JOVIAN JETS AND VORTICES AS STATISTICAL EQUILIBRIA

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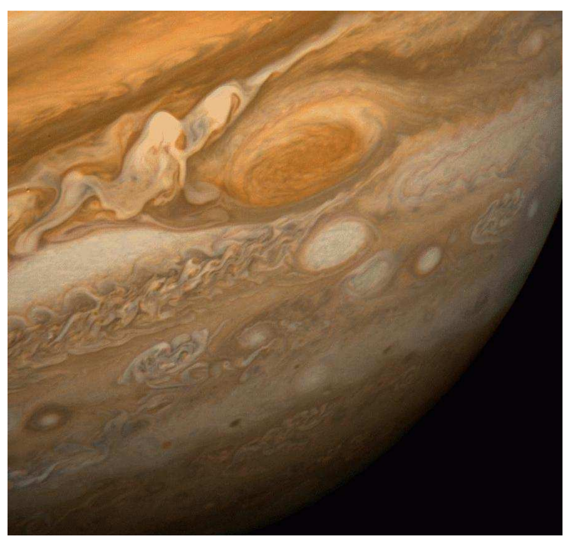
Self Organization of Large Scales of Geophysical Flows



Jupiter : The Great Red Spot

Jupiter : A Brown Barge

- Geophysical flows have the property to self-organize at large scale.
  - This is a general property (all planet atmospheres, oceans ...)
- These flows also have a turbulent nature. More quantitatively : the Reynolds' number or the number of exited degrees of freedom $R_e = \frac{UL}{\nu} \simeq 10^{12}$ and $N \simeq 10^{20}$
- Stability : the Jupiter's Great Red Spot exists from more than three century.
- Is this paradoxical ? An explanation ?

A Statistical Explanation

- Like for a usual gas example, the statistical effects leads to deterministic behaviors
- The statistical mechanics
  - Phase space : uniform density
  - A huge particle number : entropy
  - Dynamical mixing
- Statistical mechanics for geophysical flows ?

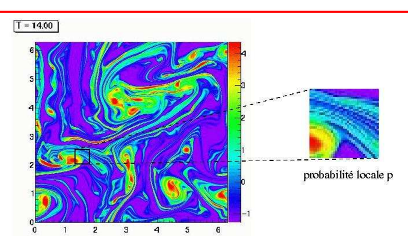
The Quasi-Geostrophic Model

$$\frac{\partial q}{\partial t} + \mathbf{u}.\nabla q = 0 \quad \text{with} \quad q = -\Delta\psi + \frac{\psi}{R^2} - h(y) \quad \text{and} \quad \mathbf{u} = -\mathbf{e}_z \wedge \nabla\psi$$

where q is the potential vorticity (PV), $\mathbf{u}$ the velocity field, ψ the stream function, and h the equivalent topography induced by the deep zonal flow.

Conservation laws : Energy : $E = \frac{1}{2}\int_D d\mathbf{r}\left(\mathbf{u}^2 + \frac{\psi^2}{R^2}\right)$ Casimirs : $C_f(q) = \int_D d\mathbf{r}\, f(q)$

Statistical Mechanics of the Quasi-Geostrophic Model



Left : Typical potential vorticity (PV) field, from a numerical simulations with initially only two PV values (red and blue) Let $p(\mathbf{r})$ be the local probability to have one of the two initial PV values (red or blue)

Entropy (proposed by Robert and Sommeria (1991), Miller (1991)) :

$$S = -\int_D [\, p(\mathbf{r})\ln p(\mathbf{r}) + (1-p(\mathbf{r}))\ln(1-p(\mathbf{r}))\,]d\mathbf{r}$$

The entropy counts the number of states corresponding to a given p: the entropy maximum is the most probable state after complete PV mixing

Statistical Equilibria : $\max\{S \mid \text{with } E = E_0 \text{ and } A = A_0\}$
where A is the area occupied by one of the potential vorticity levels, E is the energy and S the entropy (most probable state for a given energy and PV distribution)

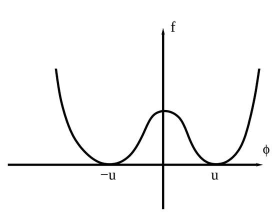
- Critical points: a stationnary state given by
$$q = -\Delta\psi + \frac{\psi}{R^2} - h(y) = f_{\alpha,\beta}(\psi)$$
- Analytical results in the limit of small Rossby deformation radius: $R \to 0$.

The Great Red Spot: Coexistence of 2 Thermodynamical Phases Separated by an Interface (Strong Jet)

Entropy maximization is equivalent to the variational problem:

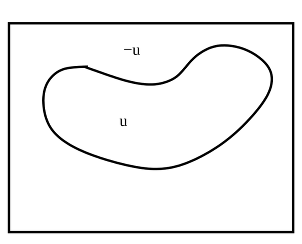
$$\begin{cases} \min\{F_R[\phi] \mid \text{with } A[\phi] \text{ given}\} \\ \text{with } F_R[\phi] = \int_D d\mathbf{r}\left[\frac{R^2(\nabla\phi)^2}{2} + f(\phi) - R\phi h_0(y)\right] \text{ and } A[\phi] = \int_D d\mathbf{r}\,\phi \end{cases}$$

This describes a first order phase transition (analogous to a gaz buble in a liquid)



f has two minima

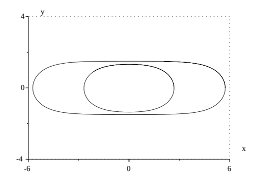
ϕ thus takes the two values where f reaches its minima, in two subdomains (phase separaration) separated by an interface (see right figure)



- With an asymptotic expansion ($R \to 0$), we describe the jet
- By analogy with usual thermodynamics, this interface should minimize its length, for a fixed area. The topography will slightly change this picture.

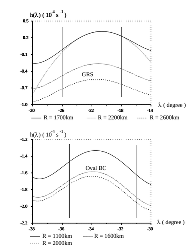
Thermodynamical Analogy

- A variational problem for the curve formed by the jet
$$\min\left\{F_R[\phi_R] = 2Re_c L - 2Ru\int_{A_1} d\mathbf{r}\, h_0(y) + o(R)\right\}$$
- Laplace equation: link between the curvature radius r and the free energy difference
$$\frac{e_c}{r} = -u(\alpha_1 - h_0(y))$$
- Prediction: the structure is located on extrema of the equivalent topography.

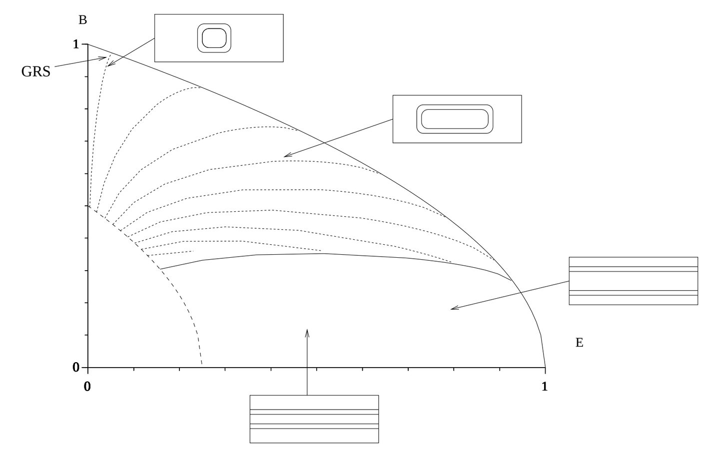


Typical vortex shape

Right : The actual equivalent topography for the GRS and White Ovals (computed using Dowling and Ingersoll (1989) analysis of the observed velocity fields)

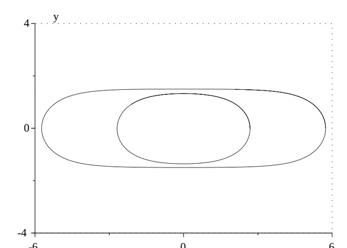


Phase Diagram : Jets and Elongated Vortices

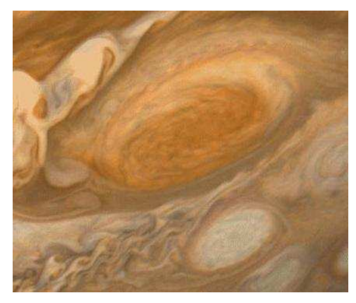


E is the energy and B measures the asymmetry of the initial PV distribution

The Shape of QG Equilibria and of Jovian Vortices



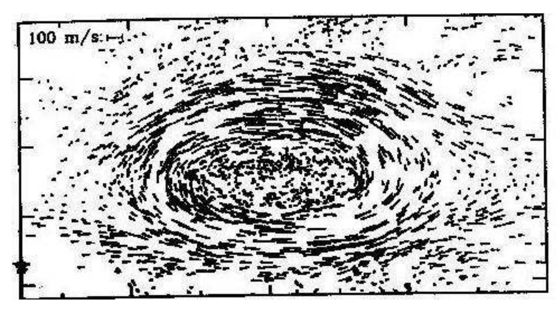
Statistical Equilibria



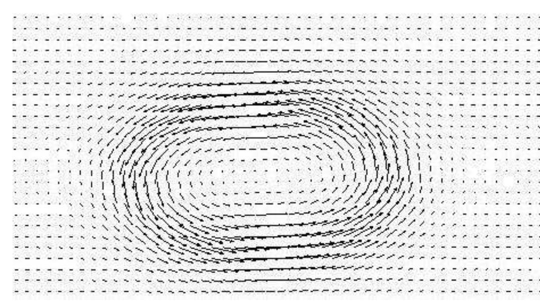
Great Red Spot and White Oval BC

A Brown Barge

Statistical Model of the Great Red Spot's Velocity Field



Observation data

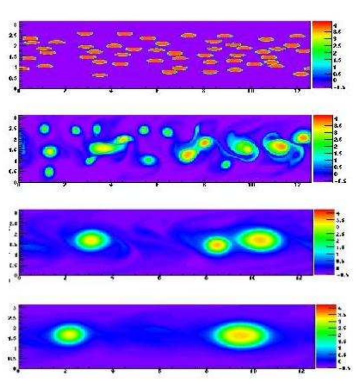


Statistical equilibrium

Observations : from Voyager data analyses (Dowling and Ingersoll 1994).

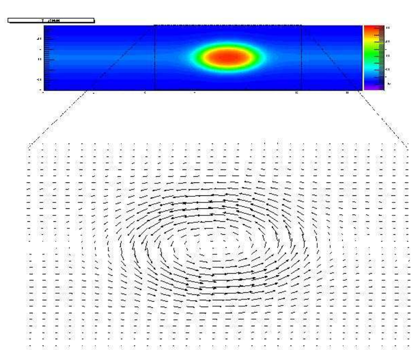
- A good quantitative agreement + Rossby deformation radius determination.

White Ovals from Random PV Distributions



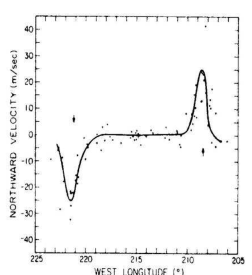
Left : Dynamical evolution from a random initial distribution

Right : This evolution converges to a statistical equilibrium, similar to a White Oval

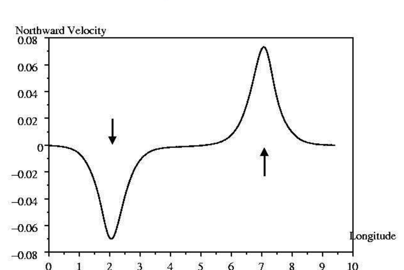


The Brown Barges' Velocity Field

Northward velocity

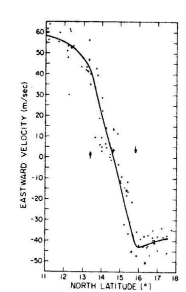


Observations

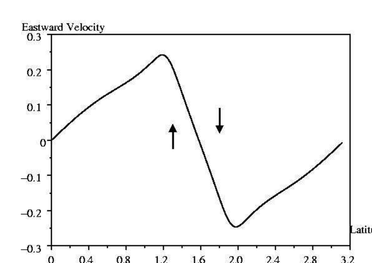


Stat. Equilibrium

Eastward velocity



Observations



Stat. Equilibrium

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- F. BOUCHET and T. DUMONT *Sub. to Journal of Atmospherical Sciences*
- F. BOUCHET, P.H. CHAVANIS and J. SOMMERIA (Shallow Water model)