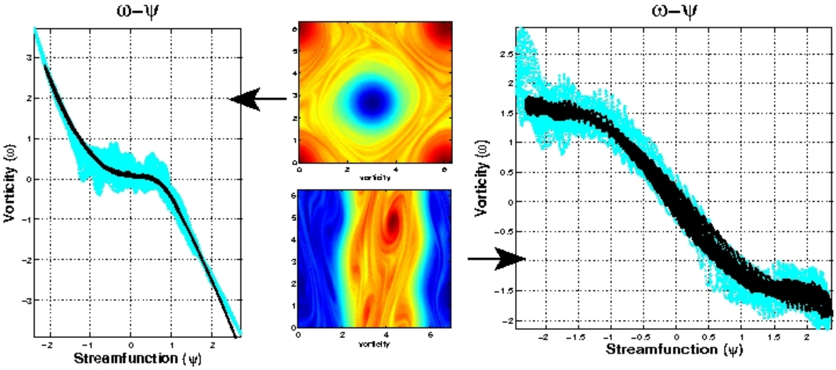
Are we close to some steady states of the Euler Eq.?

Out of Equilibrium Stationary States: Dipoles



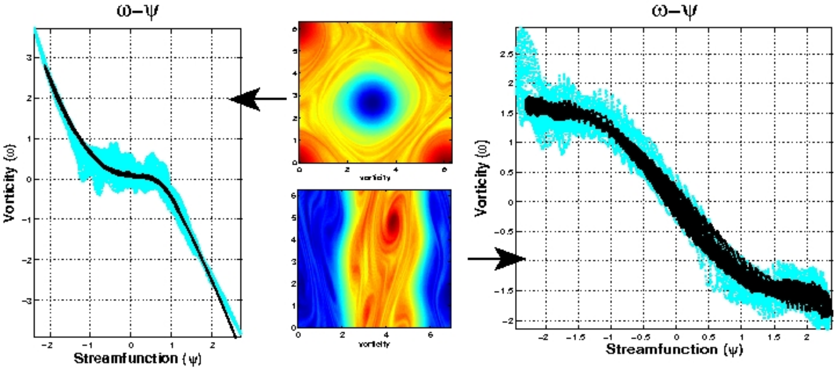
Are we close to some steady states of the Euler Eq.?

Vorticity-Streamfunction Relation



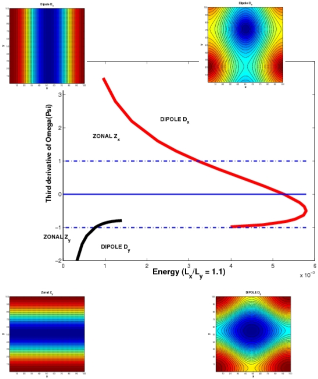
Conclusion: we are close to steady states of the Euler Eq.

Vorticity-Streamfunction Relation



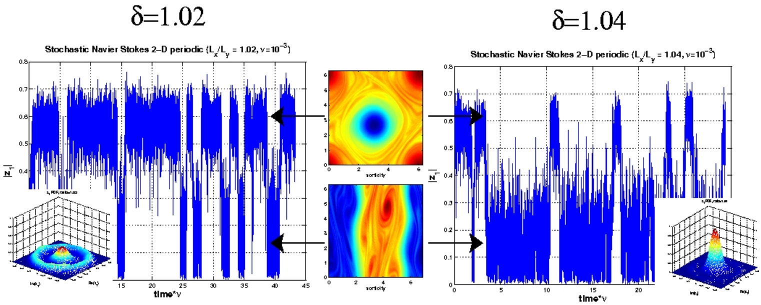
Conclusion: we are close to steady states of the Euler Eq.

Steady States for the 2D-Euler Eq. (doubly periodic)



A second order phase transition.

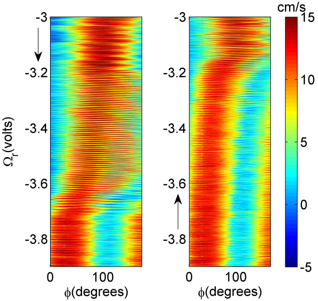
Non-Equilibrium Phase Transition
The time series and PDF of the Order Parameter



Order parameter : $z_1 = \int dxdy \exp(iy)\omega(x,y)$.

For unidirectional flows $|z_1| \simeq 0$, for dipoles $|z_1| \simeq 0.6 - 0.7$

Bistability in Rotating Tank Experiments
M. Mathur, J. Sommeria (LEGI)



Bistability (hysteresis) in rotating tank experiments

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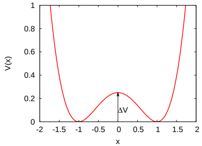
Path Integrals And Large Deviations In Non-Equilibrium Statistical Mechanics

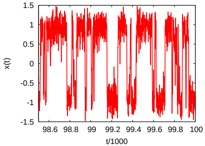
- Aim: Entropy and free energy are extremely useful in equilibrium statistical mechanics: they encode all the statistics of the system. How to compute similar quantity for out of equilibrium systems?
- Answer: Large deviations for ensembles of dynamical paths = out of equilibrium and dynamical free energies. How to compute these?

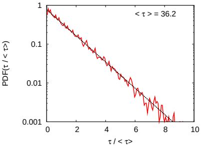
Kramer's Problem: a Pedagogical Example for Bistability

Historical example: Computation by Kramer of the Arrhenius law for a bistable mechanical system with stochastic noise

$$\frac{dx}{dt} = -\frac{dV}{dx}(x) + \sqrt{2D}\eta(t) \quad \text{Rate}: \; \lambda = A\exp\left(-\frac{\Delta V}{RT}\right) \; \text{with } RT \propto 2D$$







The problem was solved by Kramers (30'). Modern approach: path integral formulation (instanton theory, physicists) or large deviation theory (Freidlin-Wentzell, mathematicians)

Path Integrals for ODE – Onsager Machlup (50')

$$\frac{dx}{dt} = f(x) + \sqrt{2D}\eta(x,t)$$

- Path integral representation of transition probabilities:

$$P(x_0, x_T, T) = P(x = x_0, t = 0; x = x_T, t = T) = \int_{x(0)=x_0}^{x(T)=x_T} \mathscr{D}[x] \exp\left(-\frac{\mathscr{S}[T,x]}{2D}\right)$$

$$\text{with } \mathscr{S}[T,x] = \frac{1}{2}\int_0^T \mathrm{d}t \left\{[\dot{x} - f(x)]^2 - 2Df'(x)\right\}$$

- Instanton: the most probable path with fixed boundary conditions

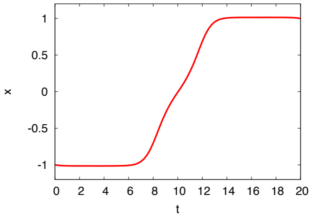
$$S(T, x_0, x_T) = \min_{x(t)} \{\mathscr{S}[T,x] \,|\, x(0) = x_0 \text{ and } x(T) = x_T\}$$

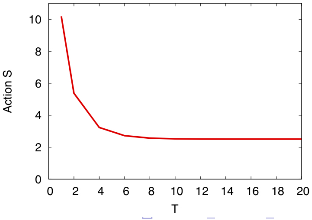
- Saddle point approximation (WKB) gives large deviations results:

$$\log P(x_0, x_T, T) \underset{D \to 0}{\sim} -\frac{S(T, x_0, x_T)}{2D}$$

What to Do with Path Integrals ?

- Solving the equations in the saddle point approximation using theory or numerical optimization (gradient methods)
- Transition rates and transition trajectories are given by minima and minimizers of the action
- It explains why most transition trajectories concentrate close to a single one (instanton trajectory)





Path Integrals And Turbulence Problems

- It has never been developed before
- Aim: compute extremely rare but essential events like transitions paths between attractors and transition rates
- This is unfeasible using conventional tools (direct numerical simulation)
- The main issue: Is it feasible for turbulence problems ? For which class of models (in terms of complexity) ?
- The route to follow:

1. Determine attractors
2. Study instantons between attractors

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$$\frac{\partial \omega}{\partial t}+\mathbf{u}.\nabla\omega=\nu\Delta\omega-\alpha\omega+\sqrt{2\alpha}f_s$$

- Time scale separation: magenta terms are small. turnover time $= 1 \ll 1/\alpha =$ forcing or dissipation time.
- At first order, the dynamics is nearly a 2D Euler dynamics.

$$\frac{\partial \omega}{\partial t}+\mathbf{u}.\nabla\omega=0$$

- where the Stream Function ψ is given by: $\mathbf{u}=\mathbf{e}_z\times\nabla\psi$.
- Steady states of the Euler equation will play a crucial role. Degeneracy : what does select f ?

The Set of Attractors of the 2D Euler Equations is Connected

A trivial consequence of the 2D Euler equation scale invariance

$$\frac{\partial \omega}{\partial t} + \mathbf{u}.\nabla \omega = 0$$

- If $\omega(\mathbf{x},t)$ is a solution of the 2D Euler Eq., then for any $\lambda > 0$, $\lambda \omega(\mathbf{x}, \lambda t)$ is also a solution (nonlinearity is homogeneous of degree 2)
- Then any steady solutions ω is connected to zero through the path $s\omega(st)$, $0 \leq s \leq 1$
- Any two steady states ω_0 and ω_1 are connected through a continuous path $\Omega(s)$, $0 \leq s \leq 1$ among the set of steady state
- The set of steady states of the 2D Euler equations is connected

Outline

The Action of the 2D Stochastic Navier-Stokes

$$\frac{\partial \omega}{\partial t}+\mathbf{u}.\nabla\omega=\nu\Delta\omega-\alpha\omega+\sqrt{2\alpha}f_s \text{ with } \left\langle f_s(\mathbf{x},t),f_s(\mathbf{x}',t')\right\rangle=C(\mathbf{x}-\mathbf{x}')\delta(t-t')$$

$$\mathscr{S}[T,x]=\frac{1}{2}\int_0^T \mathrm{d}t\int_{\mathscr{D}}\mathrm{dxdx}'\,p(\mathrm{x},\mathbf{t})C(\mathrm{x}-\mathrm{x}')p(\mathrm{x}',\mathbf{t})$$

$$\textit{with } p=\frac{\partial \omega}{\partial t}+\mathbf{u}.\nabla\omega+\alpha\omega-\nu\Delta\omega$$

- We can compute explicitly and study the stability of many instantons (parallel to parallel flows, spatial white noise, Laplacian eigenmodes, etc.)
- We can obtain explicit large deviation for rare flows (with exponential tails)

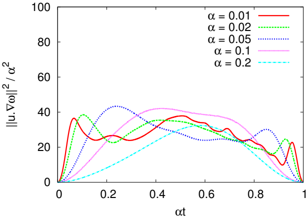
Algorithm for Action Minimization

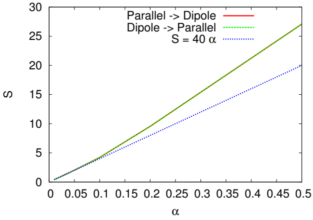
A variational approach

- We discretize action integral both in time and space (time using the central differencing scheme, and space using pseudo-spectral decomposition)
- Fix the initial and final states throughout the minimization
- Newton or quasi-Newton methods are prohibitively expensive to implement (Hessian)
- We implement a gradient method or steepest descent method:
- Then iteratively minimize an initial guess (simultaneously over space and time) in the direction of the anti-gradient:

$$\omega^{n+1} = \omega^n - c_n \frac{\delta S(\omega^n)}{\delta \omega^n}$$

Instanton from Dipole to Parallel Flows





Instanton are close to the set of attractors

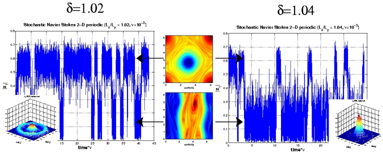
Scaling of S with α shows no large deviation

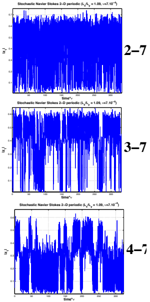
- $\log P(\omega_0, \omega_T, T) \underset{\alpha \to 0}{\sim} -\frac{S(T,\omega_0,\omega_T)}{2\alpha}$. This is not a large number. The stationary distribution or the transition probabilities are not concentrated. No large deviation. No bistability

Conclusions from Instanton Analysis

- We can numerically compute instantons for simple turbulent flows
- In the inertial limit, instantons follow the connected set of attractors. Towards an asymptotic theory?
- Definition: $C_{\mathbf{k}} = \int_{\mathscr{D}} \mathrm{d}\mathbf{x} \exp(i\mathbf{k}.\mathbf{x})\, C(\mathbf{x})$. If $C_{\mathbf{k}} = 0$ for some $\mathbf{k}$, the force is called degenerate, non-degenerate otherwise
- There is no large deviations for transitions between attractors for non-degenerate forces
- No bistability for non-degenerate forces

Degenerate Forces Prevent Bistability





Order parameter : $z_1 = \int dxdy \exp(iy)\omega(x,y)$.

For unidirectional flows $|z_1| \simeq 0$, for dipoles $|z_1| \simeq 0.6 - 0.7$.

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The Stochastic A-B Model

A toy model in order to illustrate averaging and large deviations in models with connected attractors

- A huge number of Hamiltonian PDEs have connected attractors

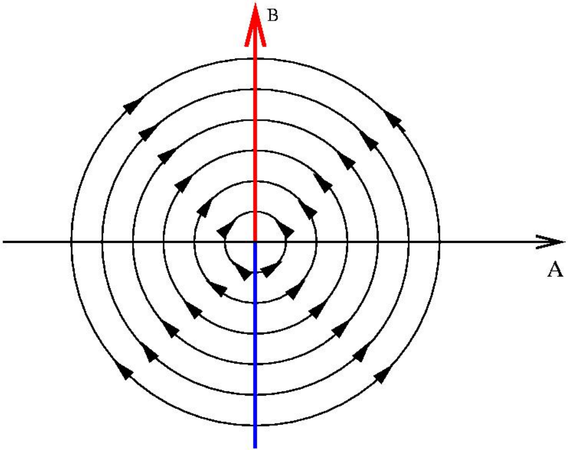
$$\left\{ \begin{array}{lcl} \frac{dA}{dt} & = & -AB \\ \frac{dB}{dt} & = & A^2 \end{array} \right.$$

- A quadratic nonlinearity. Conservation of energy

$$E = A^2 + B^2$$

- A connected set of steady states. For any B, $A = 0$ is an equilibrium

Phase Space of the A-B Model

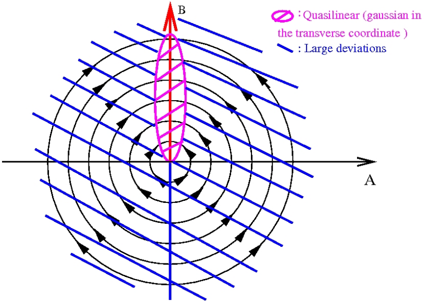


The Stochastic A-B Model

The limit of weak forces and dissipation

$$\begin{cases} dA &= \left(-AB-\nu A\right)dt+\sqrt{\nu}\sigma_1 dW_1 \\ dB &= \left(A^2-\nu B\right)dt+\sqrt{\nu}\sigma_2 dW_2 \end{cases}$$

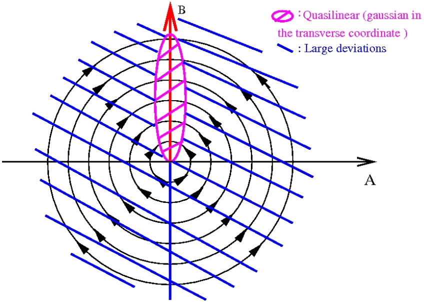
- Stationary measure in the limit $\nu \to \infty$



The Stochastic A-B Model
The limit of weak forces and dissipation

$$\begin{cases} dA = \left(-AB - \nu A\right) dt + \sqrt{\nu}\sigma_1 dW_1 \\ dB = \left(A^2 - \nu B\right) dt + \sqrt{\nu}\sigma_2 dW_2 \end{cases}$$

- Stationary measure in the limit $\nu \to \infty$



The Typical States of the A-B Model

Averaging in the limit of weak forces and dissipation

- First step of the adiabatic treatment : understand the evolution of the rapid variable A, for a fixed value of the slow variable B.
- At first order, for small ν, A is a Ornstein–Uhlenbeck process. $dA = (-AB - \nu A)\,dt + \sqrt{\nu}\sigma_1 dW_1$. Locally Gaussian :

$$P(A) = C(B)\exp\left(-\frac{BA^2}{\nu\sigma_1^2}\right)$$

$$P(A,B) = C_1 \exp\left(-\frac{BA^2}{\nu\sigma_1^2}\right) B^{\frac{\sigma_1^2}{\sigma_2^2}+\frac{1}{2}} \exp\left(-\frac{B^2}{\sigma_2^2}\right) \quad ; \quad P(E) = C_1 E^{\frac{\sigma_1^2}{\sigma_2^2}} \exp\left(-\frac{E^2}{\sigma_2^2}\right)$$

- A non trivial distribution
- The PDF is not concentrated. The weak forces and dissipation do not select a single equilibrium energy E.

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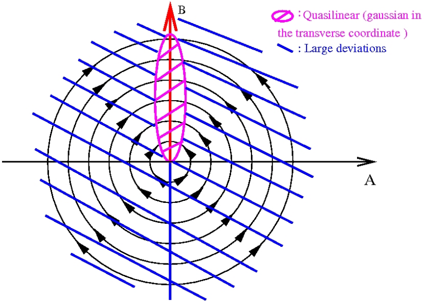


Outline

Classical Large Deviations

Freidlin–Wentzell theory or Onsager Machlup formalism

$$\mathrm{d}x = f(x)\mathrm{d}t + \sqrt{\nu}\mathrm{d}W$$

- Hypothesis: the deterministic dynamics has isolated attractors. Large deviation results:

$$P(X) \sim \exp\left(-\frac{V(X)}{\nu}\right) \text{ to mean } \lim_{\nu\to 0} \nu \log P = -V$$

$$\text{with } V(X) = \inf_{t>0} \inf_{\{x(t)|x(0)\in 0 \text{ and } x(t)=X\}} L[x]$$

$$\text{and } L[x] = \frac{1}{2}\int_0^t \mathrm{d}s\, (\dot{x} - f(x))^2$$

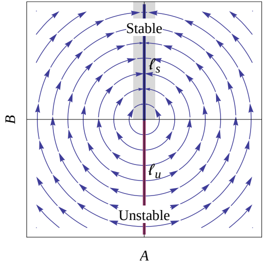
- Because of the connected attractors, the AB model does not fulfill the hypothesis of the Freidlin–Wentzell theorems

Non Classical Rate for the Large Deviations of the A-B Model

- Large deviation result:

$$P(A,B) \sim \exp\left(-\frac{V(A,B)}{\sqrt{\nu}}\right) \text{ to mean } \lim_{\nu\to 0}\sqrt{\nu}\log P = -V$$

with $V(A,B)=0$ if $A=0, B>0$ and $V(A,B)=\frac{2\sqrt{2}}{3}\left(A^2+B^2\right)^{3/4}$ otherwise

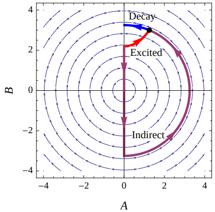


Non Classical Large Deviations of the A-B Model

Diffusion along the connected set of unstable steady states

$$L[x] = \frac{1}{2}\int_0^t \mathrm{d}s\,(\dot{x} - f(x))^2$$

- The action is zero for paths along the set of steady states and along a deterministic trajectory.



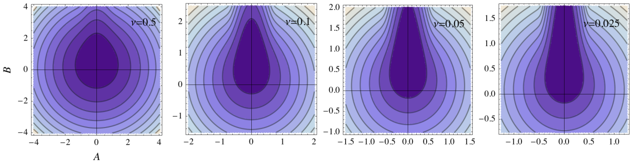
$$P(x=x_1, t=0; x=x_2, t=T) = \int_{x(0)=x_1}^{x(T)=x_2} \mathscr{D}[x]\exp\left(-\frac{1}{2\nu}L[x]\right)$$

Non Classical Large Deviations of the A-B Model

Diffusion along the connected set of unstable steady states

$$\lim_{\nu \to 0} \sqrt{\nu} \log P(A,B) = -V$$

with $V(A,B) = 0$ if $A = 0, B > 0$ and $V(A,B) = \frac{2\sqrt{2}}{3}\left(A^2 + B^2\right)^{3/4}$ otherwise



Summary

Messages :

- We predict and observe non-equilibrium phase transitions for the 2D-Stochastic Navier Stokes equations
- We can compute numerically instantons for turbulent flows
- In systems with connected attractors, large scale forces prevent bistability
- Non classical large deviation rate for dynamical systems with connected attractors

F. Bouchet, and A. Venaille, Physics Reports, 2011, Statistical mechanics of two-dimensional and geophysical flows

Publications

1. F. Bouchet, Physica D, 2008 Simplified variational problems for the statistical equilibria of 2D flows.
2. F. Bouchet and E. Simonnet, PRL (March 2009), Random changes of flow topology in 2D and geophysical turbulence.
3. A. Venaille and F. Bouchet, PRL (March 2009), Phase transitions, ensemble inequivalence and Fofonoff flows.
4. F. Bouchet and H. Morita, Physica D (April 2010), Asymptotic stability of the 2D Euler and of the 2D linearized Euler equations.
5. A. Venaille and F. Bouchet, to be submitted to J. Phys. Oceanography. Are strong mid-basin eastward jets (Gulf Stream, Kuroshio) statistical equilibria?
6. F. Bouchet and A. Venaille, to be submitted to Physics Reports: Statistical mechanics of two dimensional and geophysical flows.
7. F. Bouchet and M. Corvellec, submitted to J. Stat. Mech. Invariant measures of the 2D Euler and Vlasov equations.